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특이점이 제거된 2 차원 단일매질 주기구조에 대한 그린함수의 제안

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Proposal of Singularity-Circumvented Green's Functions for 2D Periodic Structures in Homogeneous Medium

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Abstract - In this paper, a novel method is presented for efficient calculation of the spatial-domain Green's functions of 2D electromagnetic problems. This method combines spectral and spatial domain calculation schemes and prevents the Green's functions from diverging at the singularities that complicate the process of the Method of Moment(MoM) application. For the validation of this proposed method, fields will be evaluated along the spatial distance including zero distance for 2D free-space and periodic homogeneous geometry. The numerical results show the validity of the prosed method and correspondng physics.

1. 서 론

Much of researchers'attention has been paid to two-dimensional(2D) electromagnetic radiation and scattering problems. Employing integro-differential or integral equations to model the physical phenomena, which are to be numerically solved by the frame work of the MoM, the impulse response Green's functions are the focal point of deciding the efficiency as well as validity of the solutions[1].

Green's functions are calculated in the spatial domain with the spatial basis and weighting functions during the matrix fill-up process, and they will diverge at singularities where the source and observation points coincide or one of the two is in the other's proximity. Coping with the divergence, Rao et al provided a technique that splits the original Green's function calculation into the numerically stable and analytically integrable parts[2-4]. Though this has still been used in much of concerned literature, the efficient treatment of the numerical part totally depends on trunction-related conditions.

This paper presents a novel method to efficiently figure out 2D free-space periodic problems[5, 6] expressed on the spatial domain basis. In accordance with this proposed method, the spatial domain calculation is carried out conventionally for nonsingular spatial points, and is replaced by its counterpart of the spectral domain only for the singularities to avoid the Green functions' divergence at the singularities which complicates the application of the Method of Moment(MoM) to the electromagnetic characterization. In order to validate this proposed method, the fields of 2D free-space and periodic structure are calculated. Also, the Green's functions of both the cases are handled with the adoption of the proposed method to treat the point where the distance becomes zero. The comparison shows the results obtained by the proposed method are in good ageement with those of the conventional metod, and observe physics.

2. 본 론

2.1 Theory

2D electromagnetic scattering problems have drawnmuch of researchers' concern and interest in such as radar cross-section(RCS), Nondestructive Inspection, Waves'Propagation Constant solutions, and so on. The problems are modeledas integro-differential or integral equations, and these equations are discretized and converted to matrix equations which are appropriatefor computer work by way of electromagnetic numerical analysis methods like the Method of Moment(MoM)[1]. In the MoM application, the radiation integral possesses the Green's function corresponding to impulse source-and-field relations, essential to and influencing the overall time of the MoM solution porcess[1]. The Green's function for the 2D free-space geometry is the 0th order Hankel function of the second kind as

$$G(\bar{\rho}-\bar{\rho}') = H_0^{(2)}(k_i \,|\, \bar{\rho}-\bar{\rho}'\,|) \tag{1}$$

$\bar{\rho}$, $\bar{\rho}'$, ω and μ mean the field point vector, the source point vector, the angular frequency and permeability, respectively. $k_i = \omega\sqrt{\varepsilon_i \mu}$. The position vectors are

$$\bar{\rho} = x\hat{x} + y\hat{y}, \quad \bar{\rho}' = x'\hat{x} + y'\hat{y}, \quad \bar{r} = \bar{\rho} - \bar{\rho}' \tag{2}$$

While the impedance matrix is filled via basis-expansion and weighting in a spatial domain problem, Eqn. (1) is numerically well-behaved except the singularity occurring when its argument goes to zero or the field point coincides with the source point. In Eqn. (1), the singularity point corresponds to $|\bar{\rho}-\bar{\rho}'| = 0$, where the denominator of Neumann function becomes 0. This entails cumbersome steps of mitigating singularity or reducing the order of singularity with the help of numerical extraction of singularity or approximation[2].

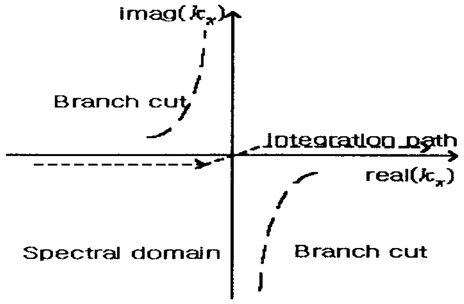
The new method is proposed to circumvent the singularity of Eqn. (1) using both the spectral form as well as spatial form regarding a spatial·domain based Green's function calculation. The detailed explanation starts as follows. Eqn.(1) of the spatial domain can be expressed in the spectral domain as

$$G(\bar{\rho}-\bar{\rho}') = \int_{-\infty}^{+\infty} \tilde{G}(k_x)dk_x = \int_{-\infty}^{+\infty} \frac{e^{-jk_{iy}|y-y'|-jk_x(x-x')}}{j2k_{iy}} dk_x \tag{3}$$

where j, $\tilde{G}(k_x)$, k_x and k_{iy} are $\sqrt{-1}$, the spectral domian Green's function, wave numbers in x and y directions, respectively [3, 4]. The corresonding dispersion relation is

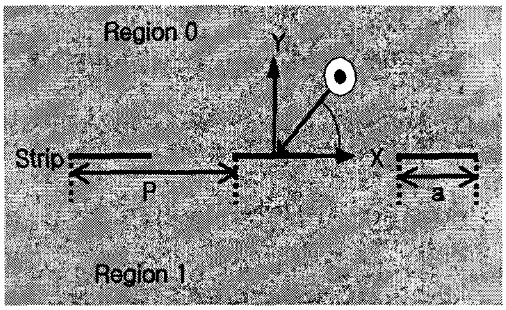
$$k_x^2 + k_{iy}^2 = k_i^2 \tag{4}$$

The Sommerfeld type integral like Eqn. (3) customarily takes the integration path in the spectral domain as in Figure 1.



