A Fuzzy Sliding Mode Control for Rotational Inverted Pendulum

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ABSTRACT

Rotational inverted pendulum is a typical under-actuated system. For its highly nonlinear characteristic, a sliding mode controller is chosen for its robustness against the system uncertainties. Tow fuzzy inference mechanisms are applied in this paper to reduce the chattering phenomenon. One is proposed to construct a time-varying sliding surface. Another one is used to obtain the minimum upper bound of the uncertainties. A comparison between the conventional sliding mode and the fuzzy sliding mode is shown by simulations.

Key Words: rotational Inverted pendulum fuzzy sliding mode control.

1. INTRODUCTION

Sliding mode control is a discontinuous control action whose primary function of each of the feedback channels is to switch between two distinctively different system structures. For its robustness againsta large class of perturbations or model uncertainties and the need for a reduced amount of information in comparison to classical control techniques, it is regarded as a useful technique on the nonlinear system controlling. However, it still has a main obstacle to the success of these techniques in the industrial community so-called chattering phenomenon. To reduce the effects of chattering, a fuzzy sliding mode is proposed. Fuzzy logic control is an algorithm witch can convert the linguistic control strategy based on expert knowledge into an automatic control strategy. It is very useful when the processes are too complex for analysis by conventional quantitative techniques or when the available sources of information are interpreted qualitatively, inexactly, or uncertainly. Rotational inverted pendulum system we chosen is a typical under-actuated system. For its highly nonlinear characteristic, compact configuration anddirect observability, it is widely used as a time variable control object in the basic control algorithm study.

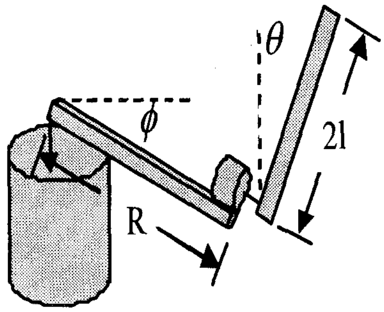

Fig.1 : Configuration to rotational inverted pendulum.

Figure 1 shows the configuration to the rotational inverted pendulum. The rotational inverted pendulum is consists of a vertical pendulum which can turn freely around the horizontal arm controlled by a DC motor. All the system parameters of the plant we used in the experiments are:

J_{arm}: Horizontal arm inertia, $0.00157[Kgm^2]$
J_{pen}: Vertical pendulum inertia, $0.000178[Kgm^2]$
m : Mass of the pendulum, $0.022[Kg]$
R : Length of the arm, $0.14[m]$
l : Length from rotation center to gravity center of the pendulum, $0.078[m]$
c : Damping coefficient, $0.126[Nms/rad]$
τ : Input torque.

When ignoring all kinds of frictions and the air resistance, the following motion equation of rotational inverted pendulum is used [1].

$$\begin{bmatrix} J_{arm}+m(R^2+l^2\sin^2\phi) & mlR\cos\phi \\ mlR\cos\phi & J_{pen}+ml^2 \end{bmatrix}\begin{bmatrix}\ddot{\theta}\\ \ddot{\phi}\end{bmatrix}+$$

$$\begin{bmatrix} c+ml^2\sin^2\theta\dot{\theta} & -mlR\sin\theta\dot{\theta} \\ -ml^2\sin\phi\cos\phi\dot{\theta} & 0 \end{bmatrix}\begin{bmatrix}\dot{\theta}\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ -mgl\sin\phi\end{bmatrix}=\begin{bmatrix}\tau\\ 0\end{bmatrix}$$

....(1)

To obtain a state space model of the rotational inverted pendulum, let us take the state variables as $[x_1\ \ x_2\ \ x_3]=[\theta\ \ \dot{\theta}\ \ \dot{\phi}]$. By linearization on the point $[x_1\ \ x_2\ \ x_3]=[0\ \ 0\ \ 0]$ then the state equation can be written into a linear part and a nonlinear part write as F_n. And E denotes the infection from all the unknown disturbances, such as the friction, starting torque, inertia. Etc. Then the state equation can be rewritten to :

$$\dot{x}=A_x+B_u+F_n \quad \text{..(2)}$$

Where

$$A=\begin{bmatrix} 0 & 1 & 0 \\ \frac{-MH}{MG-N^2} & \frac{cN}{MG-N^2} & 0 \\ \frac{NH}{MG-N^2} & \frac{-cG}{MG-N^2} & 0 \end{bmatrix},\ B=\begin{bmatrix} 0 \\ \frac{-NK}{MG-N^2} \\ \frac{GK}{MG-N^2} \end{bmatrix},$$

