Elemental image compatibility between parallax generation and Integral Imaging system for three-dimensional display

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Abstract

We have studied elemental image compatibility between integral imaging(II) and parallax generation(PG) system for three-dimensional display. The elemental images of PG can be obtained by recombination of the elemental images picked up in II system. The theoretical verification and the experimental results show that the elemental images of PG are in correspondence with the elemental images of II system with proper transformation conditions.

1. Introduction

In recent years, many people are taking interest in 3D display which will provide better reality and efficiency for recognizing visual information. There are attractive approaches to the 3D display devices using various methods, such as holography, volumetric after-imaging, PG, and II[1] etc. However, most of these methods are still not in practical stage except PG. In the case of holography, it requires super high resolution of spatial light modulator which has not been developed yet. In addition, it is not clear whether the pick up and display device can be achieved to sufficient high resolution in near future. Volumetric after imaging produces 3D image by moving or rotating screen, variable focal lens or mirror, or scanning a fluorescent spot. Although the 3D images produced by volumetric after imaging appears with high reality, its display area can not be large enough due to the limitation of mechanical moving parts. In order to overcome these obstacles, head tracking and multi-view system have been widely studied[2,3]. While head-tracking method is one of the perfect methods for a single user, it is not able to accommodate large number of viewers and generation of parallax. Multi-view system is also an attempt to increase the view points, not only to provide larger 3D image view angle but also to create more natural view points. Integral imaging produces 3D image into space by using lens, pinhole or slit arrays. Recently, it is one of the attractive methods due to its unique advantages such as full parallax, full color, and continuous perspectives etc.[4-6]. However, II is not ready for practical applications, since elemental image requires large number of pixels to represent the 3D image with adequate resolution. The method for most practical application being applied nowadays would be PG. It constructs 3D image by generating parallax to each eyes of the viewer with lenticular lenses or parallax barriers[7]. It is more practical than other methods due to the simple mechanism for implementation, being able to apply with conventional 2D display devices, and not requiring super high resolution devices. However, PG also has several unavoidable disadvantages, such as discrete view points and strain to the eyes of the viewer due to the discontinuity of images. It might have been assumed that with increase in number of view points, 3D image from PG system is same with that from II system. If PG and II system are mutually compatible either in extreme view points or in normal case, PG will have significant advantages, which are simplifying the estimation process for the displayed image and several properties of interest. In this manuscript, we have discussed the relationship between PG and II methods. The theoretical treatments and experimental results to provide comparability between PG and II system are presented and discussed.

2. Relationship between elemental images in PG and II system

A. Matching condition

In pick up and reconstruction process of II, the relation between a point on the object and the corresponding point on elemental image is given as a linear equation commonly passing through two points. In this case, all object points on the line are degenerated to the one point on elemental image. On the contrary, one point on elemental image should be spread out to the 3D images in the reconstruction process. Figure 1 shows a basic geometry for elemental image pick-up process of PG and II. If x_{en} represents the n-th image on x-coordinate, the relationship between a point (x_0, z_0) on the object and the corresponding point (x_{en}, s_0) on the elemental image is given by

$$x_{en} = \left(1 - \frac{s_0}{z_0}\right)\sum_{i=0}^{n} P_i + \frac{x_0 s_0}{z_0}, \qquad (1)$$

where s_0 is the distance between elemental image plan and the lens array. In general, lens array for II system is composed of single-size lenses.

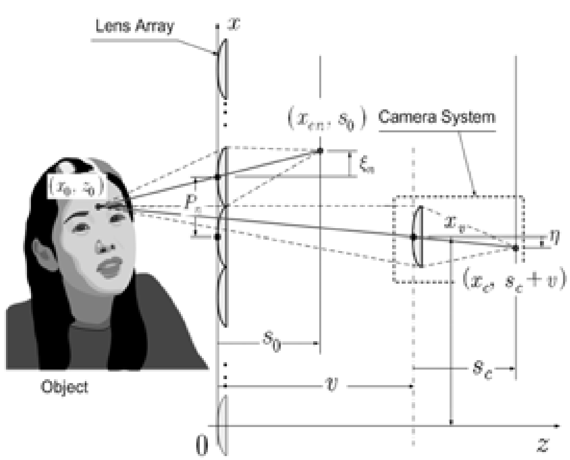

Fig. 1. Pick-up process for II and PG

However, mixed lens array, which is composed of various size lenses, can be useful for special applications. Assuming mixed lens array, P_n is the distance between *n*-th and (*n-1*)-th elemental lens and $\sum_{i=0}^{n} P_i$ shows the center location of *n*-th elemental image on x-axis. In the pick up process in PG, 3D information can be obtained with two 2D images from two different perspectives. The response relation between a point object and a responding point $(x_c, s_c + v)$ on pick-up plane can be described as

