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보조변수법을 이용한 Zwicker 라우드니스의 설계민감도

Design Sensitivity Analysis of Zwicker's Loudness Using Adjoint Variable Method

왕세명† ·권대일*·김좌일**

Semyung Wang, Daeil Kwon, Chawil Kim

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ABSTRACT

Feasibility of optimizing Zwicker's loudness has been shown by using MSC/NASTRAN, SYSNOISE, and a semi-analytical design sensitivity by Wang and Kang. Design sensitivity analysis of Zwicker's loudness is developed by using ANSYS, COMET, and an adjoint variable method in order to reduce computation. A numerical example shows significant reduction of computation time for design sensitivity analysis.

1. ITRODUCTION

When human being is exposed to product sound, he/she represents his/her feelings as detailed or comprehensive expressions. For example, in the former, there are no standard to define sound objectively. Therefore estimate sound as, 'sound is loud or not', 'sound is sharp or not', 'sound is rough or not' and so on. For expressing objectively human's subjective feelings on sound, sound quality metrics were suggested which had linear relations to perceptual feelings by many researchers.

The loudness models of Stevens and Zwicker are adopted as international standard [1]. To use the concept of loudness, several weighting curves have been used. Zwicker and Fastl defined many sound quality metrics, i.e. loudness, sharpness, roughness and fluctuation strength [2].

The current mathematical acoustic model was established in the 1950's and 1960's. Recently, Kinsler et al [3]. published intensive studies on acoustic problems including mathematical analyses. With the development of the performance of the computers, the approximate solution from numerical method has been reliable and powerful. Especially the boundary element analysis extended the acoustic problem to obtaining unmeshed exterior sound pressure.

Choi et al. [4] and Wang [5] studied a continuum sensitivity analysis for structural-acoustic system using the finite element analysis. These are done for interior acoustic problem. They extended structural continuum approach [6] to acoustic system. Acoustic sensitivity analysis for the pressure at the field points with respect to normal velocity on the structure surface was developed by Coytte [7]. They used discrete analytical method for sizing sensitivity analysis problems and semi-analytical method for shape sensitivity analysis problems. Wang and Lee [8] developed the global sizing acoustic sensitivity analysis method by using structural continuum method. They proposed more accurate method than structural semi-analytical method by using continuum method. Recently Kim et al. [9] performed the design sensitivity analysis for sequential structural acoustic problems.

2. DSA OF ZWICKER'S LOUDNESS BY ADJOINT VARIABLE METHOD

2.1 Evaluation of Zwicker's Loudness

The evaluation model of Zwicker's loudness makes a start with the concept of specific loudness. Specific loudness comes from Stevens' law that a sensation belonging to the category of intensity sensation grows with physical intensity according to a power law .

In ISO 532 B, 1/3-oct band filters are used instead of critical-band filters for the practical reason. Some compensation is needed since there is the difference between critical and 1/3-oct bandwidth. And additional specific loudness produced by the cut off slope in the abutting filter towards lower frequencies is taken into account by changing the exponent of specific loudness slightly from 0.23 to 0.25. The formula of the main specific loudness is in ISO 532 B [1]:

$$NM = \left(0.0635 \cdot 10^{0.025 L_{TQ}}\right) \cdot \left[\left[1 + 0.25 \cdot 10^{0.1(L_E - L_{TQ})}\right]^{0.25} - 1\right] \quad (1)$$

$$(sone_G \,/\, Bark)$$

NM is the main specific loudness defined in 1/3-oct band, L_{TQ} is excitation level at threshold in quite and L_E is excitation level. To calculate excitation level, some corrections are added. For 1/3-oct band filters, low frequency range is added and correction factor is used at all bands. And the logarithmic transmission factor to

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† 광주과학기술원 기전공학과 교수
E-mail : smwang@gist.ac.kr
Tel : (062)970-2390, Fax : (062)970-2384

* 광주과학기술원 기전공학과 석사과정

** 광주과학기술원 기전공학과 postdoc

represent the transmission between free field and our hearing system is incorporated.

$$L_E = P_{band} - a_0 - c_1 \tag{2}$$

where

$$P_{band} = 20log\left[\left(\int_{\Delta\omega} p^2 d\omega/\Delta\omega\right)^{0.5} /p_{ref}\right] \; dB \tag{3}$$

where $\Delta\omega$ is frequency bandwidth, p_{ref} is reference sound pressure, 20e-6 pa.

