Optimal Inventory and Price Markdown Policy for a Two-Layer Market with Demand being Price and Time Dependent

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Abstract

This paper considers a SCM issue concerned with an integrated problem of inventory control and dynamic pricing strategies when demands are price and time dependent. The associated price markdowns are conducted for inventory control in a two-layer market consisting of retailer and outlet as in fashion apparel market. The objective function consists of revenue terms (sales revenue and salvage value) and purchasing cost term. Specifically, decisions on price markdowns and order quantity are made to maximize total profit in the supply chain so as to have zero inventory level at the end of the sales horizon. To solve the proposed problem, a gradient method is applied, which shows an optimal decision on both the initial inventory level and the discount pricing policy. Sensitivity analysis is conducted on the demand parameters and the final comments on the practical use of the proposed model are presented.

I. Introduction

In the last couple of decades, the sales volume of the variety of goods in the market has significantly increased, while their life cycles have become shortened. Although improved supply chain management and production technologies may have helped to increase responsiveness to such life-cycle shortening, the associated long lead times and shorter sales horizon may have resulted in larger forecasting errors and inability of inventory level adjustment in response to demand.

Such market environment changes has incurred much difficulty in seasonal product marketing. For example, many seasonal products include fashion apparel, sports items and holiday products which are typical examples of extreme diversity as short life time products. When limiting sales season, they become out of date at the end of the predetermined season. Under such environment, the associated industry faces simultaneous problems of how much to order to satisfy any future demand and how to price any purchased seasonal product over the sales season. For these products, the objective may be set up as to maximize the profit and, at the same time, make the optimal pricing decision so that inventory approaches to zero at the end of the sales horizon. Otherwise, at the end, it will be out of style (life cycle over) and has little or no value in the marketplace. In many practical situations, the associated decision makers may need to control price markdown policies before the end of the season, hoping to stimulate sales so as to avoid the ending period with any excessive inventories.

As the seller wants to sold out the products and satisfy segmented customer needs but the customer wants to buy products at an appropriate time and at cheap price, secondary market place has been emerging to meet both sides' demand. This secondary market is generally called outlet and is coped with the first market place, retailer. However, there are few studies dealing with the secondary market place or integration of these two market places. This provides the motivation for this paper to integrate two market places and propose optimal inventory and pricing policy to maximize total profit in the whole supply chain.

Thereupon, a mathematical model is derived for any existing two-layer market place system of seasonal products, allowing the possibility of price markdowns during the sales period.

II. Problem Description

2.1 Assumptions and notation

The retailer orders a fixed quantity of product at the beginning but does not consider any additional ordering. The initial price is lasted through one price markdown after a fixed time interval. At the end of a predetermined time period, any remaining goods (left-over) are sent to outlet. At the outlet, no more price markdown is made except the one made at the starting sales point.

The customer demand rate depends on the price and the released time. The customers at the retailer is more sensitive to the released time, while customers at the outlet is more sensitive to the price. This difference is reflected in the demand function at each market place.

The inventory holding cost depends on the number of the remaining units at the end of sales horizon at each market place. However, in the case of the products with various kinds and small quantity, it is better to replace inventory holding cost for each item with another opportunity cost, salvage value. Hence, the inventory level at the end of sales horizon at each market place is represented by the associated salvage value respectively.

The following notation is introduced throughout the rest of this paper.

