Application of Coordination Policies for Fuzzy Newsvendor Model

Kwangyeol Ryu*, Youngpil Cha**, Seokwoo Lee*, Honzong Choi*, Mooyoung Jung**

* Manufacturing Process Research Division, KITECH, {ducksal; swlee; choihz}@kitech.re.kr

** Intelligent Systems and Management Lab., POSTECH, {ophil; myjung}@postech.ac.kr

Abstract

In the absence of a clear command and control structure, a key challenge in supply chain management is the coordination and alignment of the supply chain members who pursue divergent and often conflicting goals. The newsvendor model is typically used as a framework to quantify the cost of misalignment and to assess the impact of coordination initiatives. This paper considers a fuzzy approach for the newsvendor problem which includes a single manufacturer and a single retailer. We use several fuzzy parameters in the model such as the demand, the wholesale price, and the market sales price. We apply a coordination policy, referred to as buyback, to solve the fuzzy newsvendor problem. Based on the buyback policy, the optimal order quantity of the retailer can be computed, and the possible profits of the members in the supply chain can be calculated with minimum sharing of private information. Focusing on the fuzzy model with buyback policy for the newsvendor problem, we illustrate exemplary fuzzy models. We also illustrate an integration model, which extends a single-manufacturer-single-retailer model to the single-manufacturer-multiple-retailer setting. In the extended model, we consider three coordination policies including quantity discount, profit sharing, and buyback, as well as non-coordination case.

Keywords: Fuzzy Newsvendor Problem, Supply Chain Management(SCM), Supply Chain Coordination

1. Introduction

In a complex and decentralized environmental setting with no clear command and control structure, the key challenge is the coordination and alignment of the supply chain members who typically pursue divergent and often conflicting goals. When we consider the interaction between a manufacturer and a retailer, the manufacturer tries to maximize his own profit by increasing retailers' order quantity while minimizing costs to setup and produce goods. On the other hand, each retailer is primarily concerned with getting a better offer from the manufacturer in order to maximize her own profit. This self-serving focus often results in poor performance from a global supply chain point of view, ultimately deteriorating individual performance as well. Appropriate incentives must therefore be offered to both parties to ensure that individual goals are aligned with supply chain goals (Cachon 2003).

In order to quantify the cost of misalignment and to assess the impact of various coordination initiatives, the newsvendor model is typically used as a framework supporting simple, yet rich, model which captures the risk of demand-supply mismatch before the market uncertainty is resolved. Given a demand distribution along with the costs of overshooting or undershooting the demand, the newsvendor model determines the quantity that maximizes the expected profit.

The application of the newsvendor framework normally requires the specification of some probability distribution for the sources of uncertainty, and in particular, for the market demand. An alternative approach to represent and deal with all sources of uncertainty is fuzzy set theory, proposed by Zadeh (1978). This paper adopts the fuzzy model for the newsvendor problem. The classical newsvendor problem (Silver et al. 1998, Nahmias 1993) aims to find the retailer's order quantity that maximizes her expected profit. This self-serving focus, however, leads to misalignment resulting in poor supply chain performance. To align the manufacturer and the retailer, we consider several coordination policies including quantity discount, profit sharing, and buyback. The objective of this paper is thus to solve the fuzzy newsvendor problem by applying the buyback policy, and to investigate aforementioned three coordination policies as well as non-coordination case. Furthermore, we also propose integrative coordination model by extending a single-manufacturer-single retailer model to the single-manufacturer-multiple-retailer setting.

Regarding initial settings for fuzzy elements and basic assumptions as well as background information including the framework used in this paper, refer to (Ryu et al. 2005).

2. Newsvendor Problem with Uncertainties

Consider the supply chain with a manufacturer and a retailer. As depicted in Fig. 1, a single product is manufactured at the retailer and sold in the market through the retailer. As a base case, let us assume that this is a vertically integrated setting with both the manufacturer and the retailer belonging to the same firm.

Product demand in the market is uncertain. We typically reflect this uncertainty in our forecast by specifying an average demand level (μ) as well as the volatility of demand expressed by its standard deviation (σ). The product, which incurs a variable manufacturing cost of ₩ *c* can be sold in the market for a price of ₩ *p*. For now, let us ignore the fixed costs. Given the lead times in the procurement of raw materials and in the production process, the firm has to commit to a certain production level, *Q*, before observing the actual demand. In other words, the firm has to decide the optimal batch size, *Q**, to produce before the market uncertainty is resolved.

