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피보나치 후위순회 원형군

김용석
서남대학교 컴퓨터 정보통신학과
e-mail:yskim@seonam.ac.kr

A Fibonacci Posterorder Circulants

Yong-Seok Kim
Dept of Computer Science, Seonam University

Abstract

In this paper, we propose and analyze a new parallel computer topology, called the Fibonacci posterorder circulants. It connects $f_n, n \geq 2$ processing nodes, same the number of nodes used in a comparable Fibonacci cube. Yet its diameter is only $\lfloor \frac{n}{3} \rfloor$, almost one third that of the Fibonacci cube. Fibonacci cube is asymmetric, but it is a regular and symmetric static interconnection networks for large-scale, loosely coupled systems. It includes scalability and Fibonacci cube as a spanning subgraph.

1. Introduction

Need for high computing power has continued to drive the high speed computer design. One of the most straightforward and the least expensive means of achieving the end is to construct multicomputer networks that consist of nodes with local memory(no shared memory) and a communication controller, where each node is connected by a communication link to a number of nodes. Whenever a node wants to communicate with another node, it communicates through other nodes unless there exists a direct communication link between the two. The system might be represented by a communication network.

Recently the hypercube has become a popular interconnection scheme for multicomputers, and a variety of hypercube-based interconnection netwoks has been proposed[1-3]. It is known that any natural number can be uniquely represented as a sum of Fibonacci numbers (Zeckendorf's theorem[4]). The Fibonacci cube[5-8], which is inspired by the famous Fibonacci numbers, is shown to possess attractive recurrent structures, in spite of its asymmetric. For it with N nodes, the diameter, the edge connectivity, and the node connectivity of the Fibonacci cube are in the logarithmic order of N. There are features which may be taken as serious drawbacks of this new interconnection scheme. For example, one potential shortcoming of the Fibonacci cube is that (unlike the Boolean cube) the node degrees of the Fibonacci cube are not homogeneous. As such, there may be some implementation difficulties[9]. Circulant graphs give a good topology of a communication network due to the high degree of symmetry. Mader proved that every connected circulant graph has the maximum edge connec tivity[9]. In this paper ,the Σ_n is a regular and symmetric topology. It has scalability and includes Fibonacci cube as a spanning subgraph. It connects $f_n, n\geq 2$ processing nodes, same the number of nodes used in a comparable Fibonacci cube. Yet its diameter is only $\lfloor \frac{n}{3} \rfloor$, almost one third that of the Fibonacci cube. Lower diameter is desirable because it lowers the expected communication delay, thereby enhancing system performance. The low degree is a very useful factor in the construction of large scale systems.

This paper is organized as follows. Section 2 defines the Fibonacci posterorder circulants. Section 3 defines the diameter. Section 4 presents broadcasting. Section 5 briefly discusses and summarizes the results that are presented.

2. Fibonacci posterorder circulants

2.1 Fibonacci code

It is known that any natural number can be uniquely represented as a sum of Fiboancci numbers (Zeckendorf's theorem). Specifically, assume that i is an integer and $.0\leq i\leq f_n-1$, $n\geq 3$ We let $(b_{n-1},\cdots,b_3,b_2)_F$ denote the order-n Fibonacci code (or simply, *Fibonacci code,* if n is implicit) of i, where b_j is either 0 or 1 for $2\leq j\leq (n-1)$ and $i=\sum_{j=2}^{n-1} b_j\cdot f_j$. The Fibonacci representation of an integer i can be obtained by using the following greedy approach[]. First find the greatest Fibonacci number f_j that is less than or equal to i, assign a "1" to the bit that corresponds to f_j, then proceed recursively for $i-f_j$. The unassigned bits are 0's. Note that in the Fibonacci code, the rightmost bit corresponds to f_2, rather than f_1. Also notice that no consecutive 1's appeared in the Fibonacci codes, as $f_k+f_{k-1}=f_{k+1}$; to represent a number between 0 and f_n-1 requires $n-2$ bits.

Now let C_n denote the set of order-n Fibonacci codes, where $n\geq 2$. For example, $C_2=\{\lambda\}$, $C_3=\{0,1\}$, $C_4=\{00,01,10\}$. Clearly, $|C_n|=f_n$ for $n\geq 2$, where $|C_n|$ denotes the size of a set C_n. Therefore, if C_n is $\{a_1, a_2,\cdots, a_{f_n}\}$, then $[C_n]_1^i$ denotes $\{a_1, a_2,\cdots, a_i\}$, where $1\leq i\leq f_{n-1}$.

Definition 1 Let C_n be defined as above. Then $C_n = (00\cdot C_{n-2}) \bigcup (01\cdot [C_{n-2}]_1^{f_{n-3}}) \bigcup (10\cdot C_{n-2})$, where $n\geq 4$.

2.2 Definitions

The Fibonacci posterorder circulants of order n, denoted by Σ_n, has f_n nodes. When its size (i.e., the number of nodes)is a Fibonacci number, the desirable(when designing algorithms).