Parameters

T_i : sales period of product

$I_i(t)$: inventory level at the end of period i

p_1 : initial price at retailer

p_2 : discounted price at retailer

p_3 : discounted price at outlet

c : unit purchasing cost at retailer

s_r : salvage value per each item at retailer

s_o : salvage value per each item at outlet

ε_r : price elasticity at retailer

ε_o : price elasticity at outlet

α_r : coefficient of demand rate at retailer

α_o : coefficient of demand rate at outlet

γ_r : time variable at retailer

γ_o : time variable at outlet

$d_r(p_i, t)$: demand rate of retailer

$d_o(p_i, t)$: demand rate of outlet

$TP(\cdot)$: total profit

$TR(\cdot)$: total revenue

$TC(\cdot)$: total cost

Variables

Q: ordering quantity of retailer

x: the extent of markdown at retailer

y: the extent of markdown at outlet

2.2 Problem Formulation

A single-period inventory model is presented based on the assumptions of the classical, single-period model (a finite-length time period with only one opportunity for replenishment) except that the demand of the item is in a deterministic, multivariate function of price and time, in the form:

$d(p, t) = \alpha p^{-\varepsilon} t^{\gamma-1},\ \alpha>0,\ \varepsilon>1,\ 0<\gamma<1$.

Although theoretically price elasticity can be less than one, this paper considers more general situation with price elastic demand. The time variable is restricted to $\gamma<1$ which represents that the marginal increase in the demand rate will decrease as time increases.

Consequently the demand rate at the retailer is defined as follows:

$d_r(p_i, t) = \alpha_r p_i^{-\varepsilon_r} t^{\gamma_r-1},\ \alpha_r>0,\ \varepsilon_r>1,\ 0<\gamma_r<1$

Customers in different markets behave differently with regard to their own criteria. For instance, customers in an upper-income area such as department store may be less sensitive to price. On the other hand, customers in discount store or outlet may be less sensitive to time. Hence the demand rate at the outlet is defined as follows:

$d_o(p_i, t) = \alpha_o p_3^{-\varepsilon_o} t^{\gamma_o-1},\ \alpha_o>0,\ \varepsilon_o>\varepsilon_r>1,\ 0<\gamma_o<\gamma_r<1$.

Since the decrease in inventory level over time is equal to the demand rate, the inventory function for the proposed model can be determined by solving the resulting differential equation:

$\frac{dI_i}{dt} = -d(p_i),\ i=1, 2, 3$.

The initial condition is $I_0=Q$, where Q is the initial inventory level. This results in the mathematical representation of the inventory level over time:

$$I_i(t) = \begin{cases} Q - \frac{\alpha_r t^{\gamma_r}}{\gamma_r p_1^{\varepsilon_r}} & \text{if } 0 \le t \le T_1, \\ Q - \frac{\alpha_r T_1^{\gamma_r}}{\gamma_r p_1^{\varepsilon_r}} - \frac{\alpha_r (t^{\gamma_r} - T_1^{\gamma_r})}{\gamma_r p_2^{\varepsilon_r}} & \text{if } T_1 \le t \le T_2, \\ Q - \frac{\alpha_r T_1^{\gamma_r}}{\gamma_r p_1^{\varepsilon_r}} - \frac{\alpha_r (T_2^{\gamma_r} - T_1^{\gamma_r})}{\gamma_r p_2^{\varepsilon_r}} - \frac{\alpha_o (t^{\gamma_o} - T_2^{\gamma_o})}{\gamma_o p_3^{\varepsilon_o}} & \text{if } T_2 \le t \le T_3 \end{cases}$$

Let the extent of price markdown in each market be x and y which results in the form:

$p_2 = x \cdot p_1,\quad p_3 = y \cdot p_1 \quad 0 < y \le x \le 1$.

The objective of this problem is to maximize profit. Generally, the objective function is comprised of purchasing cost, inventory holding cost, and sales revenues. However, in this case, it is replaced into salvage value which is one of the opportunity cost also.

Hence, the objective function in this model is comprised of sales revenue, salvage value, and purchasing cost. Specifically, salvage value at the retailer becomes purchasing cost at the outlet which is canceled out in the integrated model.

