Procedure of drawing fragility curve as a function of material parameters

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ABSTRACT

Generally, fragility curve has been used in predicting failure of structures due to seismic actions. In this research, the method of drawing fragility curve has been applied to evaluating success/failure of structures and satisfactory/unsatisfactory of concrete mixture performance based on material parameters. In the paper, a detailed explanation of the procedure of drawing fragility curve based on material parameter has been introduced. Fragility curve generating procedure includes generation of virtual data points from limited number of actual data points by bell curve implementation, determination of success/failure status of each data point by assigned criterion, and completion of final fragility curve. For practical applications, workability of concrete mixture content based on "unit water" has been used to obtain fragility curve. Detailed explanation of fragility curve drawing procedure for material parameters is presented.

1. Introduction

Generally, a fragility curve is used as a probabilistic way of assessing the vulnerability of a bridge or any structure under a seismic event. It is presented as a conditional probability of meeting or exceeding some limit state (i.e., collapse) for a given some ground motion intensity (i.e., peak ground acceleration (PGA)). In this research, the same methodology used to develop a fragility curve for seismic performance has been applied to concrete material or construction performance based on material parameters. More specifically, a fragility curve evaluating the probability of success based on the material parameters is drawn. A detailed explanation of the procedure of developing the fragility curve is described and the example is shown.

Generally, the whole procedure includes three main parts: ① generation of virtual data points from limited number of actual data points by bell curve implementation, ② determination of success/failure status of each data point by assigned criterion, and ③ completion of final fragility curve. A practical example is used to verify the material parameter used fragility curve. A fragility curve for the workability of concrete mixture based on "unit water" has been performed.

2. Fragility curve for "unit water"

A detailed example explaining the procedure of drawing a fragility curve based on the parameter "unit water" is discussed. For drawing the fragility curve, two types of original datum have been used: (1) water-cement ratio "w/c" and (2) "unit water" (w). In this example, "w/c" and "unit water" are the parameters influencing the workability of concrete mixture content as "slump"

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measurement. A criterion of slump is used to determine the success/failure status of the workability of the concrete mixture content.

2.1. Generation of virtual data points

Fragility curve is a statistical method of deciding the success/failure of a chosen parameter. Therefore, abundant experimental data points are required for rigorously calculated results. However, the experimental data are generally insufficient for rigorous calculation of fragility curve. Therefore, in order to overcome this limitation, virtual datum are generated from limited number of actual data points by using bell curve implementation. To draw a bell curve, two parameters are needed, mean value and standard deviation. The original data used in this example are shown in Table 1.

Using the original data, the mean value and the standard deviation are calculated. The mean value and standard deviation calculation of "unit water" are as follow.

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{3 \times (155 + 165 + 175 + 185)}{12} = 170 \quad (1)$$

$$\delta = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}} \quad (2)$$
$$= \sqrt{\frac{3 \times ((155-170)^2 + (165-170)^2 + (175-170)^2 + (185-170)^2)}{12}} = 11.18$$

The data points for the bell curve are generated using mean value and standard deviation of 170 and 11.18, respectively. Table 2 shows the virtual data obtained form bell curve (Fig. 1). In Fig. 1, x-axis represents "unit water" (w) and y-axis represents the number of virtual data points generated for each "unit water" (Number).

Table 1 Data of "w/c" and "unit water"

w/c (%)	Unit Water (kg/m3)
40	155
	165
	175
	185
50	155
	165
	175
	185
60	155
	165
	175
	185

Table 2 "w" and number of data points

w	number
131.5	0
132.5	1
133.5	1
134.5	2
···	···
205.5	2
206.5	1
207.5	1
208.5	0
total	9958

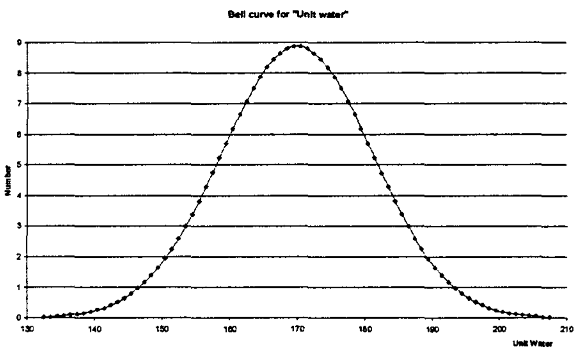

Fig. 1 Bell curve of "unit water"

The same procedure of generating virtual datum of "unit water" has been used to generate "w/c" datum. However, since the bell curve generated data points are based on mathematical equation, the generated data points may lie outside of the physically possible range. Since the data points existing outside of the physically possible range are unusable, they are eliminated in the next analysis stage of drawing fragility curve. Table 3 shows the generated virtual data points for "w/c".