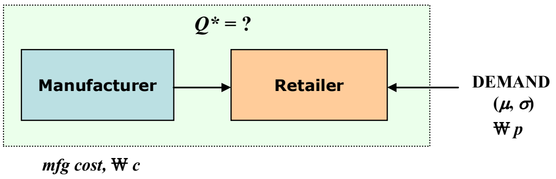

Fig. 1. An Integrated Production-Distribution System

The overall supply chain is running two types of risk: risk of underage and risk of overage. The underage risk is the risk of not producing sufficient units, leading to lost sales, while the overage risk is the risk of producing too many units, ending with extra stock at the end of the selling season. The challenge is to produce the lot size that maximizes the expected profit before observing the actual demand. This is the newsvendor problem.

Returning to our problem, we need to determine the lot optimal lot size, *Q**, that maximizes the expected profit. We follow a marginal analysis approach to solve the problem where the rule of thumb is to continue producing more units as long as the expected payoff from that extra unit exceeds the expected loss. In other words, continue producing as long as

$$[(p\text{-}c) \times Prob\{D>Q\}] - [c \times Prob\{D\leq Q\}] > 0.$$

To obtain the optimal lot size, we note that the above equation reduces to

$$Prob\{D\leq Q\}] = (p\text{-}c)/p.$$

In other words, the quantity $(p\text{-}c)/p$ represents the critical fractile of the demand distribution, which allows us to compute the optimal lot size, *Q**. Note that the optimal quantity that maximizes the expected profit depends on the average demand, the volatility of the demand, and the potential financial demand-supply mismatch risk.

This is the optimal quantity for the entire production-distribution system. Let us now embellish our base model. Suppose that we no longer have a vertically integrated setting, but an independent manufacturer and an independent retailer. The manufacturer places orders at the manufacturer. The manufacturer produces exactly the quantity ordered and delivers it to the retailer, who will later sell them in the market. In other words, the newsvendor problem is solved exclusively by the retailer who is fully bearing the demand-supply mismatch risk. Further note that the retailer does not necessarily know the manufacturer's variable production cost (₩ *c*); instead, the retailer knows the wholesale price, ₩ *w*, published by the manufacturer. The scenario is summarized in Fig. 2. Now, the problem of the retailer is to find the optimal order quantity.

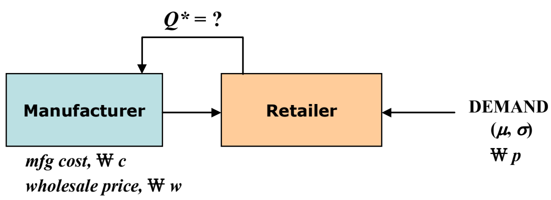

Fig. 2. A Decentralized Production-Distribution System

The retailer applies the newsvendor logic to solve the problem. Note that the only difference between the two scenarios is the replacement of the variable production cost by the wholesale price in the critical fractile. As long as $w \neq c$, the quantity ordered by the retailer will be different from the supply chain optimal quantity. In fact, if the manufacturer sets $w >> c$, then the retailer ends up ordering a significantly smaller quantity. This is the consequence of the lack of collaboration in the supply chain: double marginalization, which implies that profits shrink for all involved parties. At this point, let us consider how we can restore the profits to the level of a centralized system without having to actually vertically integrate.

Coordination is possible only by allowing returns to supply chain partners that are commensurate with the risk they carry. One should therefore design and implement *contractable* incentives. These are incentives that can be observed, verified, and enforced. They include:

- Quantity discounts
- Buyback schemes
- Profit sharing
- Two-part tariffs (selling at cost plus a flat fee)
- Profit sharing
- Cost sharing
- Volume guarantees

- Multi-year business guarantees
- Options contracts

These schemes and their variants are easy-to-implement mechanisms to align the divergent priorities of the supply chain partners. For example, offering quantity discounts is equivalent, for a manufacturer, to publishing a matrix of wholesale prices. A buyback scheme is a risk-sharing initiative that enables the retailer to return to the manufacturer unsold units at the end of the selling season. Consider the above example where the manufacturer pledges to accept returns at the end of the season at a buyback price of ₩ *b*. The resulting scenario is summarized in Fig. 3.

