완벽한 상태정합을 이용한 지능형 디지털 재설계

Intelligent Digital Redesign Via Complete State-Matching

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Abstract - In this paper, a complete solution to fuzzy-model-based digital redesign problem (IDR) for sampled-data nonlinear systems is presented. The term of intelligent digital redesign (IDR) is to design a digital fuzzy controller such that the sampled-data closed-loop fuzzy system is equivalent to the continuous-time closed-loop fuzzy system using the state matching. Its solution is simply obtained by linear transformation. Under the proposed sampled-data controller, the states of the sampled-data and continuous-time fuzzy system are completely matched at every sampling points.

Key Words : Intelligent Digital redesign (IDR), Multirate Control, Fuzzy-model-based control, Nonlinear sampled-data system, Fuzzy system

1. Introduction

The intelligent digital redesign (IDR) problem is the problem of designing a sampled-data state feedback controller such that the sampled-data closed-loop fuzzy system is equivalent to the continuous-time closed-loop fuzzy system in the sense of the state matching. There have been some researches focusing on the digital redesign [1]-[5] for complex nonlinear systems. All of these researches applied the fuzzy theory to effectively solve the digital redesign of the nonlinear systems. These approaches are referred to an intelligent digital redesign (IDR). Historically, Joo et al. [1] first defined the concept of IDR. Chang et al. [2] extended the previous result [1] by considering the uncertain T--S fuzzy systems. These methods proposed in [1,2] are so called as the local approaches. These local approaches allow us to solve the IDR problem in the analytic way, but it may lead to undesirable and/or inaccurate results. The main reasons are that because the local approaches looked into the IDR problem only for the sub-closed-loop fuzzy systems, not the whole closed-loop fuzzy systems. To circumvent these weaknesses, the global approaches were proposed in [3]-[5] Chang et al. [3] proposed the IDR method based on the genetic algorithms. However, their method incurs the computational burden. Recently, Lee [4,5] expressed the IDR conditions as the linear matrix inequalities and hence easily solved the problem with the convex optimization techniques.

Although a great deal of effort has been made on IDR such as [1-5], there still exists some matters that must be worked out. However. the IDR problem becomes the overdamped problem according as transferring the local approach to the global one in IDR problem. From that reason, their method is not able to completely match the states between the closed-loop sampled-data and continuous-time systems. It may lead to undesirable and/or inaccurate results.

Motivated by the above observations, we studies the IDR via complete state-matching. The main features of the proposed method are as follows: First, we employ the multirate control scheme to increase the dimension of input. Second, the proposed approach is to assume that the sampling period is fixed, but the exact discrete-time model can be obtained as integration step size approach zero. Finally, under the proposed sampled-data controller, the states of the discrete-time model of the sampled-data fuzzy system completely matches the state of the discrete-time model of the closed-loop continuous-time fuzzy systems are completely matched at every sampling points.

2. Preliminaries and Problem Formulation

Consider a nonlinear system described by

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$

where $x(t) \in R^n$ is the state vector, and $u_c(t) \in R^m$ is the continuous-time control input.

To facilitate the control design, we will develop a simplified model, which can represent the local linear

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input - output relations of the nonlinear system. This type of models is referred as T - S fuzzy models. The fuzzy dynamical model corresponding to the nonlinear system (1) is described by the following IF - THEN rules [1,2,3,6]:

$$R_k : IF\ z_1(t)\ is\ about\ \Gamma_{k1}\ and \cdots and\ z_p(t)\ is\ about\ \Gamma_{kp},$$
$$THEN\ \dot{x}(t) = A_k x(t) + B_k u(t) \tag{2}$$

where $R_k, k \in I_q = \{1,2,\cdots,q\}$, is the kth fuzzy rule, $z_r(t), r \in I_p = \{1,2,\cdots,p\}$, is the rth premise variable, and $\Gamma_{kr}, (k,r) \in I_q \times I_p$, is the fuzzy set. Then, given a pair $(x(t), u(t))$, using the center-average defuzzification, product inference, and singleton fuzzifier, the overall dynamics of the IF-THEN rules (2) has the form

$$\dot{x}(t) = \sum_{k=1}^{q} \theta_k(z(t))(A_k x(t) + B_k u(t)) \tag{3}$$

where $\theta_k(z(t)) = \frac{w_k(z(t))}{\sum_{k=1}^{q} w_k(z(t))}$, $w_k(z(t)) = \sum_{r=1}^{p} \Gamma_{kr}(z_r(t))$, and $\Gamma_{kr}(z_r(t))$ is the grade of membership of $z_r(t)$ in Γ_{kr}. The possibly time-varying parameter vector $\theta \in R^q$ belongs to a convex polytope Θ, where

$$\Theta := \left\{ \sum_{k=1}^{q} \theta_k = 1, \quad 0 \le \theta_k \le 1 \right\} \tag{4}$$