$$x_c = \left(1 - \frac{s_c}{z_0 - v}\right)x_v + \frac{x_0 s_c}{z_0 - v}, \qquad (2)$$

where the imaging lens has a difference of distance v from the object. s_c is a distance between imaging lens and the pick up plane. x_c represents the elemental lens location on the x-axis. x_v is a distance between lens array and the imaging lens. If we assume that P_{ck} is a distance between *k*-th camera and *(k-1)*-th camera in PG, x_v becomes $\sum_{i=0}^{k} P_{ci}$. Assuming ξ_n as the location of the elemental lens, x_{en} becomes $\sum_{i=0}^{n} P_i + \xi_n$. With this expression, a corresponding relation between the elemental images, S'_{pc} in II and S_{pc} in PG can be represented as

$$lS'_{pc} = -\frac{s_0}{z_0}\sum_{i=0}^{n} P_i + \frac{x_0 s_0}{z_0}, \qquad (4)$$

$$mS_{pc} = -\frac{s_c}{z_0 - v}\sum_{i=0}^{k} P_{ci} + \frac{x_0 s_c}{z_0 - v}, \qquad (5)$$

where *l* and *m* represent the number of pixels apart from center pixel of the elemental lens. In real imaging system, the pixels have a finite size. For a simple discussion, however, we assume that the pixel is pixel point which has no size. From Eq.(4) and (5), the elemental images in PG and II are identical in principle except the difference in point of view. The II system can provide large number of view points, while PG provide limited number of view points which is limited by the camera numbers. In order to continue detailed discussion, it is necessary to understand the relationship between dot pixel on the object and the corresponding dot on the reconstructed image in PG. Figure 2 shows the variables to explain the reconstruction process of PG. The lines show the corresponding relationship between a dot on object and a light beam corresponding to one pixel on an elemental image. In the recombination process, the pixel should be moved to new location given by

$$x_d = \gamma S''_{pc} + \sum_{i=0}^{\beta} P_{di}\,, \qquad (6)$$

where γ, β, S''_{pc}, and P_{di} are corresponding to each pixel's address number, elemental image's address number, spacing between the pixels, and spacing between *i*-th and *(i-1)*-th elemental images. Each neighboring pixel in recombined image is selected from different elemental images. When selected pixels from an elemental image are arranged in order, selection order can be denoted as γ and β. Therefore, the function represented the order can be denoted as $f(\beta)$ and $g(\gamma)$.

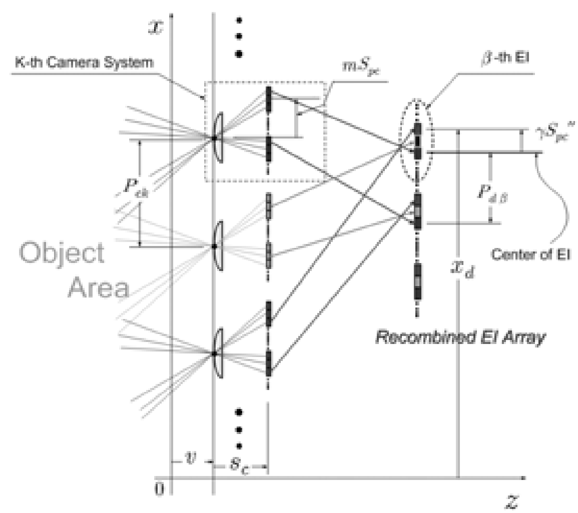

Fig. 2. Recombination of elemental images in PG system.

$$f(\beta)S_{pc} = -\frac{s_c}{z_0 - v}\sum_{i=0}^{g(\gamma)} P_{ci} + \frac{x_0 s_c}{z_0 - v} \qquad (7)$$

In order to compare the arrangement of the elemental images obtained for II and PG, it is necessary to compare term by term relationship between $x_{en} = lS'_{pc} + \sum_{i=0}^{n} P_i$ and x_d. In Eq.(6) S''_{pc} and P_{di} are optional variables that can be controlled in a reconstruction process. Therefore, the comparison is effective when γ and β in PG systems and l and n in II are compared in order. For simple comparison, $f(\beta)$ and $g(\gamma)$ are reduced as β and $\tau - \gamma$, respectively, where τ is the total number of elemental image and $\tau - \gamma$ means that the images are selected in reverse order. Regardless of the object's location (x_0, z_0), the selection order of the elemental images in II and PG must be satisfied. Therefore, to be independent of x_0, the expression can be written as

$$0 = z_0\left(\frac{\beta S_{pc}}{s_c} - \frac{lS'_{pc}}{s_0}\right) + \frac{\beta S_{pc} v}{s_c} - \sum_{i=0}^{\tau-\gamma} P_{ci} + \sum_{i=0}^{n} P_i\,. \qquad (8)$$

