Simplification of the Flexural Capacity of SFR-UHPCC Rectangular Beam

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ABSTRACT

In this paper, flexure behavior of steel fiber reinforcement ultra high performance cementations composites (SFR-UHPCC) has been analyzed by equivalent stress block. Pulling-out tensile force of steel fiber with concrete matrix was induced. An appropriate flexure evaluation formula, i.e. semi-analytical formula, was established based on rectangular cross section beam for comparing with shear capacity and ultimate load of SFR-UHPCC beam. Finally, the semi-analytical formula has been simplified for the convenience of design work. Experimental results and theoretical shear strength are shown to compare with the formula proposed by this paper. The theory formula has a good prediction of failure type of SFR-UHPCC.

1. Introduction

Several outstanding papers appeared recently on the research of steel fiber reinforcement ultra high performance cementations composites(SFR-UHPCC). The mechanical behavior and application of SFR-UHPCC has been a perplexing problem for researchers and engineers more and more. Numerous tests have been carried out to investigate it's mechanics performance (Ashour et al. 1992; Elzanaty et al. 1985; Shuaib et al. 1986; A. K. Sharma 1985; R. Narayanan et al. 1986; Samir A. Ashour 1992; Gabrielsson 1993; Kim et al. 1993; Mahmoud Imam et al. 1994)[1][2][3][4].

However, not all questions concerning the mechanical behavior and technology of this kind composite material are already answered. Rectangular cross section beam is one of the basic shapes of concrete structure. Ordinary speaking, there are also several failure types of SFR-UHPCC beam, i.e. shear failure, flexural failure, flexure-shear failure, etc.

The main aim of this paper was to establish an appropriate flexure evaluation formula based on rectangular cross section beam for comparing with shear capacity and ultimate load of SFR-UHPCC beam. By analysis of stress block, several equivalent coefficients have been assumed according to balanced beam theory. Semi-analytical formula of flexural loading capacity was derived here. From empirical evaluation of stress block, semi-analytical formula has been simplified for convenient of design work. Moreover some experimental results and shear strength from proposed formulae are shown to compare with the formula proposed by this paper. The theory formula has a good prediction for rectangular SFR-UHPCC beam flexure failure.

2. Theoretical analysis of SFR-UHPCC rectangular beam without stirrup

2.1. Exact flexural capacity analysis

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At first, stress distributions in under balanced SFR-UHPCC beam have been assumed as two stress blocks which are shown in Fig.1. Equilibrium equation can be obtained.

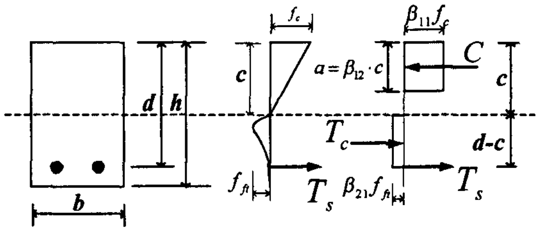
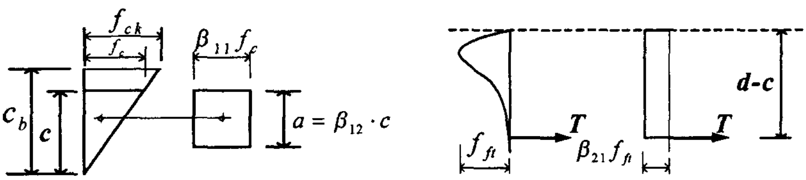


Fig. 1 Stress analysis of cross section

2.1.1 Equivalent coefficients of stress block

Equivalent coefficients of compression stress block can be obtained exactly from basic concrete theory shown in Fig.2(a). Equivalent coefficient of tensile stress block can be obtained approximately from basic concrete theory shown in Fig.2(b).



(a) Compression zone (b) Tensile zone

Fig.2 Stress block equivalent analysis

Here, $\beta_{11}=0.75, \beta_{12}=0.65$ and $\beta_{21}=0.85$. By the reason of flexural cracking displacement, the stress below reinforcement steel is ignored and the height of equivalent stress block equals d-c.

Tensile force f_{ft} is mainly induced by pulling-out force occurred between steel fiber and concrete matrix and can be expressed as [5]

$$f_{ft}=\frac{8L_f V_f d_f \tau_{fm}}{D_f(L_f+1)} \tag{1}$$

In which, V_f is steel fiber content d_f is bond factor which equals to 0.50 for smooth fibers, 0.90 for deformed fibers, and 1.00 for hooked fibers τ_{fm} is the interfacial shear strength between fiber and matrix D_f is fiber diameter and L_f is the fiber length

2.1.2 Equilibrium equation

$$\sum N = T_s - C + T_c = 0 \Rightarrow A_s f_y - \beta_{11}\beta_{12} f_c cb + \beta_{21} f_{ft}(d-c)b = 0 \tag{2}$$

$$f_c : f_{ck} = c : c_b \quad \Rightarrow \quad f_c = f_{ck}c/c_b \tag{3}$$

In which [6]

$$c_b = \frac{0.004}{0.004+f_y/E_s} = \frac{8000 \cdot d}{8000+f_y}$$

By substituting (3) into (2), equilibrium equation can be written as

$$L \cdot c^2 + N \cdot c + M = 0 \tag{4}$$

Here

$$L=\beta_{11}\beta_{12}f_{ck}b/c_b,\ N=\beta_{21}f_{ft}b \text{ and } M=-A_s f_y-\beta_{21}f_{ft}db \tag{5}$$

$$c = \left(-N + \sqrt{N^2 - 4LM}\right)/(2L) \tag{6}$$

2.1.3 Flexural loading capacity solution

$$M_n = A_s f_y(d - 0.5\beta_{12}c) + 0.5\beta_{21} f_{ft} b(d-c)(d + c - c\beta_{12}) \tag{7}$$

Where, c can be solved from expression (6). Then, flexural loading capacity can be expressed as

$$P = 2M_n/(\lambda \cdot d) \tag{8}$$

Here, λ is the shear span ration of SFR-UHPCC beam.