Definition 2 Let $\Sigma_n=(V_n, E_n)$ denote the Fibonacci circulant of order n, where

$V_n=\{0,1,\cdots,f_n-1\}$

$E_n=\{(i,j)\mid i+f_i\equiv j(\bmod f_n),\ 2\leq i\leq n-2\}$.

Example: Fig. 1 shows the Fibonacci posterorder circulants of order n.

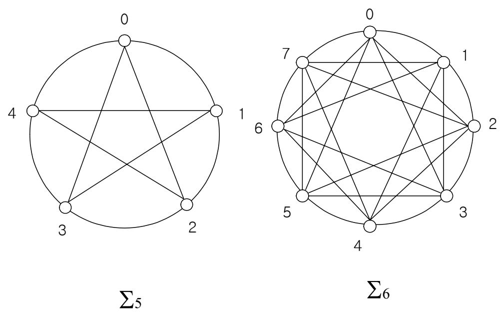

Fig. 1 Fibonacci posterorder circulants

2.3 Characterization

As it can be seen, the definition of a Fibonacci posterorder circulants is analogous to that of the Fibonacci cube. In the following, we show that the Fibonacci posterorder circulants contains the Fibonacci partial and Fibonacci complement and these are defined.

Definition 3 Let Fibonacci complement of order n denote $\Psi_n=(V'_n, E'_n)$ $V'_n=V_{n_1}\bigcup V_{n_2}=\{b_1, b_2,\cdots, b_n\}\bigcup\{c_1, c_2,\cdots c_n\}$

$=\{0,1,\cdots,f_{n-2}-1\}\cup\{f_{n-1},f_{n-2}+1,\cdots,f_n-1\}$

$E'_n=\{(b_i, c_j)\mid b_i+a_k\equiv c_k, 1\le k\le n-i+1\}$ and $a_n=\{a_1, a_2,\cdots, a_n\}=\{f_n-1, f_n-2,\cdots, f_{n-1}\}$.

Definition 4 Let Fibonacci partial of order n, denote $\delta_n, n\ge 4=(V_n, E''_n)$ $V_n=\{0,1,\cdots, f_n-1\}$ and E_n is the set of edges in Fibonacci circulant of order n, E'_n is the set of edges in Fibonacci complement of order n, then $E''_n=\{(i,j)\mid i+f_k$
$\equiv j(\le f_n-1)$
$\equiv j(\le f_n-1), 2\le kn-2\le\}=E_n-E'_n$.

Example: In Fig. 2, we show that Σ_n is induced by δ_n and Ψ_n.

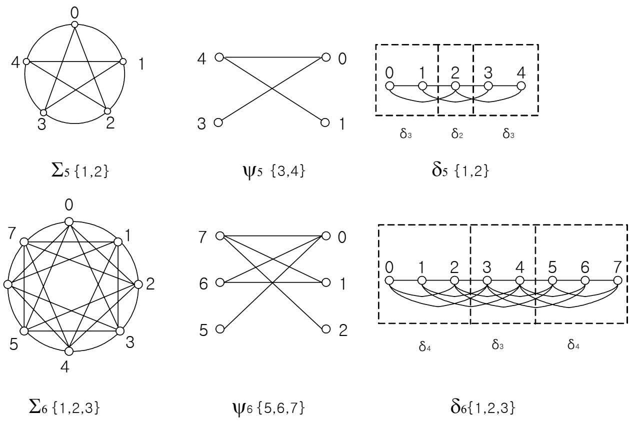

Fig. 2 the example of Σ_n, Ψ_n and δ_n

3. Diameters

Definition 5 Let $\Lambda_n=(V,E)$ denote the Fibonacci linear of order n, where $V_n=\{0,1,\cdots, f_n\}$ $E_n=\{(i,j)\mid i+f_k\equiv j(\le f_n)$. $2\le k\le n-2\}$

Definition 6 (Fibonacci distance): Let $\Delta(i,j)$ denote the distance between Node i and Node j in the Fibonacci circulant. Then $\Delta(i,j)$:= $n(|i-j|)_F$.

Lemma 1 If $dia(\Lambda_n: f_i)$ denotes the diameter of f_i distance in $\Lambda_n, n\ge 3$, then $dia(\Lambda_n: f_{n-1})= dia(\Lambda_{n+1}: f_{n-1})$.

proof) The claim is true for Λ_3 $dia(\Lambda_3: f_2)=dia(\Lambda_4: f_2)=1$ for Λ_4, $dia(\Lambda_4: f_3)= dia(\Lambda_5: f_3)=1$ and for Λ_5 $dia(\Lambda_5: f_4)=dia(\Lambda_6: f_4)=1$. Suppose that $dia(\Lambda_k: f_{k-1})= dia(\Lambda_{k+1}: f_{k-1})$, where $n\le k$. Let $n=k+1$ $\Lambda_k=\Lambda_{k-2}\cup\Lambda_{k-3}\cup\Lambda_{k-2}=\Lambda_k \cup\Lambda_{k-1}$. jump sequence is $\{f_2, f_3,\cdots, f_{k-2}\}$. and $\Lambda_{k+1}=\Lambda_{k-1}\cup\Lambda_{k-2}\cup\Lambda_{k-1}$, jump sequence is $\{f_2, f_3,\cdots, f_{k-1}\}$. Therfore, for Λ_k by definition 3, node number 0 is the same as node number f_k , it is true $\Delta(0, f_{k-1})=\Delta(f_k, f_{k-1})=1$, using maximum jump sequence. It can be seen that $\Delta(0, f_k)=1$ where the maximum sequence is f_{k-1}, for Λ_{k+1}. Then it is true that $dia(\Lambda_k: f_{k-1}) = dia(\Lambda_{k+1}: f_{k-1})$.■

