Optimal location of overwork-allowed facilities subject to choice of various equipment modes

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Abstract

This paper considers a facility location problem, which is concerned with locating facilities on a supply chain network and installing the associated equipments at the facilities to meet a given set of demands. The objective function is to minimize the sum of setup cost (facility opening cost and equipment installation cost), operation cost, and distribution cost. For the equipments, various choices of equipment modes need to be determined. Moreover, in the problem, overwork is allowed each facility but at expensive operation cost. The proposed problem is characterized as being NP-hard problem, so that a heuristic algorithm is derived. In order to evaluate the performance of the proposed algorithm, computational experiments with various numerical instances are conducted.

Ⅰ. Introduction

Under internet environment, facilities are commonly required to be flexible so as to respond well to a variety of different small-quantity demands. Thus, each facility capacity determination gets vital. In addition, producers are often required to do overwork in order to save costs if it is necessary.

Some reference papers have been studied on the facility location problem (see, for example, Geoffrion, 1974[5] and Akinc, 1997[6]). A capacitated facility location problem with choice of facility type has been analyzed (Lee, 1993[3] and Joseph B., 1999[1]). They did not, however, consider overwork. Joseph and Charles[2] has considered a similar problem, but not derived any algorithm.

This paper considers a facility location problem in which a given set of demands is to be supplied from a set of facility candidates at minimum cost. This paper introduces a new type of facility location model, which takes account of installing an equipment mode with a different capacity on each facility as well as allowing each facility to overwork once at an additional operation cost rate so as to get more flexible.

The organization of the paper is briefed as follows. Chapter 2 presents the problem description and the formulation. In Chapter 3, the solution properties are analyzed, which are used to derive an heuristic solution procedure based on Lagrangean relaxation. Chapter 4 shows the computational results of various numerical instances, and some concluding remarks are made in Chapter 5.

Ⅱ. Problem Description

The proposed problem considers a facility location problem in (M:N) supply chain where both facility location and equipment modes need to be determined (referring to Figure1). There is a set of facility candidates for which various

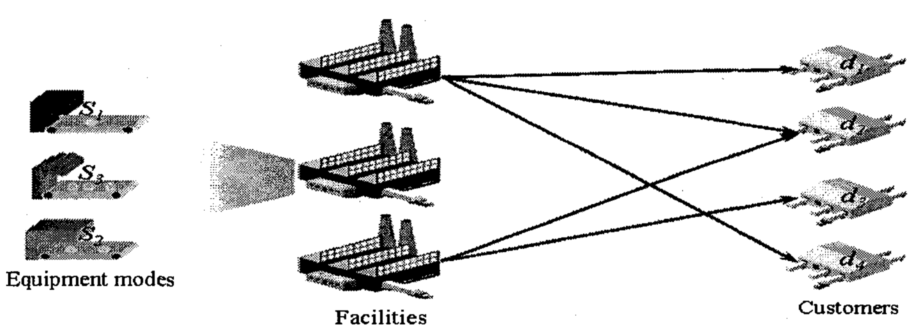

Figure1. Single-echelon Supply Chain with a choice of equipment mode

equipment modes are available. It is assumed that the varieties of the equipment modes at each facility are identical. The goal of the proposed problem is to supply every customer's demand at minimum cost. In order to achieve the goal, some decisions should be made.

The following notation is introduced throughout the rest of this paper.

Parameters :

- I : Set of customer
- J : Set of facility
- K : Set of equipment mode
- d_i : Demand of customer i
- t_{ij} : Delivery cost per unit from facility j to customer i
- o_j : Opening cost of facility j
- e_{jk} : Installation cost of kth mode of equipment at facility j
- c_k : Capacity of kth mode of equipment
- p_j : Operation cost per unit at facility j
- p_j^+ : Overwork Operation cost per unit at facility j

Variables :

- X_{ij} : Fraction of demand of customer i supplied by facility j
- Y_j : 1 if a facility j is selected to be open, or 0 otherwise
- Z_{jk} : 1 if kth mode of equipment is installed at a facility j, or 0 otherwise
- U_j^+ : Output from a facility j above the kth mode of equipment at the installed facility j

The proposed problem can then be formulated as follows.

Problem P :

min

$$\sum_I\sum_J t_{ij}d_iX_{ij} + \sum_J o_jY_j + \sum_J\sum_K e_{jk}Z_{jk} + \sum_J p_jd_iX_{ij} + \sum_J (p_j^+ - p_j)U_j^+ \qquad (1)$$

subject to

$$\sum_J X_{ij} = 1 \qquad \forall i \in I \qquad (2)$$

$$X_{ij} \le Y_j \qquad \forall i \in I,\ \forall j \in J \qquad (3)$$

$$\sum_K Z_{jk} = Y_j \qquad \forall j \in J \qquad (4)$$

$$\sum_I d_iX_{ij} \le \sum_K c_kZ_{jk} + U_j^+ \qquad \forall j \in J \qquad (5)$$