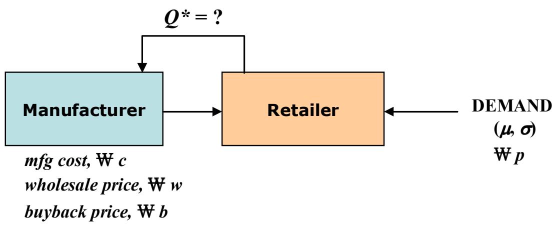

Fig. 3. A Decentralized Production-Distributed System with Buyback

As an example of profit sharing, consider movie rentals in the United States. Once a film completes its primary run in the movie theaters, the studio produces DVDs and sells them to rental shops. Until a few years ago, such DVDs were priced quite high making it virtually impossible for the rental shops to make any profit unless they rented the DVD out numerous times. This practice ultimately led to double marginalization. Under the current scheme, the movie studios have drastically cut their sales prices, encouraging the rental shops to order a large number of DVDs to rent out. At the end of the year, however, the studios claim 45% of the rental profits. These initiatives aim at distributing risks and rewards among the supply chain partners in an equitable fashion.

3. Fuzzy Newsvendor Model with Buyback

The fuzzy newsvendor model considered in this paper consists of a single manufacturer selling to a single retailer, referred to as the simple model. The extension of the simple model is also considered which deals with a single manufacturer and multiple retailers. Regardless of the model set-up, the following sequence of events occurs: the manufacturer initially proposes a fuzzy wholesale price; the retailer reports her intended order quantity; the manufacturer offers the retailer a contract; the retailer calculates her own possible profit and places an order to the manufacturer; the manufacturer makes the products and delivers them to the retailer; market demand is observed; and, finally, remaining transactions are performed corresponding to the adopted contract.

The fuzzy newsvendor problem is solved through three main steps: pre-process, pricing and ordering, and post-process (refer to Ryu *et al*. 2005). In *pre-process*, the manufacturer proposes a fuzzy wholesale price to the retailer so that the retailer can calculate her intended order quantity. To estimate the order quantity, the retailer calculates her possible profit by using fuzzy factors including overage, underage and purchasing costs. The retailer then reports her intended order quantity to the manufacturer in the form of a fuzzy number.

In *pricing and ordering*, the manufacturer receives the retailer's intended order quantity. Based on this information, the manufacturer calculates his possible profit and makes a contract offer to the retailer by using coordination policies (e.g., buyback). Once the retailer receives a contract proposal from the manufacturer, the retailer finds the optimal order quantity, which maximizes her own possible profit, and places a firm order. Because of the adopted contract is the buyback, further transactions are conducted between the manufacturer and the retailer at the end of the selling season; these are carried out in *post-process*. During this process, the manufacturer re-purchases the retailer's entire unsold inventory for a buyback price. An important implicit assumption here is that the manufacturer is able to verify the number of sold and unsold units of the retailer, and the cost of such monitoring does not negate the benefits created by the contract. Even if the manufacturer does not monitor such information, it is assumed that the retailer truthfully reports the results of her sales to the manufacturer, not pursuing self-interest. At the end of the post-process, the manufacturer and the retailer(s) calculate their own profits and, finally, compute the total profit of the supply chain.

Recall that the following are the main assumptions of our model: 1) delivery costs are not considered, 2) no loss or damage occur during delivery, 3) unit production cost and ordering cost will not change with the quantity to produce, and 4) an incremental discount policy is applied for quantity discount (this is used for the extended model).

The fuzzy newsvendor model with buyback includes 1)retailer's fuzzy model for *pre-process*, 2)manufacturer's fuzzy model for *pricing*, and 3)description of *post-process*. Note that we omit the illustration of retailer's fuzzy models for *ordering* step because it is similar to the model used in *pre-process*.

3.1 Retailer's Fuzzy Model for Pre-Process

Note that even though we have illustrated the pre-process of the retailer in Ryu *et al*. 2005, we summarize them again in this paper in order to facilitate understanding of the paper. The manufacturer selects the initial wholesale price ($\tilde{w}^{(\alpha)}$) and announces it to the retailer. Then the retailer calculates her intended order quantity ($\grave{Q}$) and her possible profit ($\pi_R^{(\alpha)}$) as follows.