The requirement for Eq.(8) to be independent of z_0 can be obtained from the following expressions:

$$\sum_{i=0}^{n} P_i + \frac{\beta S_{pc} v}{s_c} = \sum_{i=0}^{\tau-\gamma} P_{ci}\,, \qquad (9)$$

$$\frac{\beta S_{pc}}{s_c} = \frac{lS'_{pc}}{s_0} \qquad (10)$$

Therefore, Eq.(9) and (10) must be satisfied for all of γ, β, l and n in sequence. For a pixel on the elemental image, when γ is fixed, the Eq.(9) and (10) must be satisfied with any n and β in sequence. Assigned pixel address for the image in pickup process of PG corresponds to the elemental image address of the II. With the correspondence of elemental image of II, total number of β and n have to be the same for all the pixels in PG system's γ -th image. For one to one correspondence of the pixels in PG and II, the number of pixels in II has to be the same as number of total images τ in PG. In this case, the elemental images in PG can be partially included in that of II. It is the results of corresponding Eq.(9) and (10) which should be valid for all γ. By using equation (10), $\beta S_{pc} / s_c$ in equation (9) can be transformed to lS'_{pc} / s_0. Hence the following expression has to be realized:

$$\sum_{i=0}^{n} P_i + \frac{lS'_{pc} v}{s_0} = \sum_{i=0}^{\tau-\gamma} P_{ci} \qquad (11)$$

The pixel address of elemental images in II corresponds to the address of the cameras in PG contrary to the case of Eq. (9). Equation (9), (10) and (11) are matching condition between II and PG, because the elemental images in II and PG can be matched pixel by pixel using these equations. From the matching condition, we can see the elemental image of II is identical with PG's, when PG is located at distance v apart from II. Therefore, the depth of the reconstructed image can be controlled by using recombination of the elemental images with the matching condition.

B. Identical condition

Regardless of n and β, Eq.(9) must be satisfied, Therefore, the following equation can be obtained by substituting $n-1$ and $\beta-1$ to n and β, respectively:

$$P_n = -\frac{S_{pc}v}{s_c} \qquad (12)$$

By the same way for l and γ, Eq.(11) is reduced to

$$P_{c(\tau-\gamma)} = \frac{S'_{pc}v}{s_0}. \qquad (13)$$

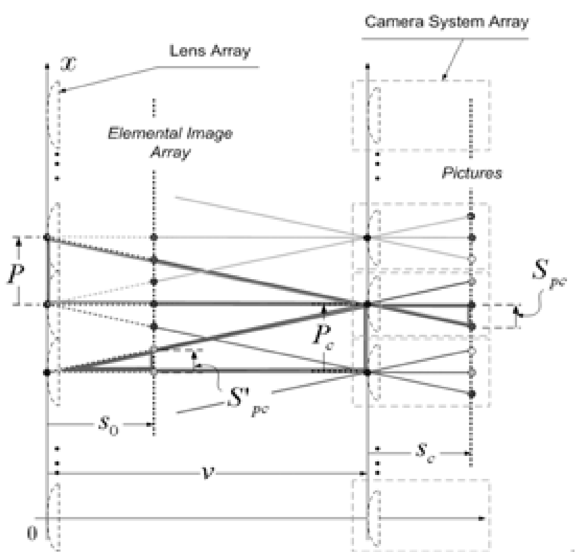

Fig. 3. Diagram representing the geometrical relations for the identical conditions.

Regardless of n and γ, Eq. (12) and (13) must be satisfied simultaneously, which means that P_n and $P_{c(\tau-\gamma)}$ should be constant. Assuming ideal system with unlimited number of lenses, the following expressions are the condition that the elemental images in II and PG make one to one correspondence:

$$\begin{cases} P = -\dfrac{S_{pc}v}{s_c} \\ P_c = \dfrac{S'_{pc}v}{s_0} \end{cases} \qquad (14)$$

Fig. 3 shows schematic diagram for the conditions expressed in Eq. (14). The small and large triangles with thick solid line denote the proportional relations among the variables in Eq. (14). From the diagram, it is possible to find nonlinear solutions as shown in Fig. 4.