As each market has different value added, it is profitable to consider first each market separately. Then, the total profit function at the retailer is as follows:

$$TP_r = \left[p_1(Q-I_{T_1}) + p_2(I_{T_1}-I_{T_2}) + s_r I_{T_2} \right] - [Qc]$$
$$= \frac{\alpha_r p_1^{1-\varepsilon_r}(T_2^{\gamma_r}-T_1^{\gamma_r})}{\gamma_r} x^{1-\varepsilon_r} - s_r \frac{\alpha_r p_1^{-\varepsilon_r}(T_2^{\gamma_r}-T_1^{\gamma_r})}{\gamma_r} x^{-\varepsilon_r}$$
$$+ \frac{\alpha_r p_1^{1-\varepsilon_r} T_1^{\gamma_r}}{\gamma_r} - s_r \frac{\alpha_r p_1^{-\varepsilon_r} T_1^{\gamma_r}}{\gamma_r} + (s_r - c)Q$$

The total profit function at the outlet is in the same form:

$$TP_o = \left[p_3(I_{T_2}-I_{T_3}) + s_o I_{T_3} \right] - [s_r I_{T_2}]$$
$$= \frac{\alpha_o p_1^{1-\varepsilon_o}(T_3^{\gamma_o}-T_2^{\gamma_o})}{\gamma_o} y^{1-\varepsilon_o} - s_o \frac{\alpha_o p_1^{-\varepsilon_o}(T_3^{\gamma_o}-T_2^{\gamma_o})}{\gamma_o} y^{-\varepsilon_o}$$
$$+ (s_o - s_r)\left[Q - \frac{\alpha_r p_1^{-\varepsilon_r} T_1^{\gamma_r}}{\gamma_r} - \frac{\alpha_r p_2^{-\varepsilon_r}(T_2^{\gamma_r}-T_1^{\gamma_r})}{\gamma_r} \right]$$

Consequently, the total profit function at the retailer and outlet is as follows:

$$TP = \left[p_1(Q-I_{T_1}) + p_2(I_{T_1}-I_{T_2}) + p_3(I_{T_2}-I_{T_3}) + s_o I_{T_3} \right] - [Qc]$$
$$= \frac{\alpha_r p_1^{1-\varepsilon_r}(T_2^{\gamma_r}-T_1^{\gamma_r})}{\gamma_r} x^{1-\varepsilon_r} - s_o \frac{\alpha_r p_1^{-\varepsilon_r}(T_2^{\gamma_r}-T_1^{\gamma_r})}{\gamma_r} x^{-\varepsilon_r}$$
$$+ \frac{\alpha_o p_1^{1-\varepsilon_o}(T_3^{\gamma_o}-T_2^{\gamma_o})}{\gamma_o} y^{1-\varepsilon_o} - s_o \frac{\alpha_o p_1^{-\varepsilon_o}(T_3^{\gamma_o}-T_2^{\gamma_o})}{\gamma_o} y^{-\varepsilon_o}$$
$$+ \frac{\alpha_r p_1^{1-\varepsilon_r} T_1^{\gamma_r}}{\gamma_r} - s_o \frac{\alpha_r p_1^{-\varepsilon_r} T_1^{\gamma_r}}{\gamma_r} + (s_o - c)Q$$

III. Solution Approach

To obtain the optimal solution, gradient method is applied. Although the gradient vector and the Hessian matrix of the profit function can be easily calculated, restrictions on parameters make this problem difficult. Hence, to get the optimal solution, several solution properties will be characterized in this chapter.

Property 1. TP_r decreases with Q.

Proof)

$\frac{\partial TP_r}{\partial Q} = s_r - c$. As $s_r < c$, $\frac{\partial TP_r}{\partial Q} < 0$.

This completes the proof.

Property 2. TP_o decreases with Q.

Proof)

$\frac{\partial TP_o}{\partial Q} = s_o - s_r$. As $s_o < s_r$, $\frac{\partial TP_o}{\partial Q} < 0$.

This completes the proof.

Property 3. TP decreases with Q.

Proof)

$\frac{\partial TP}{\partial Q} = s_o - c$. As $s_o < c$, $\frac{\partial TP}{\partial Q} < 0$.