It is clear that as θ varies inside Θ, $\sum_{k=1}^{q} \theta_k(z(t)) A_k$ and $\sum_{k=1}^{q} \theta_k(z(t)) B_k$ range over a matrix polytope

$$\left[\sum_{k=1}^{q} \theta_k(z(t)) A_k, \sum_{k=1}^{q} \theta_k(z(t)) B_k \right] \in \mathrm{Co}\{(A_k, B_k), k \in I_q\}$$

where Co denotes the convex hull. In this note, the stabilization of the polytopic model (3) is equivalent to the simultaneous stabilization of its vertices $(A_k, B_k), k \in I_q$.

In this paper, a well-constructed continuous-time state feedback controller, which will be employed in redesigning the digital controller, is given. The controller is described by the following IF-THEN rules:

$$R_k : IF\ z_1(t)\ is\ about\ \Gamma_{k1}\ and \cdots and\ z_p(t)\ is\ about\ \Gamma_{kp},$$
$$THEN\ u_c(t) = K_k x_c(t), \tag{5}$$

and its defuzzified output is

$$u_c(t) = \sum_{k=1}^{q} \theta_k(z(t)) K_k x_c(t) \tag{6}$$

Therefore, main purpose of this paper is to find the digital equivalent of the following continuous-time closed-loop system:

$$\dot{x}_c(t) = \sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(t)) \theta_l(z(t))(A_k + B_k K_l) x_c(t) \tag{7}$$

3. New Intelligent Digital Redesign Using Complete State-Matching

The task of the sampled-data controller is to stabilize the origin of the closed-loop system. We follow the digital redesign approach to this problem. First, we assume that the stabilizable continuous-time controller is predesigned. Then, we design the sampled-data controller such that the responses of the closed-loop continuous-time and discrete-time systems are closely matched for the same initial conditions. For convenience, we rewrite the continuous-time closed-loop system (7) as

$$\dot{x}_c(t) = f(x_c(t)) x_c(t) \tag{8}$$

We consider a zero-order-hold system where u is held constant over the time interval of τ. This type of the system is called as multirate control system [9]. Let T be the sampling period and $\tau = T/M$ for some integers $M > 0$. To descretize the plant dynamics, we rewrite (3) as

$$\dot{x}(t) = F(x(t), u(k + j\tau)) \tag{9}$$

Following the discretization procedure of [8], the discrete-time model of (8) over the period $[kT + j\tau, kT + j\tau + \tau]$, $k \times j \in Z_{\ge 0} \times Z_{[0,M-1]}$ is given by

$$x(k + j\tau + \tau) = F_{\tau,h}(x(k + j\tau), u(k + j\tau)) \tag{10}$$

where $h = \tau/N$ for some integers $N > 0$.

Remark 1. *A more realistic approach is to assume that the sampling period is fixed (or has positive a lower bound), since the required sampling rate may not be implementable due to hardware limitations. In [6], it is shown that integration step size h determines the accuracy of the discrete-time model.*

By recursive application of (10) defined as

$$F_{\tau,h}^{1}(x(k), u(k)) := F_{\tau,h}(x(k), u(k))$$
$$F_{\tau,h}^{j+1}(x(k), U_{[kT, kT+j\tau]})$$
$$:= F_{\tau,h}(F_{\tau,h}^{j}(x(k), U_{[kT, kT+j\tau-\tau]}), u(k + j\tau))$$
$$F_{T,\tau,h}(x(k), U_{[kT, kT+M\tau-\tau]}) := F_{\tau,h}^{M}(x(k), U_{[kT, kT+M\tau-\tau]}),$$

where

$$U_{[kT, kT+M\tau-\tau]} = \begin{bmatrix} u(k) \\ u(k+\tau) \\ \vdots \\ u(k + M\tau - \tau) \end{bmatrix},$$