2.2 DSA of Zwicker's Loudness

In many cases loudness pattern diagram clearly shows which partial area is dominant. It is efficient to reduce the dominant part of the noise that produces the largest area in the loudness pattern. So reducing the main specific loudness contributes largely reducing total loudness. This procedure is efficient especially because of making effect.

In Eq. (1), the main specific loudness has not structural design variables. To calculate sensitivity with respect to structural design variable, chain rule is used as below.

$$\frac{\partial NM}{\partial u} = \frac{\partial NM}{\partial L_E} \cdot \frac{\partial L_E}{\partial u} \tag{4}$$

The first derivative can be derived directly from Eq. (1). Because L_{TQ} is constant in the specific octave band, $\partial NM / \partial L_E$ is constituted by the excitation level. And the second term is equal to $\partial P_{band} / \partial u$ because of a_0 and c_1 which are constant in Eq. (2). By chain rule, $\partial P_{band} / \partial u$ can be obtained as

$$\frac{\partial P_{band}}{\partial u} = \frac{10}{\ln 10} \cdot \frac{\left(\int_{\Delta\omega} p^2 d\omega/\Delta\omega\right)^{-1}}{\Delta\omega} \cdot \int_{\Delta\omega} p \cdot p' d\omega \tag{5}$$

where p' is global acoustic design sensitivity.

If the critical band is selected, constants depend on the critical band are obtained, i.e. a_0, c_1 and L_{TQ}. And L_E, pressures and global acoustic design sensitivity of each frequency in critical band can be calculated. With these values the sensitivity of the main specific loudness is calculated.

2.3 Global Acoustic DSA by Adjoint Varible Method

Assume that $\psi(\mathbf{u})$ is continuously differentiable with respect to design $\mathbf{u}$. The perturbation of the design is $\delta\mathbf{u}$ (arbitrary), and τ is a parameter that controls the perturbation size, then the variation of $\psi(\mathbf{u})$ in the direction of $\delta\mathbf{u}$ is defined as [10]

$$\psi'_{\delta\mathbf{u}} \equiv \frac{d}{d\tau}\psi(\mathbf{u}+\tau\delta\mathbf{u})\bigg|_{\tau=0} = \frac{\partial\psi^T}{\partial\mathbf{u}}\delta\mathbf{u} \tag{6}$$

If the variation of a function is continuous and linear with respect to $\delta\mathbf{u}$, the function is differentiable.

$$\mathbf{v}' = \frac{d}{d\tau}\psi[v(x,\mathbf{u}+\tau\delta\mathbf{u})]\bigg|_{\tau=0} = \frac{\partial\mathbf{v}^T}{\partial\mathbf{u}}\delta\mathbf{u} \tag{7}$$

$$\mathbf{p}' \equiv \frac{d}{d\tau}\psi[p(x,\mathbf{u}+\tau\delta\mathbf{u})]\bigg|_{\tau=0} = \frac{\partial p^T}{\partial\mathbf{u}}\delta\mathbf{u} \tag{8}$$

A direct differentiation method computes the variation of state variables by differentiating the state equations (7) and (8) with respect to the design. Considering the structural part, the variations are defined as

$$a'_{\delta u}(\mathbf{v},\bar{\mathbf{z}}) \equiv \frac{d}{d\tau}[a_{\mathbf{u}+\tau\delta\mathbf{u}}(\tilde{\mathbf{v}},\bar{\mathbf{z}})]\bigg|_{\tau=0} \tag{9}$$

$$d'_{\delta u}(\mathbf{v},\bar{\mathbf{z}}) \equiv \frac{d}{d\tau}[d_{\mathbf{u}+\tau\delta\mathbf{u}}(\tilde{\mathbf{v}},\bar{\mathbf{z}})]\bigg|_{\tau=0} \tag{10}$$

$$l'_{\delta u}(\bar{\mathbf{z}}) \equiv \frac{d}{d\tau}[l_{\mathbf{u}+\tau\delta\mathbf{u}}(\bar{\mathbf{z}})]\bigg|_{\tau=0} \tag{11}$$

where $\tilde{\mathbf{v}}$ denotes state variable $\mathbf{v}$ with the dependence on τ being suppressed, and $\bar{\mathbf{z}}$ and its complex conjugate are independent of the design.