Let the fuzzy demand ($\widetilde{d}^{(\alpha)}$) be given by a domain $\widetilde{d}^{(\alpha)} = \left(\underline{d}^{(\varepsilon)}, \underline{d}^{(\gamma)}, \underline{d}^{(1)}, \overline{d}^{(1)}, \overline{d}^{(\gamma)}, \overline{d}^{(\varepsilon)}\right)$ and the membership function $\mu_{\widetilde{d}^{(\alpha)}}$, where α indicates the membership level ($\alpha \in \{\varepsilon, \gamma, 1\}$). The fuzzy unit market sales price ($\widetilde{p}^{(\alpha)}$) has a domain $\widetilde{p}^{(\alpha)} = \left(\underline{p}^{(\varepsilon)}, \underline{p}^{(\gamma)}, \underline{p}^{(1)}, \overline{p}^{(1)}, \overline{p}^{(\gamma)}, \overline{p}^{(\varepsilon)}\right)$. Constant unit ordering costs is c_o. Also, let the fuzzy wholesale price announced by the manufacturer be given by a domain $\widetilde{w}^{(\alpha)} = \left(\underline{w}^{(\varepsilon)}, \underline{w}^{(\gamma)}, \underline{w}^{(1)}, \overline{w}^{(1)}, \overline{w}^{(\gamma)}, \overline{w}^{(\varepsilon)}\right)$ and the membership function $\mu_{\widetilde{w}^{(\alpha)}}$.

For each membership level of $\alpha \in \{\varepsilon, \gamma, 1\}$, and for each order quantity Q and demand $D \in \widetilde{d}^{(\alpha)}$, the uncertain purchasing cost is $\widetilde{F}_p^{(\alpha)} = \widetilde{w}^{(\alpha)} \cdot Q$ with a membership function, $\mu_{\widetilde{F}_p^{(\alpha)}}(\widetilde{w}^{(\alpha)})$. Uncertain demand causes uncertain overage cost ($\widetilde{F}_h^{(\alpha)}$), uncertain underage cost ($\widetilde{F}_s^{(\alpha)}$), and uncertain revenue ($\widetilde{R}_R^{(\alpha)}$). $\widetilde{F}_h^{(\alpha)}$, $\widetilde{F}_s^{(\alpha)}$, $\widetilde{R}_R^{(\alpha)}$ are given by:

$$\widetilde{F}_h^{(\alpha)} = \widetilde{w}^{(\alpha)}\, max(Q - D, 0), \quad D \in \widetilde{d}^{(\alpha)}, \tag{1}$$

$$\widetilde{F}_s^{(\alpha)} = (\widetilde{p}^{(\alpha)} - \widetilde{w}^{(\alpha)})\, max(D - Q, 0), \quad D \in \widetilde{d}^{(\alpha)}, \tag{2}$$

$$\widetilde{R}_R^{(\alpha)} = \widetilde{p}^{(\alpha)} \cdot min(D, Q), \quad D \in \widetilde{d}^{(\alpha)}, \tag{3}$$

with membership functions

$$\mu_{\widetilde{F}_h^{(\alpha)}}(\widetilde{w}^{(\alpha)}\, max(Q - D, 0)), \quad \mu_{\widetilde{d}^{(\alpha)}}(D), \quad D \in \widetilde{d}^{(\alpha)}, \tag{4}$$

$$\mu_{\widetilde{F}_s^{(\alpha)}}((\widetilde{p}^{(\alpha)} - \widetilde{w}^{(\alpha)})\, max(D - Q, 0)), \quad \mu_{\widetilde{d}^{(\alpha)}}(D), \quad D \in \widetilde{d}^{(\alpha)}, \tag{5}$$

$$\begin{cases} \mu_{\widetilde{R}_R^{(\alpha)}}(\widetilde{p}^{(\alpha)} \cdot Q) = \mu_{\widetilde{R}_R^{(\alpha)}}(\widetilde{p}^{(\alpha)}), & \text{if } D > Q,\ D \in \widetilde{d}^{(\alpha)} \\ \mu_{\widetilde{R}_R^{(\alpha)}}(\widetilde{p}^{(\alpha)} \cdot D),\ \mu_{\widetilde{d}^{(\alpha)}}(D),\ D \in \widetilde{d}^{(\alpha)}, & \text{if } D \le Q,\ D \in \widetilde{d}^{(\alpha)}. \end{cases} \tag{6}$$