$$X_{ij},\ U_j^+ \ge 0 \qquad \forall i \in I,\ \forall j \in J \qquad (6)$$

$$Y_j,\ Z_{jk} \in \{0,1\} \qquad \forall i \in I,\ \forall j \in J \qquad (7)$$

In the formulation, the objective function (1) is to minimize sum of the variable (delivery and operation) costs and the fixed (facility opening and equipment mode installation) costs. Constraints (2) requires that all of customer i's demand should be met. Constraints (3) requires that each facility j should be equipped first in order to be supplied from it. Constraints (4) requires that at most one equipment mode should be installed at the associated selected facilities from the set K of equipment modes. Constraints (5) then makes sure that the total demand required to produce at each facility j is less than the total amount produced by the facility with the equipment mode installed at. Finally, constraints (6) and (7) are provided for the non-negativity and integrality of the decision variables.

Ⅲ. Solution Procedure

The objective of the proposed problem is to find the optimal locations of the facilities Y_j^*, their optimal equipment mode E_{jk}^*, the optimal fraction of allocation X_{ij}^*, and finally two variables $(U_j^+)^*$ related to the amount of production together, which minimizes the total supply chain cost.

Property 1.

The proposed problem is *NP-hard* problem.

Proof) The capacitated facility location problems are strongly NP-hard[4]. Moreover, the proposed problem is an extension of the facility location problem.

This completes the proof.

Accordingly, a Lagrangean heuristic algorithm is suggested in the paper.

Constraints (3) and (5) in problem P are relaxed by using multipliers μ_{ij} and λ_j for all $i \in I$ and $j \in J$, respectively.

Problem LR :

min

$$\sum_I\sum_J ((t_{ij}+p_j)d_i + \mu_{ij} + \lambda_jd_i)X_{ij} + \sum_J (o_j - \sum_I \mu_{ij})Y_j + \sum_J\sum_K (e_{jk} - \lambda_jc_k)Z_{jk} + \sum_J (p_j^+ - p_j + \lambda_j)U_j^+$$

subject to (2), (4), (6), (7).

This subproblem LR can be decomposed into three subproblems.

Subproblem LR1 :

min

$$\sum_I\sum_J ((t_{ij}+p_j+\lambda_j)d_i + \mu_{ij})X_{ij}$$

subject to

$$\sum_J X_{ij} = 1 \qquad \text{for } i \in I$$

$$X_{ij} \ge 0 \qquad \text{for } i \in I,\ j \in J$$

This subproblem LR1 can be solved in $O(|I|\cdot|J|)$ by finding $j^* = \mathop{Arg\min}_{j\in J}((t_{ij}+p_j+\lambda_j)d_i + \mu_{ij})$ for $\forall i \in I$, and set $X_{ij} = \begin{cases} 1 & \text{if } j \text{ is equal to } j^* \\ 0 & \text{otherwise} \end{cases}$

Subproblem LR2 :

min

$$\sum_J (o_j - \sum_I \mu_{ij}) Y_j + \sum_J \sum_K (e_{jk} - \lambda_j c_k) Z_{jk}$$

subject to

$$\sum_K Z_{jk} = Y_j \qquad \text{for } j \in J$$

$$Y_j \in \{0, 1\} \qquad \text{for } j \in J$$

$$Z_{jk} \in \{0, 1\} \qquad \text{for } j \in J,\ k \in K$$

This subproblem LR2 can be solved in O(|J|·|K|) as follows.

If $(O_j - \sum_I \mu_{ij}) + \min_{k \in K}(e_{jk} - \lambda_j c_k) \geq 0$ for $\forall j \in J$, then $Y_j = 0$ and $Z_{jk} = 0$ for $\forall k \in K$. Otherwise, $Y_j = 1$ and

$$Z_{jk} = \begin{cases} 1 & \text{if } k = Arg \min_{k \in K}(e_{jk} - \lambda_j c_k) \\ 0 & \text{otherwise} \end{cases}$$

Subproblem LR3 :

min

$$\sum_J (p_j^+ - p_j - \lambda_j) U_j^+$$

subject to

$$U_j^+ \geq 0 \qquad \text{for } j \in J$$

Property 2.

The following constraint is valid to the original problem P.

$$\sum_J U_j^+ \leq \sum_I di - \min_{k \in K} c_k$$

Proof) Suppose $\sum_J U_j^+ > \sum_I d_i - \min_{k \in K} c_k$ is satisfied at the optimal solutions.

However the better objective value can be obtained at the optimal solutions when $\sum_J U_j^+ \leq \sum_I di - \min_{k \in K} c_k$ is satisfied, because $\sum_J (p_j^+ - p_j) U_j^+$ is included in the objective function and $(p_j^+ - p_j)$ is always positive. Therefore the supposition is a contradiction, and then the Property2 proves to be true.

This completes the proof.

By adding the constraint in the Property2, the bound of sub problem LR3 can be tightened. Then this sub problem LR3 can be solved in $O(|J|)$ as follows.