This completes the proof.

Property 4. According to the assumption $I_{T_3} \geq 0$ which means $Q \geq \frac{\alpha_r T_1^{\gamma_r}}{\gamma_r p_1^{\varepsilon_r}} - \frac{\alpha_r (T_2^{\gamma_r} - T_1^{\gamma_r})}{\gamma_r p_2^{\varepsilon_r}} - \frac{\alpha_o (t^{\gamma_o} - T_2^{\gamma_o})}{\gamma_o p_3^{\varepsilon_o}}$

TP is maximized at

$$Q^* = \frac{\alpha_r T_1^{\gamma_r}}{\gamma_r p_1^{\varepsilon_r}} - \frac{\alpha_r (T_2^{\gamma_r} - T_1^{\gamma_r})}{\gamma_r p_2^{\varepsilon_r}} - \frac{\alpha_o (t^{\gamma_o} - T_2^{\gamma_o})}{\gamma_o p_3^{\varepsilon_o}}$$

Since TP decreases with Q.

This completes the proof.

Substituting property 4, the total profit function at the retailer with Q^* is as follows:

$$TP_r = \left[p_1(Q^*-I_{T_1}) + p_2(I_{T_1}-I_{T_2}) + s_r I_{T_2} \right] - [Q^* c]$$
$$= \frac{\alpha_r p_1^{1-\varepsilon_r}(T_2^{\gamma_r}-T_1^{\gamma_r})}{\gamma_r} x^{1-\varepsilon_r} - c \frac{\alpha_r p_1^{-\varepsilon_r}(T_2^{\gamma_r}-T_1^{\gamma_r})}{\gamma_r} x^{-\varepsilon_r}$$
$$+ \frac{\alpha_r p_1^{1-\varepsilon_r} T_1^{\gamma_r}}{\gamma_r} - c \frac{\alpha_r p_1^{-\varepsilon_r} T_1^{\gamma_r}}{\gamma_r} + (s_r - c) \frac{\alpha_o p_1^{-\varepsilon_o}(T_3^{\gamma_o}-T_2^{\gamma_o})}{\gamma_o} y^{-\varepsilon_o}$$

In the same way, the total profit function at the outlet with Q^* is as follows:

$$TP_o = \left[p_3(I_{T_2}-I_{T_3}) + s_o I_{T_3} \right] - [s_r I_{T_2}]$$
$$= \frac{\alpha_o p_1^{1-\varepsilon_o}(T_3^{\gamma_o}-T_2^{\gamma_o})}{\gamma_o} y^{1-\varepsilon_o} - s_r \frac{\alpha_o p_1^{-\varepsilon_o}(T_3^{\gamma_o}-T_2^{\gamma_o})}{\gamma_o} y^{-\varepsilon_o}$$

Consequently, the total profit function at the retailer and outlet with Q^* is as follows:

$$TP = \left[p_1(Q^*-I_{T_1}) + p_2(I_{T_1}-I_{T_2}) + p_3(I_{T_2}-I_{T_3}) + s_o I_{T_3} \right] - [Q^* c]$$

$$= \frac{\alpha_r p_1^{1-\varepsilon_r}(T_2^{\gamma_r}-T_1^{\gamma_r})}{\gamma_r} x^{1-\varepsilon_r} - c \frac{\alpha_r p_1^{-\varepsilon_r}(T_2^{\gamma_r}-T_1^{\gamma_r})}{\gamma_r} x^{-\varepsilon_r}$$
$$+ \frac{\alpha_o p_1^{1-\varepsilon_o}(T_3^{\gamma_o}-T_2^{\gamma_o})}{\gamma_o} y^{1-\varepsilon_o} - c \frac{\alpha_o p_1^{-\varepsilon_o}(T_3^{\gamma_o}-T_2^{\gamma_o})}{\gamma_o} y^{-\varepsilon_o}$$
$$+ \frac{\alpha_r p_1^{1-\varepsilon_r} T_1^{\gamma_r}}{\gamma_r} - c \frac{\alpha_r p_1^{-\varepsilon_r} T_1^{\gamma_r}}{\gamma_r}$$

When y is given, optimal decision at the retailer can be made exclusively.

