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Reliability P(Y<X), Ratio X/(X+Y), and a Skewed-Symmetric Distribution of Two Independent Random Variables

Jungsoo Woo[1)]

Abstract

We shall consider an inference of the reliability P(Y<X), distributions and moments of the ratio X/(X+Y), and derive a skewed-symmetric distribution and its moment for two independent random variables X and Y. We shall also consider an exponentiated distribution and a complementary distribution of a given continuous distribution.

1. Reliability

Fact 1. Let X and Y be two independent random variables with its density $f_X(x;\theta_1)$ and $f_Y(x;\theta_2)$,respectively. where θ_1 , θ_2 are the scale parameters. Then the reliability P(Y<X) is a function of $\rho \equiv \frac{\theta_1}{\theta_2}$ or $\theta_1 \cdot \theta_2$ only when X and Y have the same distributions or different distributions

Fact 2 (McCool(1991)). If the reliability P(Y<X) is a monotone function of $\rho \equiv \frac{\theta_1}{\theta_2}$ or $\theta_1 \cdot \theta_2$, then an inference on reliability P(Y<X) is equivalent to an inference on $\rho \equiv \frac{\theta_1}{\theta_2}$ or $\theta_1 \cdot \theta_2$.

Reference. McCool, J.I.(1991), Commun. Statist.-Simula.,, 20(1), 129-148

Problem 1. Assume the reliability P(Y<X) is a function of ρ_1, ρ_2 . If the reliability P(Y<X) is a monotone function of each ρ_i, i=1,2 then an inference on

1) Professor, Department of Statistics, Yeungnam University, Gyeongsan, 712-749, Korea

the reliability is equivalent to each ρ_i, i=1,2 , where each ρ_i, i=1,2 can't depend each other.

Example. Let X, Y and Z have independent exponential distributions with means $\theta_1, \theta_2, \theta_3$,respectively. Then $P(X<Y<Z)=\frac{1}{1+\rho}-\frac{1}{1+\rho+\eta}$, where $\rho=\frac{\theta_2}{\theta_1}$ $\eta=\frac{\theta_2}{\theta_3}$.

Published Papers

1. Manisha Pal, M. Masoom Ali, and Jungsoo Woo(2005), Estimation and testing of P(Y>X) in two-parameter exponential distribution, Statistics(영국), 39-5 ,415-428

2. M. Masoom Ali & Jungsoo Woo(2005), Inference on reliability P(Y<X) in a power function distribution, Journal of Statistics & Management Systems(인도), 8-3, 681-686

3. M. Masoom Ali and Jungsoo Woo(2005), Inference on Reliability P(Y<X) in the Levy Distribution, Mathematical and Computer Modelling(미국) 41, 965-971

4. M. Masoom Ali and Jungsoo Woo(2005), Inference on P(Y<X) in a Pareto Distribution, J. of Modern Applied Statistical Methods(미국) 4-2, 583-586

5. M. Masom Ali, Manisha Pal and Jungsoo Woo(2005), Inference on P(Y<X) in Generalized Uniform Distributions, Calcutta Statistical Association Bulletin 57,(인도) march & June, 35-48

6. M. Masoom Ali and Jungsoo Woo(2005), Inference on Reliability P(Y<X) in a p-Dimensional Rayleigh Distribution, Mathematical and Computer Modelling(미국) 42, 367-373

7. M. Masoom Ali, Jungsoo Woo and Manisha Pal(2004), Inference on Reliability P(Y>X) in Two-Parameter Exponential Distribution, International Journal of Statistical Sciences(뱅그라데시), 3, 119-125

Unpublished Papers (submitting papers by coworkers, M. M. Ali ,)

1. UMVUE of Reliability in Discrete Distributions.
2. Estimation of Reliability P(Y<X) in two different distributions-1 (and 2)
3. Reliability in an alpha-Exponentiated Burr distribution
4. Inference on Reliability in a Burr distribution

5. Inference on reliability P(Y<X) in a Truncated Arcsine Distribution
6. Inference on Reliability in a Generalized Pareto Distribution
7. Inference on reliability P(Y<X) in a Gamma Distribution
8. Estimation of Reliability P(Y<X) in a Weibull-Uniform Distributions
9. Estimation of Reliability in a Levy-Uniform and Exponential distribution

2. Ratio

Let X and Y be two independent random variables each having densities $f_X(x)$ $f_Y(y)$, respectively. Then the distributions and moments of V=X/(X+Y) will be derived when X and Y have several distributions

Reference. Bowman, K.O. and Shenton, I.R.(1998), Distribution of the ratio of Gamma Variates, Commun. Statist.-Simula.,, 27(1), 1-19

Problem 2. Let $X_1, X_2, \ldots, X_n$ be independent random variables each having density$f_{X_i}(x)$, I=1,2,...n. Then we may consider distributions and moments of the ratio $V \equiv X_i/(X_1+, X_2+, \ldots + X_n)$, when $X_1, X_2, \ldots, X_n$ have several distributions