Thus, by taking a variation of both sides of equation (7) with respect to the design

$$j\omega d_{\mathbf{u}}(\mathbf{v}',\bar{\mathbf{z}})+\kappa a_{\mathbf{u}}(\mathbf{v}',\bar{\mathbf{z}}) \equiv l'_{\delta\mathbf{u}}(\bar{\mathbf{z}})-j\omega d'_{\delta\mathbf{u}}(\mathbf{v},\bar{\mathbf{z}})-\kappa a'_{\delta\mathbf{u}}(\mathbf{v},\bar{\mathbf{z}}), \quad \forall\bar{\mathbf{z}}\in Z \tag{12}$$

If a design perturbation $\delta\mathbf{u}$ is defined, and if the right-hand side of equation (12) is evaluated with the solution to equation (2), then equation. (12) can be numerically solved to obtain using the finite element method. By interpreting the right-hand side of equation (12) as another load form, equation (12) can be solved by using the same solution process as the frequency response problem in equation (2)

In acoustic point of view, the equation (13) yields the following sensitivity equation

$$[A]\{p\} = [B]\{\partial_n p\} \tag{13}$$

$$b(\mathbf{x}_0;\mathbf{v}') + e(\mathbf{x}_0;\mathbf{p}'_s) = \alpha p'(\mathbf{x}_0) \tag{14}$$

Since integral forms $b(\mathbf{x}_0;\bullet)$ and $e(\mathbf{x}_0;\bullet)$ are independent of the design, the above equation has exactly the same form as equation (13). Thus, using the solution $(\mathbf{v}')$ of the structural sensitivity equation (12), equation (14) can be used by following the same solution process as BEM

$$[\mathbf{A}]\{\mathbf{p}_s^{'}\} = [\mathbf{B}]\{\mathbf{v}'\} \tag{15}$$

Then, the pressure sensitivity at point can be obtained as

$$p'(\mathbf{x}_0) = \{b(\mathbf{x}_0)\}^T\{\mathbf{v}'\} + \{e(\mathbf{x}_0)\}^T\{\mathbf{p}_s^{'}\} \tag{16}$$

The acoustic performance ψ_2 is defined at point x_0. And its sensitivity, p', should be explicitly expressed in terms of $\delta\mathbf{u}$. The objective is to express p' in terms of $\mathbf{v}'$. By using an relation in equation (13), adjoint equation is obtained as

$$\begin{aligned}\psi_2^{'} &= h_{,\mathbf{u}}^T\delta\mathbf{u} + h_{,p}p' \\ &= h_{,\mathbf{u}}^T\delta\mathbf{u} + h_{,p}[b(\mathbf{x}_0;\mathbf{v}') + e(\mathbf{x}_0; A^{-1}\circ B(\bar{\boldsymbol{\lambda}}))]\end{aligned} \tag{17}$$

ψ_2 is expressed in terms of $\mathbf{v}'$. The second term on the right-hand side of the above equation can be used to define the adjoint load by substituting $\bar{\boldsymbol{\lambda}}$ for $\mathbf{v}'$. Therefore, the adjoint equation is obtained

$$\begin{aligned} j\omega d_{\mathbf{u}}(\mathbf{v}',\bar{\mathbf{z}}) + \kappa a_{\mathbf{u}}(\mathbf{v}',\bar{\mathbf{z}}) \equiv h_{,p}[b(\mathbf{x}_0;\mathbf{v}') + e(\mathbf{x}_0; A^{-1}\circ B(\bar{\boldsymbol{\lambda}}))], \\ \boldsymbol{\lambda}\in Z \end{aligned} \tag{18}$$