2.2 Simplification of the exact flexural capacity of SFR-UHPCC

By the reason of inconvenient the solution process of c for engineering design, stress distribution in compression zone is simplified with f_c substituted by f_{ck} here. And the flexural moment expression is revised as

$$M_n = A_s f_y(d - 0.5a) + 0.5\beta_{21} f_{ft} b(d - a/\beta_{12})(d - a + a/\beta_{12}) \tag{9}$$

In which

$$a = (A_s f_y + \beta_{21} f_{ft} db)/(b\beta_{21} f_{ft}/\beta_{12} + b\beta_{11} f_c) \tag{10}$$

One reductive coefficient $\eta = 0.8$ is considered here as a safety design factor. Then, flexural loading capacity can be expressed as

$$P = 2\eta M_n/(\lambda \cdot d) \tag{11}$$

3. Comparison between test results and theoretical results

Ten beams have been tested here. Some of factors of beam and steel fiber are L_f=13mm, D_f=0.2mm, d_f=0.5, b=125mm and h=250mm, d=220mm, and other factors are shown in Tab.1. Comparisons between test results, theoretical results and shear strength of the ten test SFR-UHPCC beams are shown in Fig.3.

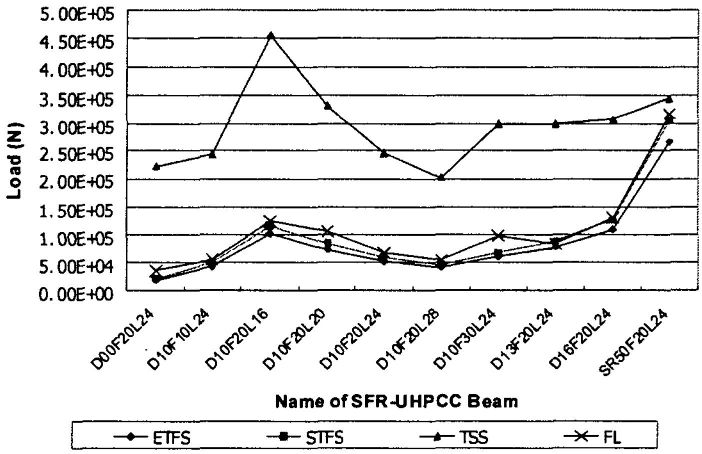


Fig.3 Comparison analysis between test results and theoretical results of rectangular beam

Table 1. Comparison between test and theoretical flexural loading capacity of the ten beams

	SFRC-UHPCC Beam	Some Coefficients				Test Result (N)	Theoretical Solution (N)		
		a/d	L	A_s	f_{ck}	FL	Flexural Strength		TSS
							ETFS	STFS	
1	D00F20L24	3.5	2400	0	146	3.61E+04	1.93E+04	2.15E+04	2.24E+05
2	D10F10L24	3.5	2400	142.6	144	5.64E+04	4.39E+04	5.09E+04	2.45E+05
3	D10F20L16	1.8	1600	142.6	146	1.24E+05	1.03E+05	1.17E+05	4.56E+05
4	D10F20L20	2.5	2000	142.6	146	1.06E+05	7.41E+04	8.42E+04	3.31E+05
5	D10F20L24	3.5	2400	142.6	146	6.85E+04	5.29E+04	6.02E+04	2.48E+05
6	D10F20L28	4.5	2800	142.6	146	5.49E+04	4.12E+04	4.68E+04	2.03E+05
7	D10F30L24	3.5	2400	142.6	152	9.76E+04	6.18E+04	6.93E+04	2.99E+05
8	D13F20L24	3.5	2400	253.4	146	8.39E+04	7.85E+04	8.92E+04	2.99E+05
9	D16F20L24	3.5	2400	397.2	146	1.31E+05	1.11E+05	1.26E+05	3.07E+05
10	SR50F20L24	3.5	2400	1161	146	3.18E+05	2.68E+05	3.04E+05	3.43E+05

In which, FL means test failure load, ETFS means exact theoretical flexural strength, STFS means simplified theoretical flexural strength, and TSS means theoretical shear strength.

4. Conclusion

(1) There are large difference almost $4\sim8$ times between Shear strength from proposed theoretical formula and failure load of test. This point proves that the failure models of the rectangular beams were not in shear, but in flexure. This is also corresponding to the test investigation. In certain degree, the exact theoretical flexural strength formula is reliable.

(2) Numerical results from simplified theoretical flexural strength formula are almost close to test failure load, and more safely comparing with the results from the exact theoretical flexural strength formula. This point indicates that the reductive coefficient $\eta=0.8$ is reasonable and the simplified formula is reliable to evaluate flexural strength of SFR-UHPCC beam because it has no complicate derivation and can be used conveniently.

(3) The Tensile force formula of f_{ft}proposed in this paper is reliable. It is mainly induced by the pulling-out force between steel fiber and concrete matrix. This formula can be also used to analysis other behaviors of SFR-UHPCC structure.

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