<그림 1> Sommerfeld integration path

The simple numerical experiment revelas the the phase of the integrand of Eqn. (3) grows and tends to prevent convergence with the increasing distance between the source and field points. Conversley, it is noteworthy that Eqn. (3) converges very fast and makes claculation most efficient when $\bar{\rho}$ approaches $\bar{\rho}'$. Thus, it is proposed that Eqn.(3) is adopted for only the spatial singularity point and Eqn.(1) for all the other spatial points, taking only the advantages of Eqn.'s (1) and (3).
Now strips are periodically positioned in one medium as in Figure 2, in common with [5, 6].



<그림 2> TM incidence on periodic strips interfacing regions 1 and 2 that have the same permittivity ε_1 : Incidence angle is ϕ_0 $G(\bar{\rho}-\bar{\rho}')$ can be expressed as

$$G(\bar{\rho}-\bar{\rho}')=\sum_{n=-\infty}^{\infty}H_0^{(2)}(k_0\sqrt{(\Delta_x-nP)^2+\Delta_y^2})e^{-n\beta P} \quad (5)$$

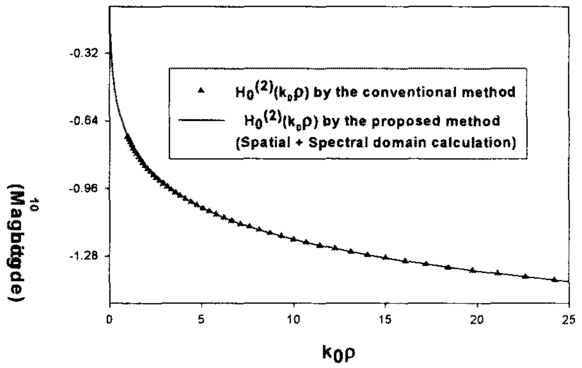
where

$\beta=-k_0\cos\phi_0$, $\Delta_x=x-x'$ and $\Delta_y=y-y'$

P and a menas periodicity and width of the strips. The computauion time of Eq. (2) grows due to the oscillatory phase - and sigularity-oriented slow convergence of the Green's function.

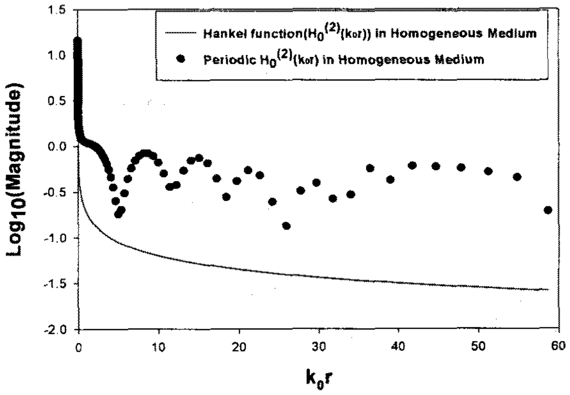
2.2 Numerical Results

Two examples are presented for proving the validity of the proposed method. As the first example, the 2D Free-space Green's function as the field is calculated with frequency of 300MHz by the proposed and conventional methods.



<그림 3> Green' s function of the 2D free-space

As seen in Figure 3, the conventional method produces discontinuity due to the avoidance of the spatial domain logarithmic singularity, but the proposed method presents the finite field value.
The second example regards the structure of 2D periodic strips s pecified with P of 1λ (wavelength) and frequnecy of 1GHz. With regard to this, Eq. (5) is used as the Green's function whose co mpution method can be verfied, compared with $H_0^{(2)}(k_0|\bar{\rho}-\bar{\rho}'|)$ as is usually done. The results of them are given in Figure 4.



<그림 4> Comparing the Green' s functions of the 2D free- space and a periodic structure in the homogeneous media example with P of 1λ and 1GHz

The free space Green's function shows the typical behavior of $r^{-1/2}$ variation withthe increasing distance. On the contrary, the result of Eq. (3) is calculable at distance zero and varies up and down, representing the mutual couplings of nearby strips, and will approach that of the free space if the periodicity is assumed infinite.

3. 결 론

A new and efficient method is proposed to calculate the spatial-domain 2D free-space periodic Green's functions for electromagnetic radiation and scattering problems. This method can simply circumvent the divergence problems at the spatial singularities by combining spectral and spatial domain calculation schemes and extended from simple free-space solution to solving periodic structures in homogeneous media.

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