we arrive at

$$x(kT + T) = F_{T,\tau,h}(x(k), U_{[kT, kT+M\tau-\tau]}). \tag{11}$$

In the same manner, we can generate the following discrete-time model of (8):

$$x_c(k+1) = f_{T,\tau,h}(x_c(k)) x_c(k) \tag{12}$$

by defining

$$f_{\tau,h}^{1}(x_c(k)) := f_{\tau,h}(x_c(k))$$
$$f_{\tau,h}^{j+1}(x_c(k)) := f_{\tau,h}(f_{\tau,h}^{j}(x_c(k))) f_{\tau,h}^{j}(x_c(k))$$
$$f_{T,\tau,h}(x_c(k)) := f_{\tau,h}^{M}(x_c(k)).$$

From (11) and (12), it remains to determine how to construct the sampled-data controller such that

$$F_{T,\tau,h}(x(k),U_{[kT,kT+M\tau-\tau]}) = f_{T,\tau,k}(x_c(k))x_c(k) \quad (13)$$

Here, the nonlinear interpolation between $x(k)$ and $U_{[kT,kT+M\tau+\tau]}$ makes the state matching based on the linear transformation impossible, which leads us to make the following assumption.

Assumption 1. $x(k+j\tau+ih+h) = x_c(k+j\tau+ih+h)$ *for* $k\times j\times h \in Z_{\geq 0}\times Z_{[1,M-1]}\times Z_{[0,N-1]}$.

The decompose model of (11) is given by the following theorem.

Theorem 1. *Let Assumption 1 hold; then the dynamics of (11) can be decomposed as*

$$x(k+1) = A_{T,\tau,h}(x(k))x(k) + B_{T,\tau,h}(x(k))U_{[kT,kT+M\tau-\tau]} \quad (14)$$

where $n\times n$ *matrix* $A_{T,\tau,h}(x(k))$ *and* $n\times mM$ *matrix* $B_{T,\tau,h}(x(k))$ *are given by the following recursive procedure:*

$$A^1_{\tau,h}(x(k)) := A_{\tau,h}(x(k))$$
$$A^{j+1}_{\tau,h}(x_c(k)) := A_{\tau,h}(f^j_{\tau,h}(x(k)))A^j_{\tau,h}(x(k))$$
$$A_{T,\tau,h}(x(k)) := A^M_{\tau,h}(x(k))$$

and

$$B^1_{\tau,h}(x(k)) := B_{\tau,h}(x(k))$$
$$B^{j+1}_{\tau,h}(x(k))$$
$$:= \left[A_{\tau,h}(f^j_{\tau,h}(x(k)))B^j_{\tau,h}(x(k)) \quad B_{\tau,h}(f^j_{\tau,h}(x(k)))\right]$$
$$B_{T,\tau,h}(x(k)) := B^M_{\tau,h}(x(k)),$$

respectively.

The following assumption is introduced for ease of control synthesis.

Assumption 2. $\mathrm{rank}\left(B_{T,h,\tau}(x(k))\right) = n$

Remark 2. *From the fact that* $\mathrm{rank}\left(B_{T,h,\tau}(x(k))\right) \leq \min\{mM, n\}$, *the* $mM\times n$ *matrix* $B_{T,h,\tau}(x(k))$, $mM < n$ *is necessarily singular. In the proposed approach, this circumstance does not happen because it is possible to set the input multiplicity* M *such that* $mM \geq n$.

Theorem 2. *Under the sampled-data controller defined as*

$$U_{[kT,kT+M\tau-\tau]} = B^-_{T,h,\tau}(x(k))[f_{T,\tau,h}(x(k)) - A_{T,\tau,h}(x(k))]x(k), \quad (15)$$

where $B^-_{T,h,\tau}(x(k))$ *is the generalized inverse of matrix* $B_{T,h,\tau}(x(k))$ *[7], the state* $x(k)$ *of the discrete-time model of sampled-data system (11) completely matches the state* $x_c(k)$ *of the discrete-time model of closed-loop continuous-time system (12).*

4. Closing Remarks

In this paper, a new IDR method has been presented for multirate sampled-data fuzzy systems Under some assumption, we develop the sampled-data controller for completely state matching. In addition, the proposed sampled-data controller is available for the long and fixed sampling limit because a family of discrete-time models is employed to deign the controller.

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