Where an adjoint solution $\boldsymbol{\lambda}^*$ is required. The sensitivity of ψ_2 can be obtained as

$$\psi_1^{'} = h_{,\mathbf{u}}\delta\mathbf{u} + l_{\delta\mathbf{u}}^{'}(\bar{\mathbf{z}}) - j\omega d_{\delta\mathbf{u}}^{'}(\mathbf{v},\bar{\mathbf{z}}) - \kappa a_{\delta\mathbf{u}}^{'}(\mathbf{v},\bar{\mathbf{z}}) \tag{19}$$

Even though ψ_2 is a function of pressure p, its sensitivity expression in equation (19) does not require the value of p, only the structural solution $\mathbf{v}$ and the adjoint solution $\boldsymbol{\lambda}^*$ are required in the calculation of $\psi_2^{'}$.

Form of the adjoint load, equation (19) can be written in the discrete system as

$$[j\omega\mathbf{M} + \kappa\mathbf{K}]\{\boldsymbol{\lambda}^*\} = \{\mathbf{b}\} + [\mathbf{B}]^T[\mathbf{A}]^{-T}\{\mathbf{e}\} \tag{20}$$

Where, the right-hand side corresponds to the adjoint load in the discrete system. Instead of computing the inverse matrix, an acoustic adjoint problem in BEM is defined as

$$[\mathbf{A}]^T\{\boldsymbol{\eta}\} = \{\mathbf{e}\} \tag{21}$$

Where the acoustic adjoint solution $\{\boldsymbol{\eta}\}$ is desired. By substituting $\{\boldsymbol{\eta}\}$ into equation (4.19), the structural adjoint problem is obtained as

$$[j\omega\mathbf{M} + \kappa\mathbf{K}]\{\boldsymbol{\lambda}^*\} = \{\mathbf{b}\} + [\mathbf{B}]^T\{\boldsymbol{\eta}\} \tag{22}$$

Acoustic adjoint solution $\{\eta\}$, which is obtained from BEM, is required to compute the structural adjoint load, and frequency-response re-analysis then provides the structural adjoint solution $\{\boldsymbol{\lambda}^*\}$. Thus, two different adjoint problems are defined. The first is similar to BEM, and is used to compute the adjoint load, the second is similar to the structural frequency-response problem.

3. CALCULATION PROCEDURE

Procedures related to calculating glabal acoustic sensitivity and loudness sensitivity are shown in Figure 1.

4. NUMERICAL EXAMPLE

A structural–acoustic system is solved using both finite element and the boundary element methods. Even if FEM and BEM are used to evaluate the acoustic performance measure, only the structural response v is required to perform design sensitivity analysis. The adjoint load is calculated from the transposed boundary element analysis, and the adjoint equations are then numerically solved using the FEA code with the same finite element model used for the original structural analysis.

The frequency range of interest is decided by the criterion that the smallest edge length has to be less than wavelength over six. This criterion is following equation.

$$f \le \frac{c}{6\lambda_E}. \tag{23}$$

Where, $c = 340\ m/s$, $\lambda_E = 0.1\ m$

From the equation (23), the frequency range of interest is less than 566 Hz. This range covers 14^{th} 1/3-octave band of which boundary is 560 Hz.

The panel is an aluminum plate with thickness of 0.01 m, mass density of $\rho_s = 2700 kg/m^3$, Young's modulus of $E = 70GPa$, a Poisson's ratio of $\upsilon = 0.334$ and a structural-damping coefficient $\varphi = 0.055$ of A harmonic force $f = 1N$ in the Z direction is applied at four points on the plate as shown in Figure 2. The whole structure is discretized by 864 elements and 866 nodes.