The fuzzy overage and underage costs (i.e., $\widetilde{F}_h^{(\alpha)}$ and $\widetilde{F}_s^{(\alpha)}$) are level 2 fuzzy sets, and the fuzzy revenue ($\widetilde{R}_R^{(\alpha)}$) is also a level 2 fuzzy set only when $D \le Q$, $D \in \widetilde{d}^{(\alpha)}$. When the retailer's intended order quantity is $\grave{Q}$, she can compute her possible profit by using Eqs.(7)-(8).

i) $\widetilde{d}^{(\alpha)} > \grave{Q}$

$$\pi_R^{(\alpha)}(\grave{Q})\Big|_{\widetilde{d}^{(\alpha)} > \grave{Q}} = \text{defuzz}(\widetilde{R}_R^{(\alpha)} - \widetilde{F}_p^{(\alpha)}) - \text{defuzz}(s\text{-}fuzzif(\widetilde{F}_s^{(\alpha)})) - c_o, \tag{7}$$

ii) $\widetilde{d}^{(\alpha)} \le \grave{Q}$

$$\pi_R^{(\alpha)}(\grave{Q})\Big|_{\widetilde{d}^{(\alpha)} \le \grave{Q}} = \text{defuzz}(s\text{-}fuzzif(\widetilde{R}_R^{(\alpha)} - \widetilde{F}_h^{(\alpha)})) - \text{defuzz}(\widetilde{F}_p^{(\alpha)}) - c_o, \tag{8}$$

The retailer can use two methods to compute her possible profit according to her expectation for demand. She can either use the possibility measure for the case of $D > Q$ and the necessity measure for $D \le Q$ (Eq.(9)), or she can use the necessity measure when $D > Q$ and the possibility measure when $D \le Q$ (Eq.(10)). As a consequence, the retailer uses the proper method to determine the possible profit based on the future expectation of demand.

$$\pi_R^{(\alpha)^*}(\grave{Q}^*) = max_{\grave{Q}}\Big[M(\widetilde{d}^{(\alpha)} > \grave{Q}) \cdot \pi_R^{(\alpha)}(\grave{Q})\Big|_{\widetilde{d}^{(\alpha)} > \grave{Q}}, + N(\widetilde{d}^{(\alpha)} \le \grave{Q}) \cdot \pi_R^{(\alpha)}(\grave{Q})\Big|_{\widetilde{d}^{(\alpha)} \le \grave{Q}}\Big] \tag{9}$$

$$\pi_R^{(\alpha)^*}(\grave{Q}^*) = max_{\grave{Q}}\Big[N(\widetilde{d}^{(\alpha)} > \grave{Q}) \cdot \pi_R^{(\alpha)}(\grave{Q})\Big|_{\widetilde{d}^{(\alpha)} > \grave{Q}}, + M(\widetilde{d}^{(\alpha)} \le \grave{Q}) \cdot \pi_R^{(\alpha)}(\grave{Q})\Big|_{\widetilde{d}^{(\alpha)} \le \grave{Q}}\Big] \tag{10}$$

where

$M(\widetilde{d}^{(\alpha)} > \grave{Q}) + N(\widetilde{d}^{(\alpha)} \le \grave{Q}) = 1,\ N(\widetilde{d}^{(\alpha)} > \grave{Q}) + M(\widetilde{d}^{(\alpha)} \le \grave{Q}) = 1$.

The retailer then fuzzifies $\grave{Q}^*$ and reports the fuzzy order quantity to the manufacturer. The retailer also reports $\widetilde{d}^{(\alpha)}$ so that the manufacturer can calculate his possible profits while making the pricing offer. The fuzzification of $\grave{Q}^*$ is plausible because the retailer might change the final order quantity (for instance, due to internal budget constraints). The retailer may also change the order quantity after considering the pricing offer from the manufacturer. The value of $\grave{Q}^*$ is fuzzified into a six-point fuzzy number by picking values that result in a positive profit. As a result, $\grave{Q}^*$ is fuzzified to the fuzzy number $\widetilde{Q}^{(\alpha)} = \left(\underline{Q}^{(\varepsilon)}, \underline{Q}^{(\gamma)}, \underline{Q}^{(1)}, \overline{Q}^{(1)}, \overline{Q}^{(\gamma)}, \overline{Q}^{(\varepsilon)}\right)$.

3.2 Manufacturer's Fuzzy Model for Pricing with Buyback

Let us define net manufacturing cost (c_{net}) as following.

$$c_{net} = c_p + k / \text{defuzz}(\widetilde{Q}^{(\alpha)}) \tag{11}$$

where c_p and k are the unit production cost and the setup cost, respectively.