Property 5.

(a) When $\varepsilon_r \geq \frac{p_1}{p_1 - c}$, TP_r is maximized with optimal value $x^* = \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1}$.

(b) When $\varepsilon_r \le \frac{p_1}{p_1 - c}$, TP_r is maximized with optimal value $x^* = 1$.

Proof)

When $\frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1} \le \frac{(\varepsilon_r + 1)c}{(\varepsilon_r - 1)p_1} \le 1$, i.e., $\varepsilon_r \ge \frac{p_1 + c}{p_1 - c}$,

$x^* = \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1}$.

When $\frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1} \le 1 \le \frac{(\varepsilon_r + 1)c}{(\varepsilon_r - 1)p_1}$, i.e., $\frac{p_1}{p_1 - c} \le \varepsilon_r \le \frac{p_1 + c}{p_1 - c}$,

$x^* = \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1}$.

When $1 \le \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1} \le \frac{(\varepsilon_r + 1)c}{(\varepsilon_r - 1)p_1}$, i.e., $\varepsilon_r \le \frac{p_1}{p_1 - c}$, $x^* = 1$.

This completes the proof.

In the same way, optimal decision at the outlet can be determined exclusively as follows.

Property 6.

(a) When $\varepsilon_o \ge \frac{p_2}{p_2 - s_r}$, TP_o is maximized with optimal value $y^* = \frac{\varepsilon_o s_r}{(\varepsilon_o - 1)p_1}$.

(b) When $\varepsilon_o \le \frac{p_2}{p_2 - s_r}$, TP_o is maximized with optimal value $y^* = x$.

Proof)

When $\frac{\varepsilon_o s_r}{(\varepsilon_o - 1)p_2} \le \frac{(\varepsilon_o + 1)s_r}{(\varepsilon_o - 1)p_2} \le x$, i.e., $\varepsilon_o \ge \frac{p_2 + s_r}{p_2 - s_r}$,

$y^* = \frac{\varepsilon_o s_r}{(\varepsilon_o - 1)p_1}$.

When $\frac{\varepsilon_o s_o}{(\varepsilon_o - 1)p_2} \le x \le \frac{(\varepsilon_o + 1)s_o}{(\varepsilon_o - 1)p_2}$, i.e., $\frac{p_2}{p_2 - s_o} \le \varepsilon_o \le \frac{p_2 + s_o}{p_2 - s_o}$,

$y^* = \frac{\varepsilon_o s_o}{(\varepsilon_o - 1)p_1}$.

When $x \le \frac{\varepsilon_o s_o}{(\varepsilon_o - 1)p_2} \le \frac{(\varepsilon_o + 1)s_o}{(\varepsilon_o - 1)p_2}$, i.e., $\varepsilon_o \le \frac{p_2}{p_2 - s_o}$, $y^* = x$.

This completes the proof.

Accordingly, consider the integrated model of the total profit at the retailer and outlet.

Property 7.

(a) When $\frac{p_1}{p_1 - c} \le \varepsilon_r \le \varepsilon_o$, $TP(x, y)$ is maximized with optimal value $x^* = \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1}$, $y^* = \frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1}$.

(b) When $\varepsilon_r \le \frac{p_1}{p_1 - c} \le \varepsilon_o$, $TP(x, y)$ is maximized with optimal value $x^* = 1$, $y^* = \frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1}$.

(c) When $\varepsilon_r \le \varepsilon_o \le \frac{p_1}{p_1 - c}$, $TP(x, y)$ is maximized with optimal value $x^* = y^* = 1$.