Table 3 Selected data of "w", "w/c"

w	number	w/c	number
132.5	1	40.5	248
133.5	1	41.5	284
134.5	2	42.5	320
···	···	···	···
205.5	2	57.5	320
206.5	1	58.5	284
207.5	1	59.5	248

The next step is to combine these two groups of data points as combinations. Here, for combining "w" data points with "w/c" data points, the total number of "w" is set equal to the number of each "w/c" value. For example, when "w/c" is equal to 40.5, the number of data points is 248. So the bell curve for "w" is redrawn with the total number of "w" data points as 248. This procedure can be described as calibrating the curve data points using the equation of $\text{recalculated number of } w = \frac{\text{original number of } w}{\text{total number of } w} \times \text{number of each w/c value}$. When w is equal to 132.5, calibrate number of "w" is $\frac{1}{1+1+2+\cdots+2+1+1} \times 248 \approx 0$. By using the calibration of data points, all of "w/c" data points have corresponding "w" combinations. Table 4 shows the complete (w/c, w) combinations.

2.2. Determination of success/failure status of each data point by assigned criterion

The next step in the procedure is to check the status of data points by applying a predetermined thrash-hold criterion, success or failure. In this case, "w/c" and "w" should be checked with slump criterion to check whether (w/c,w) combination satisfies the success/failure status of slump criterion. In this example, the experimentally obtained curves of slump based on w/c and w (Fig. 2) and the thrash-hold value of 15cm slump are used as the slump criterion.

Table 4 Combined data

w/c	w	number
40.5	132.5	0
40.5	133.5	0
40.5	134.5	0
···	···	···
41.5	157.5	5
41.5	158.5	6
41.5	159.5	7
···	···	···
59.5	205.5	0
59.5	206.5	0
59.5	207.5	0

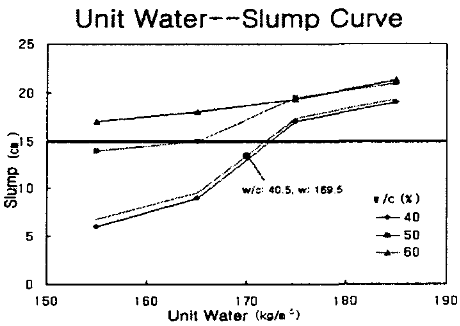

Fig. 2 "w/c" and "w"-Slump curve

Table 5 success/failure status of "w"

w	number of data points	s or f
139.5	1	1
140.5	1	1
140.5	1	1
140.5	1	1
···	···	···
198.5	1	0
198.5	1	0
198.5	1	0

In this criterion, (w/c,w) data points above the 15cm line are success, and the points below the line are failure. For example, when "w/c" is equal to 40.5 and "w" is equal to 169.5, the slump is approximately 13cm, which lies under the line. Therefore, for this (w/c,w) data point, it is determined as failure. In determination of success/failure, "0" stands for success, and "1" stands for failure. This process of checking success/failure are performed for all of data points. Table 5 shows the completed result of success/failure determination for "w".

2.3. Completion of final fragility curve

To calculate the probability, the cumulative log normal distribution method is used. "LOGNORMDIST" command in "MS Excel" is used to calculate the value of cumulative lognormal distribution. The function requires the three parameters: parameter value, mean value, and standard deviation. In our case, x is the parameter "w", "w" data points number, and the success/failure status ("0" or "1") is inputted to obtain the mean value and standard deviation using a computer program that calculates log normal distribution. Fig. 3 is the fragility curve drawn using obtained mean value and standard deviation. X-axis is "w" and y-axis is the "probability" of success/failure of the task.

Table 6 Probability of "w"

w	probability
140.5	0.994142228
140.5	0.994142228
...	...
162.5	0.469276278
162.5	0.469276278
...	...
198.5	0.000130864
198.5	0.000130864

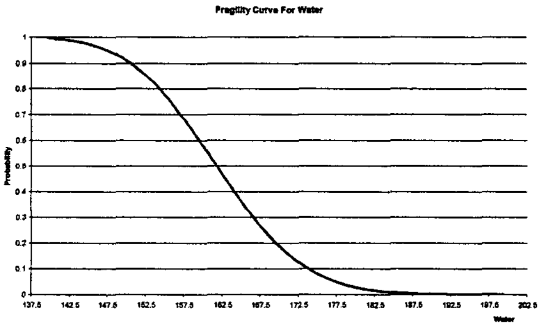

Fig. 3 Fragility curve for "w"

3. Conclusion

The procedures of drawing fragility curve has been applied to concrete material quality success/failure determination as a function of material parameter. The fragility curve is developed by generating virtual data points, determination of success/failure status of data points by using given criterion, and completion of final fragility curve. The example shows that fragility curve method used for a structure performance under earthquake loading can be applied to concrete material quality performance determination based on material parameter.

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Reference

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