Numerical Simulation of Square Driven Cavity Flows Using Preconditioned Lattice Boltzmann Method

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1. Introduction

In recent years, the lattice Boltzmann method (LBM) has received considerable attention as an alternative tool for computational fluid dynamics (CFD). In a traditional method in CFD, the macroscopic variables, such as velocity **u** and pressure p are obtained by solving the Navier-Stokes equations. In the LBM, the dynamic behavior of a fluid is described by a lattice Boltzmann equation (LBE) for the single-particle distribution function. The LBM can be regarded as a special discrete form of the Boltzmann equation from kinetic theory.

The simplest lattice Boltzmann model is the lattice Bhatnager-Gross-Krook (LBGK) model [1]. The compressible Navier-Stokes equations can be derived from the LBGK equation through the Chapmann-Enskog procedure. The compressible effect in the standard LBGK model may produce some serious errors in numerical simulations of incompressible flow. To overcome this difficulty, He and Luo [2] proposed a LBM for the incompressible flow by introducing the pressure distribution function instead of the mass density one. As applied to steady flows, however, this method usually converges rather slowly.

In this paper, we incorporate a preconditioning proposed by Guo, *et al.* [3] into the LBM and applied for simulations of square driven cavity flows. Detailed comparison of efficiency between the standard and preconditioned LBM is presented.

2. Preconditioned Lattice Boltzmann Method

The evolution equation of the so- called pseudo-compressibility LBM can be written:

$$p_\alpha\left(x+e_\alpha\delta_t,t+\delta t\right)-p_\alpha(x,t)=-\frac{1}{\tau}[p_\alpha(x,t)-p_\alpha^{(eq)}(x,t)], \qquad (1)$$

where, τ is the dimensionless relaxation time, and the equilibrium distribution function, $p_\alpha^{(eq)}$ for d2q9 model modified by introducing preconditioning proposed by Guo *et al.* is given by:

$$p_\alpha^{(eq)}=w_\alpha\left[p+\rho_0\left((e_\alpha\cdot u)+\frac{1}{\gamma}\left(\frac{3(e_\alpha\cdot u)^2}{2c^2}-\frac{u^2}{2}\right)\right)\right], \qquad (2)$$

where, $c=\delta_x/\delta_t$, and δ_x and δ_t are the lattice spacing and the time step size, respectively. $\gamma\le 1$, and is defined as $\gamma=\left(M/M^*\right)^2$, where M^* is the effective Mach number.
The dimensionless relaxation time τ should also be modified as,

$$\tau_p-0.5=\left(\tau-0.5\right)/\gamma \qquad (3)$$

Where τ_p is the relaxation time for the preconditioned LBM(PLBM).

3. Numerical Results

Numerical simulations of square cavity flows were carried out using pseudo-compressible LBM and PLBM for Re=100 and 400 on 129×129 and 257×257 square lattice (129 and 257 lattice nodes and 128 and 256 lattice units in one side). The Reynolds number used in the present simulation is defined as $Re=UN/\nu$, where U is the uniform velocity of the top wall, N is the number of lattice units along one side of the cavity, and ν is the kinematic viscosity given as $\nu=\left(2\tau-1\right)c^2\delta t/6$.

Figure 1 shows the velocity profiles of u and v through the geometric centers of the cavity, whereas Figure 2 shows the convergence histories of pseudo-compressibility LBM computed with M=0.52 and PLBM computed with M=0.1. The velocity profiles are almost indistinguishable between two methods. The PLBM with a small Mach number converges faster than the pseudo-compressibility LBM with much larger Mach number.

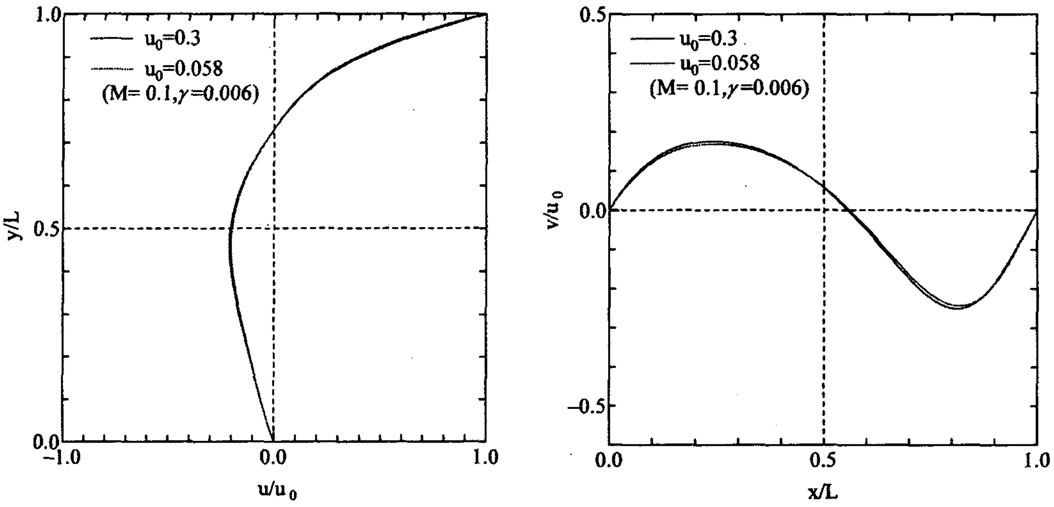

Fig. 1 Velocity profiles of u and v through the geometric centers of the cavity

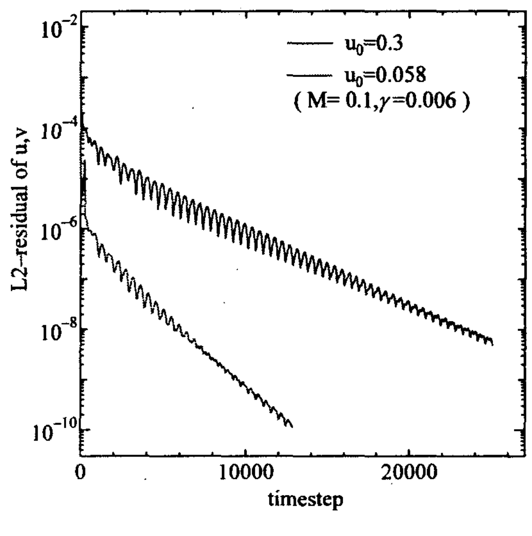

Fig. 2 Convergence histories of pseudo-compressibility LBM computed with M=0.52 and PLBM computed with M=0.1

References

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