While the manufacturer may incur an additional cost for re-purchasing the retailer's unsold units at the end of the selling period, this strategy is aimed at reducing the retailer's overage risk. The manufacturer proposes to the retailer a wholesale price of w_b. As long as the manufacturer has an additional cost factor (i.e., the buyback cost) under this strategy, it is reasonable for the manufacturer to try to compensate for his possible loss by increasing his minimum margin, thereby offering a higher wholesale price. Similar to the case of profit sharing, the manufacturer should determine jointly both the wholesale price and the buyback price in order to encourage the retailer to purchase additional units. For simplicity, we assume that the wholesale price is set at $w_b = (1 + \beta_2)c_{net}$, which satisfies the condition in Eq.(12).

$$\underline{w}^{(\varepsilon)} \le (1 + \beta_2)c_{net} \le w_b \,. \tag{12}$$

where $\beta_2 \cdot c_{net}$ is the minimum margin.

When the retailer's order quantity is $Q' \in \widetilde{Q}^{(\alpha)}$, the revenue ($R_M^{(\alpha)}$) and production costs ($F_m^{(\alpha)}$) are $R_M^{(\alpha)} = w_b \cdot Q'$, and $F_m^{(\alpha)} = c_p \cdot Q'$, respectively. If the manufacturer re-purchases unsold units from the retailer at the unit price of $y(Q')\cdot w_b$, $0 \le y(Q') \le 1$, then the possible buyback cost ($\tilde{F}_b^{(\alpha)}$) becomes :

$$\tilde{F}_b^{(\alpha)} = y(Q')\cdot w_b \cdot max(Q' - D, 0),\ \ D \in \tilde{d}^{(\alpha)}, \tag{13}$$

with membership function

$$\mu_{\tilde{F}_b^{(\alpha)}}(y(Q')\cdot w_b \cdot max(Q' - D, 0)) = \mu_{\tilde{d}^{(\alpha)}}(D),\ D \in \tilde{d}^{(\alpha)}. \tag{14}$$

The buyback rate ($y(Q')$) and the order quantity are assumed to have a piecewise linear relationship, as illustrated in Fig. 4. This indicates that the manufacturer is willing to compensate for the retailer's unsold units with higher unit buyback price at the end of the selling period as the retailer purchases more units at the beginning of the period. For example, assume that the fuzzy order quantity $\tilde{Q}^{(\alpha)} = (10, 15, 25, 25, 32, 40)$ and the retailer purchased 36 units at the wholesale price of ₩20 at the beginning of the selling period. When $y(32)$ and $y(40)$ are 0.7 and 0.9, respectively, then the manufacturer should repurchase the retailer's unsold units, if any, at the buyback price of ₩16 per unit, which is 80% of the initial wholesale price, at the end of the selling period.

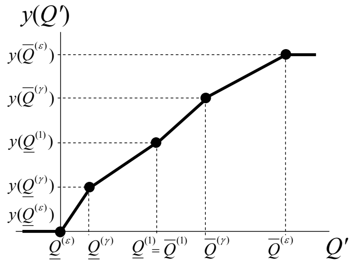

Fig. 4. Piecewise Linear Relationship between $y(\cdot)$ and Q'

When the order quantity is Q', the manufacturer's possible profit ($\phi_b^{(\alpha)}(Q')$) becomes:

$$\begin{aligned}\phi_b^{(\alpha)}(Q') &= R_M^{(\alpha)} - F_m^{(\alpha)} - \text{defuzz}(\tilde{F}_b^{(\alpha)}) - k \\ &= (w_b - c_p)\cdot Q' - \text{defuzz}(\tilde{F}_b^{(\alpha)}) - k.\end{aligned} \tag{15}$$

Note that $\tilde{\phi}_b^{(\alpha)}$ has the membership function of $\mu_{\tilde{\phi}_b^{(\alpha)}}(Q') = \mu_{\tilde{Q}^{(\alpha)}}(Q')$. When the buyback rate is y and the wholesale price is w_b, the possible profit of the manufacturer is:

$$\pi_M^{(\alpha)}(y, w_b) = \text{defuzz}(\tilde{\phi}_b^{(\alpha)}). \tag{16}$$

The manufacturer can then compute his possible profit using Eq.(17).

$$\pi_M^{(\alpha)*}(y^*, w_b^*) = \max_{y, w_b}\left[\text{defuzz}(\tilde{\phi}_b^{(\alpha)})\right]. \tag{17}$$