Proof)

When $\frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1} \le \frac{(\varepsilon_r + 1)c}{(\varepsilon_r - 1)p_1} \le 1$ and $\frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1} \le \frac{(\varepsilon_o + 1)c}{(\varepsilon_o - 1)p_1} \le 1$,

i.e., $\varepsilon_r \ge \frac{p_1 + c}{p_1 - c}$ and $\varepsilon_o \ge \frac{p_1 + c}{p_1 - c}$,

$x^* = \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1}$, $y^* = \frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1}$.

When $\frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1} \le 1 \le \frac{(\varepsilon_r + 1)c}{(\varepsilon_r - 1)p_1}$ and $\frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1} \le \frac{(\varepsilon_o + 1)c}{(\varepsilon_o - 1)p_1} \le 1$

i.e., $\frac{p_1}{p_1 - c} \le \varepsilon_r \le \frac{p_1 + c}{p_1 - c}$ and $\varepsilon_o \ge \frac{p_1 + c}{p_1 - c}$,

$x^* = \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1}$, $y^* = \frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1}$.

When $\frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1} \le 1 \le \frac{(\varepsilon_r + 1)c}{(\varepsilon_r - 1)p_1}$ and $\frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1} \le 1 \le \frac{(\varepsilon_o + 1)c}{(\varepsilon_o - 1)p_1}$

i.e., $\frac{p_1}{p_1 - c} \le \varepsilon_r \le \frac{p_1 + c}{p_1 - c}$ and $\frac{p_1}{p_1 - c} \le \varepsilon_o \le \frac{p_1 + c}{p_1 - c}$,

$x^* = \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1}$, $y^* = \frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1}$.

When $1 \le \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1} \le \frac{(\varepsilon_r + 1)c}{(\varepsilon_r - 1)p_1}$ and $\frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1} \le 1 \le \frac{(\varepsilon_o + 1)c}{(\varepsilon_o - 1)p_1}$,

i.e., $\varepsilon_r \le \frac{p_1}{p_1 - c}$ and $\frac{p_1}{p_1 - c} \le \varepsilon_o \le \frac{p_1 + c}{p_1 - c}$,

$x^* = 1$, $y^* = \frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1}$.

When $1 \le \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1} \le \frac{(\varepsilon_r + 1)c}{(\varepsilon_r - 1)p_1}$ and $1 \le \frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1} \le \frac{(\varepsilon_o + 1)c}{(\varepsilon_o - 1)p_1}$,

i.e., $\varepsilon_r \le \frac{p_1}{p_1 - c}$ and $\varepsilon_o \le \frac{p_1}{p_1 - c}$, $x^* = 1$, $y^* = 1$.

This completes the proof.

As the reciprocal of $\frac{p_1}{p_1 - c}$ means the sales return rate which is represented as $\frac{p_1 - c}{p_1}$, property 5, 6, 7 can be replaced as follows.

Property 5'.

(a) When $\frac{p_1 - c}{p_1} \ge \frac{1}{\varepsilon_r}$, TP_r is maximized with optimal value $x^* = \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1}$.

(b) When $\frac{p_1 - c}{p_1} \le \frac{1}{\varepsilon_r}$, TP_r is maximized with optimal value $x^* = 1$.

Property 6'.

(a) When $\frac{p_2 - s_r}{p_2} \ge \frac{1}{\varepsilon_o}$, TP_o is maximized with optimal value $y^* = \frac{\varepsilon_o s_r}{(\varepsilon_o - 1)p_1}$.

(b) When $\frac{p_2 - s_r}{p_2} \le \frac{1}{\varepsilon_o}$, TP_o is maximized with optimal value $y^*=x$.

Property 7'.

(a) When $\frac{1}{\varepsilon_o} \le \frac{1}{\varepsilon_r} \le \frac{p_1-c}{p_1}$, $TP(x,y)$ is maximized with optimal value $x^*=\frac{\varepsilon_r c}{(\varepsilon_r-1)p_1}$, $y^*=\frac{\varepsilon_o c}{(\varepsilon_o-1)p_1}$.