Lemma 2 If $dia(\Lambda_n)$ denotes the diameter of Λ_n, then $dia(\Lambda_n: f_{n-1})=1+dia(\Lambda_{n-3})$.

Proof) It is true that $dia(\Lambda_0)= dia(\Lambda_1)= dia(\Lambda_2)=0$, then it can be easily verified $dia(\Lambda_3: f_2)= 1+\Lambda_0=1$, $dia(\Lambda_4: f_3)= 1+\Lambda_1=1$ and $dia(\Lambda_5: f_4)=1+\Lambda_2=1$. Assume that $dia(\Lambda_k: f_{k-1})=1+dia(\Lambda_{k-3})$ holds for $n\le k$, $k\ge 5$. Consider the case $n=k+1$, be cause $\Lambda_{k+1}=\Lambda_{k-1}\uplus\Lambda_{k-2}\uplus\Lambda_{k-1}$ and its maximum jump sequence is f_{k-1}, which com pletes the proof.■

Theorem 1 The diameter of Σ_n, $n\ge 2$ denotes $dia(\Sigma_n)=\lfloor \frac{n}{3}\rfloor$.

Proof) It should be clear that $dia(\Lambda_3)= dia(\Lambda_4)=dia(\Lambda_5)=1$. Suppose that $dia(\Lambda_k: f_k)=\lfloor \frac{n}{3}\rfloor$, where $n\le k$, $k\ge 6$. Consider the case $n=k+1$, It is easily verified that

$dia(\Lambda_{k+1}) = dia(\Lambda_{k+1}; f_k) = 1 + dia(\Lambda_{k-2})$ $dia(\Lambda_{k+2}) = dia(\Lambda_{k+2}; f_{k+1}) = 1 + dia(\Lambda_{k-1})$. $dia(\Lambda_{k+3}) = dia(\Lambda_{k+3}; f_{k+2}) = 1 + dia(\Lambda_k)$. It is straightforward that $dia(\Lambda_{k-2}) = dia(\Lambda_{k-1}) = dia(\Lambda_k) = 1$. Therefore, by lemma $dia(\Lambda_n) = dia(\Sigma_n) = \lfloor \frac{n}{3} \rfloor$ can be pro ved.

4. Broadcasting

Lemma 3 Broadcasting from node 0 to all nodes can be performed in $n-2$ steps on Σ_n, which is optimal.

proof) It is easy to verify the proposition for $n \le 2$. Assume that the proposition is true for $n \le k$, where $k \ge 3$ denote an integer. Let l_n denote the number of steps required in trans mitting a unit of data in T_n. while $l_k = MAX\{l_{k-1}+1, l_{k-2}\} = k-2$. Now consider the case $n = k+1$. Note that Σ_{k+1} encompasses a T_{k+1} and Node 0 is the root of T_{k+1}. We shall send the data along the branches of T_{k+1}, since T_{k+1} is a spanning tree of Σ_{k+1}. Based on the definition of Fibonacci trees T_{k+1} consists of a T_k and T'_{k-1} ($\cong T_{k-1}$); notice that the root of T_{k+1} (Node 0) is also the root of the subtree T_k. When sending a value from Node 0 to all nodes, we first send it from Node 0 to the root of T'_{k-1}, then proceed recursively(and concurrently) in both T_k and T'_{k-1}.■

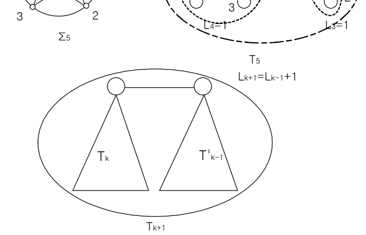

Fig. 4 broadcasting

5. Conclusion

One potential shortcoming of the Fibonacci cube is that (unlike the Boolean cube) its node degrees are not homogeneous. As such, there may be some implementation difficults. The Fibonacci posterorder circulants is a regular and symmetric topology. It has scalability and Fibonacci cube as a spanning subgraph. It connects $f_n, n \ge 2$ pro cessing nodes, same the number of nodes used in a comparable Fibonacci cube. Yet its diameter is only $\lfloor \frac{n}{3} \rfloor$, almost one third that of the Fibona cci cube. Lower diameter is desirable because it lowers the expected communication delay, thereby enhancing system performance. The low degree is a very useful factor in the construction of large scale systems. We need to conider em beddings for the hypercube, the binary and gene ral trees.

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