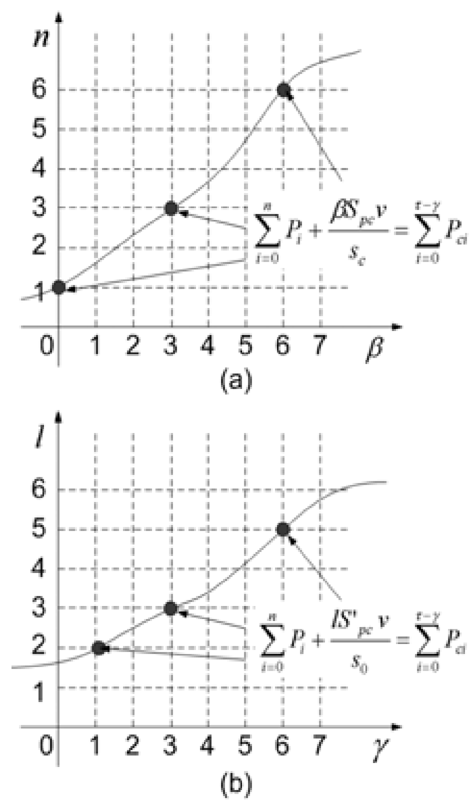

Fig. 4. Conceptual diagram for plotting Eq. (9) (a) and (11) (b) on (β, n) and (γ, l) plane.

In Fig. 4, Eq.(9) and (11) are plotted on (β, n) and (γ, l) plane, respectively. Supposing that the pixels on elemental image of II and PG have a size, the solution points marked in diagram can be considered as pixel area. If the pixel area is big enough to cover the vicinity of the pixels, the solution is one of the identical conditions under the distortion terms. In the large number of n, β, γ and l, the solution is closed on the plan. If the pitch P_i, P_{ci}, S'_{pc} and S_{pc} are small and the numbers n, β, γ and l are sufficiently large, $\sum_{i=0}^{n} P_i$, $\sum_{i=0}^{\tau-\gamma} P_{ci}$, lS'_{pc} and βS_{pc} can be supposed as continuous and the pixel relations in elemental images of II and PG can be considered as identical under the distortion terms. From the matching conditions and the identical conditions for PG and II, it is clear that the elemental images in PG and II are obviously compatible.

3. Experiment and the results

Experiments to conform the relationship between elemental images in PG and II are performed with the arrangement shown in Fig. 5. The elemental images for II are picked up using lens array and recombined for PG. In Fig. 5, the elemental image array is picked up by the camera moving with 40 steps with 4mm interval as one-dimensional array. Therefore, the pick up elemental images could be considered as 40 elemental images for II or 40 views for PG. For the recombination of elemental images, which can be used in PG system, matching conditions are applied to the image selection.

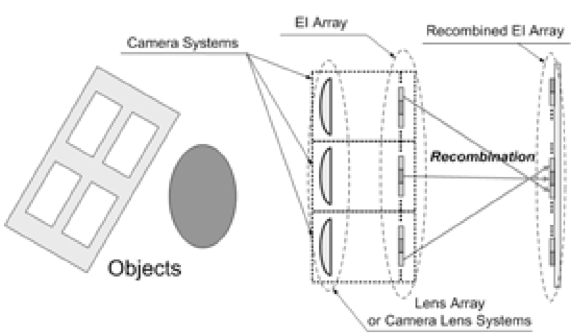

Fig. 5. Experiment scheme for pick up and recombination process for elemental image array.

Figure 6(a) shows the objects which are used in the experiment. The objects are composed of a circle located at 5cm apart from the camera and an inclined square window located at 15cm apart from the camera. Figure 6(b) and (c) show elemental image array and its recombination, respectively. Figure 6(d) and (e) present enlarged images of dotted rectangular line in Fig. 6(b) and (c), respectively. As shown in Fig. 6(d) and (e), the recombined elemental images are identical with original one, when we compare two images in due consideration of horizontal size reducing.

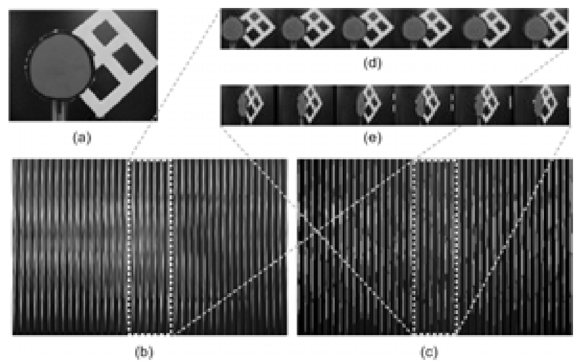

Fig. 6. Comparison between original elemental image(EI) and recombined elemental image. (a) objects, (b) EI array, (c) recombined EI array; (d) and (e) are the images in the dot line of (b) and (c) respectively; (d) and (e) are resized with horizontally magnified and vertically reduced.

4. Conclusion

We have carried out studies on compatibility of the elemental images of II and PG for practical 3D display system. From the analysis, it is cleared that the elemental images for PG is in correspondence with the one for II. In addition, the elemental images in PG could also be generated by recombination of the elemental images in II on the specific condition. The results are useful to develop and enhance 3D display system using lens or slit array system.

5. References

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