(b) When $\frac{1}{\varepsilon_o} \le \frac{p_1-c}{p_1} \le \frac{1}{\varepsilon_r}$, $TP(x,y)$ is maximized with optimal value $x^*=1$, $y^*=\frac{\varepsilon_o c}{(\varepsilon_o-1)p_1}$.

(c) When $\frac{p_1}{p_1-c} \le \frac{1}{\varepsilon_o} \le \frac{1}{\varepsilon_r}$, $TP(x,y)$ is maximized with optimal value $x^*=y^*=1$.

Under all situations, if price elasticity is greater than reciprocal of the sales return rate, optimal policy is to discount price by proposed rate. Otherwise selling at the fixed price without any price markdown is the optimal policy. The integrated model gives better total profit than the separated model. Optimal initial inventory level to sell out of products is as follows:

$$Q^* = \frac{\alpha_r T_1^{\gamma_r}}{\gamma_r p_1^{\varepsilon_r}} - \frac{\alpha_r (T_2^{\gamma_r} - T_1^{\gamma_r})}{\gamma_r p_2^{\varepsilon_r}} - \frac{\alpha_o (t^{\gamma_o} - T_2^{\gamma_o})}{\gamma_o p_3^{\varepsilon_o}}$$

IV. Computational Results

4.1 Numerical example

To illustrate the impact of this type of model, consider the following example in which the demand is of the functional form described by section 3.2 with the values of the parameters as follows:

$\alpha_r = 50$, $\varepsilon_r = 2$, $\gamma_r = 0.8$,

$\alpha_o = 100$, $\varepsilon_o = 2.5$, $\gamma_o = 0.5$,

$p_1 = 8$, $c = 3$, $s_r = 2.5$, $s_o = 1.5$ (*dollars*),

$T_1 = 60$, $T_2 = 90$, $T_3 = 120$ (*days*)

If consider the retailer's profit only, then the total profit at the retailer is maximized with

$$x^* = \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1} = 0.75$$

If consider the outlet's profit only, then the total profit at the outlet is maximized with

$$y^* = \frac{\varepsilon_o s_r}{(\varepsilon_o - 1)p_1} = 0.52$$

If consider the retailer's and outlet's profit together, than the total profit is maximized with

$$x^* = \frac{\varepsilon_r c}{(\varepsilon_r - 1)p_1} = 0.75,\ y^* = \frac{\varepsilon_o c}{(\varepsilon_o - 1)p_1} = 0.625$$

4.2 Sensitivity analysis

Sensitivity of the proposed model to errors made in estimating the parameters of demand function (all else equal) is investigated. This analysis is based on the example presented previously. The error in the estimation of the outlet's demand parameter is negligible, as an error of 50% will result in a deviation in profit of no more than 7%. On the other hand, it is estimated that errors in retailer's demand parameter is fairy significant which results in maximum 370% deviation.

V. Conclusion

This paper considers a combined model of dynamic pricing and inventory policy for a seasonal product with price and time dependent demand pattern and price markdowns in two market places. Decisions on price markdowns and order quantity are made to maximize total profit in the supply chain so as to have zero inventory level at the end of the sales horizon. To solve the proposed problem, a gradient method is applied, which shows an optimal decision on both the initial inventory level and the discount pricing policy.

This model can be applied for most seasonal products which have predetermined sales horizon and a secondary market place, for those including fashion apparel, sports items, holiday products like Christmas. However, for perishable items which have circulation period, including agricultural products and diary goods, their demand function and sales horizon should be modified carefully.

For further study, it may be interested in extensions of the proposed model to complex demand function and multiple price markdowns. Another interesting research issue is to consider any difference in the primary and secondary market demand functions, or apply totally different customer demand functions in each market place.