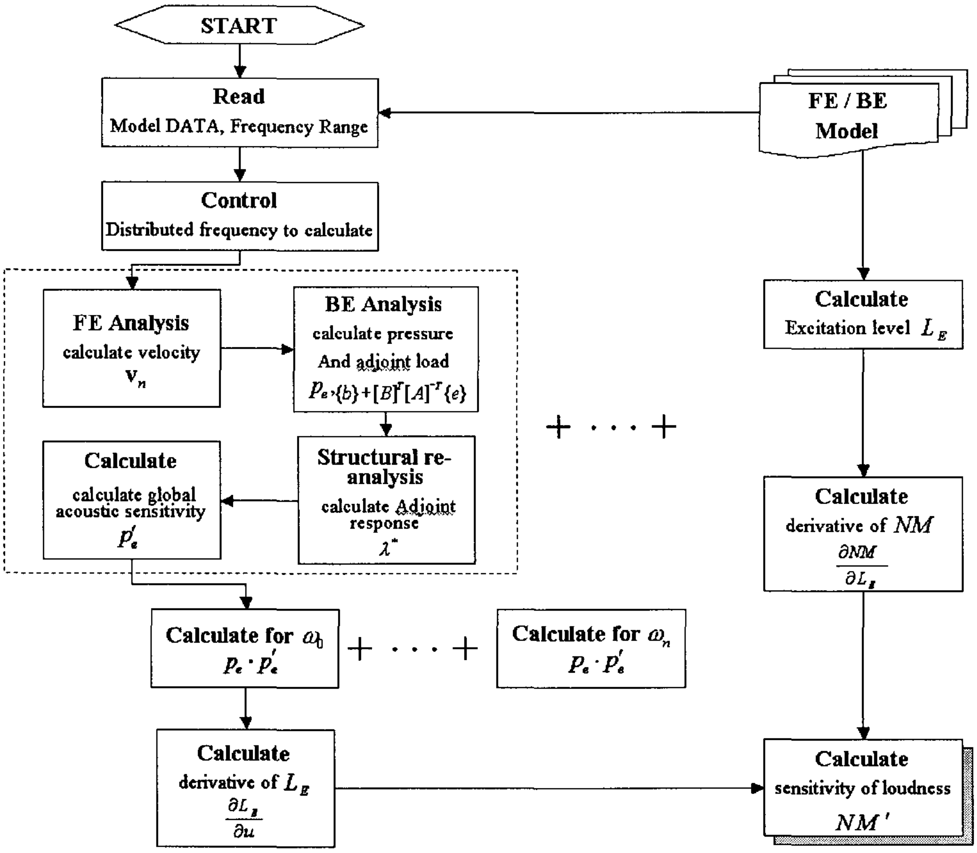

Figure 1 Computational procedures of the sizing sensitivity of main specific loudness

Table 1 Global acoustic sensitivity result of the acoustic box model

Element Number	Perturbation δd [%]	$\psi(d+\delta d)$	$\psi(d-\delta d)$	$\Delta\psi$	ψ'	Accuracy $\psi'/\Delta\psi$
210	10	0.380502	0.388376	-3.9370	-4.3691	110.97
	1	0.383538	0.384385	-4.2380		103.09
174	10	0.415502	0.428376	-6.4370	-5.2429	81.45
	1	0.423385	0.4243856	-5.0030		104.79

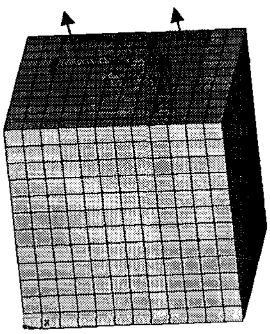

Figure 2 An acoustic box model

In the harmonic analysis, the five sides of the structure are fixed to simulate the rigid wall, only the upper panel is allowed to move. In the acoustic analysis, the pressure value of each node is calculated from the structural velocity data and the pressure is evaluated. The panel thickness is chosen as the design variable. The following design sensitivities are considered: the acoustic pressure at $A(0.6, 0.6, 1.3)$. The ANSYS program is used for Harmonic analysis of the primary and adjoint structural problems, whereas BEM is performed by using the COMET/ACOUSTIC. The design sensitivities are computed at 60 Hz, which is close to the resonant frequency, as shown in Figure 3.