Published papers

1. M. Masoom Ali, Jungsoo Woo and Manisha Pal(2006), Distribution of the ratio of generalized uniform variates, Pakistan J. Statistics(파키스탄통계학회), 22-1, 11-19

2. M. Masoom Ali, Jungsoo Woo, and Saralees Nadarajah(2005), On the Ratio X/(X+Y) for the Power Function Distribution, Pakistan J. Statistics(파키스탄통계학회) 21-2, 131-138

3. M. Masoom Ali, Saralees Nadarajah, and Jungsoo Woo(2005), On the Ratio X/(X+Y) for the Weibull and Levy Distributions, Journal of the Korean Statistical Society, (한국 통계학회) 34-1, 11-20

Unpublished papers(submitting papers by M. M. Ali, coworker)

1. Reliability and Distribution of the ratio in a Mixed distribution
2. Reliability and Ratio in a truncated exponential distribution
3. On Reliability and Ratio of a Beta Distribution
4. On the Ratio in a p-dimensional Rayleigh Distribution

3. A Skewed-Symmetric Distribution

Let X and Y be two independent continuous random variables ech having $X \sim G'(x) = g(x)$, $Y \sim F'(x) = f(x)$, the same (or different) densities which have symmetric about zero.

Since $\frac{1}{2} = P(X - c \cdot Y) < 0) = \int_{-\infty}^{\infty} G(cx)f(x)dx$,

Fact 3. $f(z;c) \equiv 2f(z) \cdot G(c \cdot z)$ is a skewed density of a continue random variable Z.

Especially if c=0, then asymmetric density f(z;c) becomes the original density f(x)

Definition. $f(z;c) \equiv 2f(z) \cdot G(c \cdot z)$ is a skewed density of a continue random variable Z. which is called a skewed-symmetric density of F generated by a kernel G. And especially if G(x)=F(x), then it's called a skewed-symmetric density of f(x).

Fact 3. If the cdf's F(x)=G(x), then

(1) $f_{-Z}(z; -c) = f_Z(z;c)$

(2) $F(z;1) = F^2(z)$

(3) For positive z, $\lim_{c} f(z;c)$ becomes a half-distribution of f(x).

Define $S(z;c) \equiv \int_z^{\infty} \int_0^{ct} f(t)f(s)dtds$, when G(x)=F(x)

Fact 4.Suppose F(x)=G(x). Then we have the following

(1) S(z;c) is a decreasing function of z.

(2) S(z;c)=-S(z;-c)

(3) S(-z;c) =S(z;c)

(4) 2S(z;1)=F(z)F(-z)

Remark 1. From Fact 4, it's sufficient for us to consider S(z;c) only when z and c are positive

Reference. Azzalini, A.(1985), A Class of distributions which includes the normal ones, Scand. J. Statist., 12, 171-178

Problem 3. Let (X, Y) be a bivariate random variable which has a symmetric about the origin. Then we may define a asymmetric distribution and derive its moment and others.

Problem 4. Inference on the parameter c based on a random sample from a skewed distribution based on method of AMLE

Published Paper

1. M. Masoom Ali and Jungsoo Woo(2006), Skewed-symmetric reflected distributions, Soochow J. Math(대만), 32-2 pp. 233-240

2. M. Masoom Ali, Jungsoo Woo and Manisha Pal(2006), Some Skew Symmetric Reflected Distributions, To appear it in American Journal of Mathematical and Management Sciences(미국), American Sciences Press Inc.(ASP)

Unpublished Papers, (submitting papers by M. M. Ali ,coworker)

1. Skew reflected distribution generated by a reflected gamma kernel distribution
2. Skewed distributions generated by a double Rayleigh kernel distribution.
3. Skew reflected distribution generated by a Laplace kernel distribution.
4. On a skew truncated Cauchy distribution.
6. Some skew-symmetric double inverted distributions.
7. On some beta-kernel distributions.

4. An exponentiated distribution and a complementary distribution

Let F(x) be a continuous distribution of a random variable. Then $F^{\alpha}(x)$ is also a distribution of a random variable, which distribution is called an α-exponentiated distribution.

Let F(x) be a continuous distribution of a random variable having its support (0.1). Then $F^{-1}(x)$ is also a distribution of a random variable, which distribution is called a complementary distribution.

References

Gupta, R.D. and Kundu, D.(2001), Exponentiated exponential family, Alternative to Gamma and Weibull, Biometrical Journal 43-1, 117-130

Jones, M.C.(2002), The complementary beta distribution, J. of Statistical Planning and Inference 104, 329-337

Unpublished Papers, (submitting papers by M. M. Ali ,coworker)

1. On α-exponentiated exponential distribution
2. Reliability estimation in α-exponentiated Burr distribution
3. On α-exponentiated complementary truncated exponential distribution
4. Inference on Reliability in α-exponentiated Pareto distribution