3.3 Post-Process

When the manufacturer and the retailer have a buyback contract, the interaction between them is not over upon the delivery of the ordered units. For the buyback case, the manufacturer will repurchase the retailer's unsold units at the end of the selling period. Consider the situation where the actual demand was 25 during the selling period. The retailer, however, had purchased 36 units at the wholesale price of ₩300 per unit and had sold them to the customers at the price of ₩400. When the buyback rate y(36) is 0.8, then the manufacturer should repurchase 11 units from the retailer and pay for 11× ₩300×0.8 = ₩2,640 at the end of the selling period to compensate for the overage cost of the retailer.

4. Extension to the multiple-retailer setting

The simple model can be readily extended to the single-manufacturer-multiple-retailer setting, referred to as the 1/K model. The pre-process of 1/K model remains the same as described in this paper since the pre-process is conducted by each retailer. Let the outcome of the pre-process (i.e., the intended order quantity of the i^{th} retailer) be $\tilde{Q}_i^{(\alpha)}$. Here we assume that each retailer uses the same membership function level to simplify the calculation. For fuzzy numbers $\tilde{x}_i$, the fuzzy summation operation is defined as follows:

$$\tilde{\bigoplus}_{i=1}^{n} \tilde{x}_i = \tilde{x}_1 \oplus \tilde{x}_2 \oplus \cdots \oplus \tilde{x}_n. \tag{18}$$

In the 1/K model, there are four sets of retailers according to the coordination policy. Let us define by R_q, R_p, R_b, and R_n the mutually exclusive and collectively exhaustive sets of retailers using quantity discount, profit sharing, buyback, and non-coordination, respectively. The retailer set, R, is then the union of three sets, i.e., $R = R_q \cup R_p \cup R_b \cup R_n$, with $n(R_q) + n(R_p) + n(R_b) + n(R_n) = K$.

The manufacturer uses the value of $\tilde{Q}_{sum}^{(\alpha)} = \bigoplus_{i \in R} \tilde{Q}_i^{(\alpha)}$ for calculating the net manufacturing cost, c_{net} as well as w_b. However, the manufacturer may use different pricing policies to satisfactorily deal with each retailer. For example, the manufacturer may assign a wholesale price with the different range to retailers under quantity discount, a different sharing rate under profit sharing, and a different buyback rate under the buyback policy.

Under the quantity discount policy, the manufacturer sets the wholesale price for an individual retailer based on the retailer's intended order quantity determined in pre-process. In this case, the purchasing cost of two retailers may be different even though they order the same number of units, since the relationship between the wholesale price and the order quantity might differ between the two retailers. Since the net manufacturing cost in the 1/K model, $c'_{net} = c_p + k / defuzz(\tilde{Q}_{sum}^{(\alpha)})$, considers the order quantities of all retailers, the threshold value of the wholesale price for each retailer (i.e., $(1+\beta_1)c'_{net}$) is smaller than that of the simple model. Let the wholesale price offered to the i^{th} retailer be $w_{q,i}(Q'_i)$ when the order quantity is Q'_i. Then the manufacturer's profit excluding the setup cost becomes $\phi'^{(\alpha)}_{q,i}(Q'_i) = (w_{q,i}(Q'_i) - c_p) \cdot Q'_i$. Therefore, the possible profit of the manufacturer from the transactions with those retailers having a quantity discount contract can be calculated using Eq.(19). Note that the setup cost is incorporated in Eq.(23).

$$\pi^{(\alpha)}_{M,q}(w_q) = \text{defuzz}(\widetilde{\Upsilon}_{i \in R_q} \widetilde{\phi}'^{(\alpha)}_{q,i}) \,. \qquad (19)$$

Similar to the quantity discount contract, each retailer included in R_p has her own sharing rate under the profit sharing policy. Let the profit share of the manufacturer from the i^{th} retailer be $\pi^{(\alpha)}_{shared,i}(Q'_i)$ when the order quantity is Q'_i. Then the manufacturer's profit $\phi'^{(\alpha)}_{p,i}(Q'_i)$ excluding the setup cost becomes $\phi'^{(\alpha)}_{p,i}(Q'_i) = (c'_{net} - c_p) \cdot Q'_i + \pi^{(\alpha)}_{shared,i}(Q'_i)$. Even though we have simply used c'_{net} as a wholesale price under the profit sharing strategy, the optimal wholesale price should be determined so as to maximize the manufacturer's possible profit. Let x and w_p be the sharing rate and the wholesale price under the profit sharing strategy. Then, the possible profit of the manufacturer from the transactions with retailers having a profit sharing contract becomes:

$$\pi^{(\alpha)}_{M,p}(x, w_p) = \text{defuzz}(\widetilde{\Upsilon}_{i \in R_p} \widetilde{\phi}'^{(\alpha)}_{p,i}) \,. \qquad (20)$$