If $(p_j^+ - p_j - \lambda_j)$ is always positive for $j \in J$, then $U_j^+ = 0$ for $j \in J$. Otherwise, $j^* = Arg \min_{j \in J}(p_j^+ - p_j - \lambda_j)$ and

$$U_j^+ = \begin{cases} \sum_I d_i - \min_{k \in K} c_k & \text{if } j = j^* \\ 0 & \text{otherwise} \end{cases}$$

A good lower bound is to be obtained from a good set of multipliers, which is known to be a very difficult task in general. One of the most popular methods to select values for the Lagrangean multipliers is known as the subgradient optimization algorithm, which is used in this paper.

Primal Heuristic Procedure (PHP) :

When a solution to the Lagrangean relaxed problem LR is infeasible to the primal problem, this procedure is used to make the solution feasible.

Step0: Classify every possible instances of lower bound solutions into four cases according to the facility.

- Case1 is that a facility is close, however some demands are allocated to the facility.
- Case2 is that a facility is close, and no demand is allocated to the facility.
- Case3 is that a facility is open, however no demand is allocated to the facility.
- Case4 is that a facility is open, and some demands are allocated to the facility.

Step1: Perform the following move operation for the first two cases, Case1 and Case2, respectively.

- Case1 : Compare the costs of two possibilities; The first one is to open the facility and supply the allocated demand, and the second one is to transfer the allocated demand to the other facilities which are already open.
- Case2 : No move is needed.
- If all of Case1 and Case2 are performed, then go to *Step0* and *Step2*; otherwise go to *Step0* and come to *Step1* again.

Step2: Perform the following move operation for the last two cases, Case3 and Case4, respectively.

- Case3 : Close the facility, which imply uninstall the equipment and no U_j^+ is assigned.
- Case4 : No move is needed.
- If all of Case3 and Case4 are performed, then Stop; otherwise go to *Step0* and come to *Step2* again.

Overall Procedure :

The overall procedure is terminated after a specific number, set at the value 1000 in this paper, of iterations.

Step1: Initialize Lagrangean multipliers, at the value 1, and parameters.

Step2: Generate an initial primal feasible solution.

step2-1: Open a facility with the least open cost, and install an equipment mode at the facility.

step2-2: Allocate the customers in order of the least delivery cost till the capacity of the facility.

step2-3: If all customer demands are satisfied, then stop. Otherwise, open a facility with the next least open cost and install an equipment mode there, and go to *step2-2*.

Step3: Solve the Lagrangean Problem LR, using $(\mu_{ij})^t$ and $(\lambda_j)^t$, and obtain Z_{LB}.

Step4: Update the lower bound and generate an feasible solution by using the procedure PHP.

Step5: Update the upper bound, and the best feasible solution until now.

Step6: Terminate this procedure when the iteration counter t exceeds a prespecified limit (1000 iterations).

Step7: Compute a new subgradient and update the Lagrangean multipliers.

Ⅳ. Computational Experiments

A set of computational experiments was performed to evaluate the performance of the proposed algorithm. The proposed heuristic algorithm was coded in Visual C and run on a 2.8GHz pentium processor with 1GB RAM using a Microsoft compiler.

A total of 4 problems with various dimensions were generated, and each problem was then replicated 30 times. Each data was randomly generated as follows; Customer demand, d_i, from U[300, 500], delivery cost, t_{ij}, from U[10, 25], facility open cost, o_j, from U[2000, 3000], equipment installation cost, e_{jk}, from U[1000, 1500], *operation cost*, p_j, *from U[15, 25], over-operation* cost, p_j^+, from U[20, 30], and equipment capacity, c_k, from U[400, 800]. Note that all the instances were generated such that the relation $p_j^+ > p_j$ holds.

Table 1. reports the performance of the Lagrangean-relaxed heuristic algorithm. The first thress columns of the table specify the problem dimension. The information on the fourth column indicates the average Percent Optimality of the heuristic, as measured by $100[1-(Z_{UB}-Z_{LB})/Z_{LB}]$, and the average CPU time (in seconds) is reported on the last column.

n_I	n_J	n_K	Percent Optimality	CPU (s)
10	5	5	93.30 %	0.0255
10	10	5	92.33 %	0.0506
20	10	5	91.95 %	0.1094
20	10	10	92.26 %	0.1583
20	20	10	91.09 %	0.2500

Table 1. Lagrangean-relaxed heuristic algorithm performance

The table shows that the proposed heuristic algorithm is effective in identifying high quality solutions, within 10% of optimality. Moreover, it also shows that the elapsed time of the algorithm increases as the size of the problem increases, however it is still within a modest amount of computational effort.

Ⅴ. Concluding Remarks

This paper considers a facility location, which is to make decisions on which facility to be open, and how big equipment mode to be installed in order to satisfy the given set of demands at minimum costs. Moreover, the facility is allowed to be overworked, but with a higher unit-production cost.

The proposed problem is *NP-hard* problem, so that a Lagrangean-based heuristic algorithm is suggested in this paper. The computational experiments demonstrated that the proposed algorithm offers high-quality solutions in a short time.

For a further study, it may be interested in the extension of the proposed problem to produce multiple products and to allow multiple stage of overwork.

Ⅵ. References

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