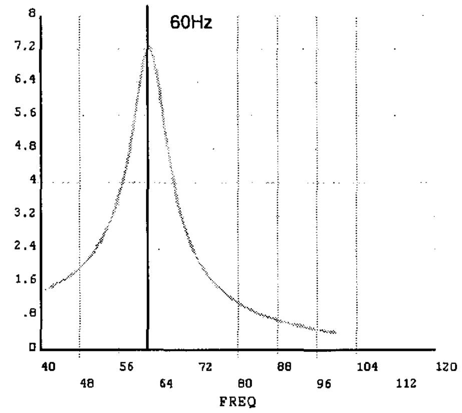

Figure 3 Structural analysis result of the acousticbox box model

$$|p|' = \frac{p_r p_r' + p_i p_i'}{|p|} \tag{24}$$

Acoustic peak values of the pressure appear at frequencies corresponding to natural frequencies of the plate. The design sensitivity results are shown in Table 1. Since the pressure $p = p_r + ip_i$ is a complex variable, the sensitivity of its amplitude can be calculated from the formula

Where, $p' = p_r' + ip_i'$ *is obtained from the design* sensitivity analysis.

Whole analysis is performed in the frequency range of interest to validate sensitivity result.

And Table 1 shows the sensitivity result of the elements.

5. CONCLUSION

In this thesis, the design sensitivity analysis using adjoint variable method is applied to the Zwicker's loudness. Loudness is more appropriate to express the human hearing than the sound pressure level, because it contains human hearing system characteristics and the basic metric for sound quality engineering. The design sensitivity proposed in this thesis is formulated especially considering sizing design variable, i.e. thickness. Accuracy test is conducted for checking sensitivity formula.

The formula of the sensitivity of main specific loudness, adjoint variable method is proposed. Structural FEM and acoustic BEM are used to calculate structural velocities and sound pressure, respectively. Especially, through the acoustic BEM, adjoint load can be obtained. The sensitivity of the main specific loudness includes the derivative of the loudness with respect to excitation level and the derivative of excitation level with respect to the design variable, i.e. thickness. The derivative of excitation level is calculated with global acoustic sensitivities. ANSENS, in-house code developed in this research controls all procedures for DSA and optimization with ANSYS and COMET/ACOUSTICS. Using the code global acoustic sensitivity is calculated. These sensitivity results give engineers the guideline to reduce loudness directly. Also sensitivity information plays the gradient role of optimization. Just simple box cavity model is used for numerical example to verify acoustic sensitivity formulation. By the proposed method, the analysis time can be reduced very much.

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REFERENCES

(1) ISO R532, 1966, "Method for Calculating Loudness Level", International Organization for Standardization, Geneva.

(2) E. Zwicker and H. Fastl, 1990, Psychoacoustics. Facts and models, Springer, Heidelberg, New York

(3) L. Kinsler, A. Frey, A. Coppens, J. Sanders, 2000, Fundamentals of Acoustics, John Wiley & Sons Inc., 4th ed., New York.

(4) K. K. Choi, I. Shim, S. Wang, 1997, "Design sensitivity analysis of structure-induced noise and vibration", Journal of Vibration and Acoustics, vol.119, pp.173-179.

(5) S. Wang, K.K Choi, and H.T. Kulkarni, 1996, Application of Continuum Sensitivity Analysis and Optimization to Automobile Structures, IUTAM Symposium in Optimization of Mechanical Systems, Kluwer Academic Publishers, pp. 317-324.

(6) K. K. Choi, J. Lee, 1992, "Sizing design sensitivity analysis of dynamic frequency response of vibrating structures", Journal of Mechanical Design, vol.114, pp.166-173.

(7) J. P. Coyette, H. Wynendaele and M. Chargin, 1993, "Evaluation of global acoustic sensitivities using a combined finite element/boundary element", Noise-Con 93 (INCE/USA) in Willamsburg, USA, May2-5.

(8)J. Lee, S. Wang, 2003 , "Shape design sensitivity analysis for the radiated noise from the thin body", Journal of Sound and Vibration

(9) N.H. Kim, J. Dong, K.K. Choi, N. Vlahopoulos, Z-D. Ma, M.P. Castanier, C. Pierre, "Design Sensitivity Analysis for a Sequential Structural-Acoustic Problem," Journal of Sound and Vibration, Vol. 263, No. 3, 2003, pp. 569-591.

(10) E. Haug, K.K. Choi, V. Komkov, 1986, Design Sensitivity Analysis of Structural Systems, Academic Press, Inc..