Under the buyback policy, the wholesale price, w_b, is set to satisfy the condition, $\underline{w}^{(\varepsilon)} \le (1+\beta_2)c'_{net} \le w_b$, which is similar to Eq.(12). Let the buyback cost to the manufacturer induced by the unsold units of the i^{th} retailer be $F^{(\alpha)}_{b,i}$. Then the manufacturer's profit $\phi'^{(\alpha)}_{b,i}(Q'_i)$, excluding the setup cost, becomes $\phi'^{(\alpha)}_{b,i}(Q'_i) = (w_b - c_p) \cdot Q'_i - \text{defuzz}(F^{(\alpha)}_{b,i})$. Note that y is the buyback rate. Then the possible profit of the manufacturer from the transactions with retailers having a buyback contract becomes:

$$\pi^{(\alpha)}_{M,b}(y, w_b) = \text{defuzz}(\widetilde{\Upsilon}_{i \in R_b} \widetilde{\phi}'^{(\alpha)}_{b,i}) \,. \qquad (21)$$

In the absence of any coordinating policy, an appropriate wholesale price, w_n, is set by the manufacturer and communicated to the retailers in R_n. Note that we assume the wholesale price with no coordinating policy to be $\underline{w}^{(1)}$ in our examples because it has the highest possibility of the membership function. The manufacturer's profit $\phi'^{(\alpha)}_{n,i}(Q'_i)$, excluding the setup cost, becomes $\phi'^{(\alpha)}_{n,i}(Q'_i) = (w_n - c_p) \cdot Q'_i$. Then, the possible profit of the manufacturer without any coordinating policy becomes:

$$\pi^{(\alpha)}_{M,n}(w_n) = \text{defuzz}(\widetilde{\Upsilon}_{i \in R_n} \widetilde{\phi}'^{(\alpha)}_{n,i}) \,. \qquad (22)$$

From Eqs.(19)-(22), the total possible profit of the manufacturer becomes:

$$\begin{aligned} &\pi^{(\alpha)}_{M}(x, y, w_q, w_p, w_b, w_n) \\ &= \pi^{(\alpha)}_{M,q}(w_q) + \pi^{(\alpha)}_{M,p}(x, w_p) + \pi^{(\alpha)}_{M,b}(y, w_b) + \pi^{(\alpha)}_{M,n}(w_n) - k \qquad (23) \\ &= \text{defuzz}(\widetilde{\Upsilon}_{i \in R_q} \widetilde{\phi}'^{(\alpha)}_{q,i}) + \text{defuzz}(\widetilde{\Upsilon}_{j \in R_p} \widetilde{\phi}'^{(\alpha)}_{p,j}) \\ &\quad + \text{defuzz}(\widetilde{\Upsilon}_{k \in R_b} \widetilde{\phi}'^{(\alpha)}_{b,k}) + \text{defuzz}(\widetilde{\Upsilon}_{l \in R_n} \widetilde{\phi}'^{(\alpha)}_{n,l}) - k \end{aligned}$$

Further analysis of the 1/K model is the focus of on-going research.

5. Conclusion

A fuzzy approach provides an alternative mechanism to characterize many uncertain parameters including demand, the wholesale price, and market sales price. This paper introduced the fuzzy newsvendor problem and proposed a framework for solving the problem under the buyback policy with the numerical model. Furthermore, we also proposed an integration model for solving the fuzzy newsvendor problem with a single-manufacturer-multiple-retailer setting based on three coordination policies (quantity discount, profit sharing, and buyback as well as non-coordination case).

By using the framework we proposed, the retailer can determine her optimal order quantity that maximizes the global profit. For the manufacturer, it provides guidelines for pricing offers since he can calculate his possible profit with uncertain data. Fuzzy newsvendor model should still be extended to deal with multiple-manufacturer-multiple-retailer setting problem, and this is one of research topics for further study.

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