$M=J_{arm}+mR^2,\qquad N=mlR,\qquad G=J_{pen}+ml^2,$
$H=-mgl.$

2. SLIDING MODE CONTROLLER

Assume the state equation (2) can be transformed into the following from

$$\dot{\hat{x}}_i=\hat{x}_{i+1}$$
$$i=1,2,K,n-1$$
$$\dot{\hat{x}}_i=a_1\hat{x}_1+a_2\hat{x}_2+\cdots+a_n\hat{x}_n+bu+f(x,u,t)$$

....(3)

Where $|f(x,u,t)|\le f_{\max}$, $|e(x,u,t)|\le e_{\max}$
Use $\tilde{x}$ to denote the error between the state variable $\hat{x}$ and the desired value x_d.
Then the sliding surface is defined as

$$s(x)=C^T\tilde{X}=\sum_{i=1}^{n}c_i\tilde{x}_i,\ c_i=constan(t),\ c_n=1 \quad \text{....(4)}$$

Based on the hitting condition and the characteristics of the u_{eq}, the controller is proposed as follows

$$u=\widehat{u_{eq}}+u_N+u_S \quad \text{...(5)}$$

Where we defined

$$\widehat{u_{eq}}=-b^{-1}\left\{\sum_{i=1}^{n-1}c_i\widetilde{x^{(i)}}-x_d^{(n)}+\sum_{i=1}^{n}a_i\hat{x}_i\right\} \quad \text{.......................(6)}$$

$$u_s=-K_SS \quad \text{...(7)}$$

And assume

$$u_N=-K_Nsgn(s) \quad \text{..(8)}$$

to compensate the uncertainty, $f(x,u,t)$ and $e(x,u,t)$.
From equations (5) (6) (7) (8), and due to the hitting condition $s\dot{s}\leftarrow\eta s^2$ (η is a positive constant) we got the following control laws :

$$K_S>\frac{\eta}{b}$$

$$K_N>\frac{f_{\max}+e_{\max}}{b\cdot s\cdot sgn(s)}$$

....(9)

Therefore, if K_S and K_N is big enough to satisfy the hitting condition the system will be stable. However, in order to rejection the uncertainty and disturbance a larger C and K_N is needed. Enlarging the value these parameters causes the chattering increasing.

3. FUZZY SLIDING MODE CONTROLLER

For dealing with the deficiency of the conventional sliding surface, we proposed a time varying sliding surface by applying the fuzzy algorithm which is changing when error changes as the following form:

R^j : if x_1 is A_1^j, x_2 is A_2^j, ... ,x_n is A_n^j,

Then $S^j=\sum_{i=1}^{n}c_i^j\tilde{x}_i,\ c_n^j=1$

Where $R^j\ (j=1,2,\cdots,k)$ denotes the jth fuzzy implication, k is the number of fuzzy implications, $x_1,x_2,\cdots,x_n$ are input variables, S_j is the jth switching surface, $A_1,A_2,\cdots,A_n$ are fuzzy membership functions [2].
Then S can be denoted as

$$S=\sum_{j=1}^{k}(c_1^j\tilde{x}_1\beta_j+c_2^j\tilde{x}_2\beta_j+\cdots c_n^j\tilde{x}_n\beta_j) \quad \text{.....................(10)}$$

$$\text{Where } \beta_j=\frac{A_1^j(x_1)\wedge\cdots\wedge A_n^j(x_n)}{\sum_{j=1}^{k}(A_1^j(x_1)\wedge\cdots\wedge A_n^j(x_n))} \quad \text{..............(11)}$$

Defined $\hat{c_i} = \sum_{j=1}^{k} c_i^j \beta_j$ ....(12)

Then $\widehat{u_{eq}}$ can be rewritten into

$$\widehat{u_{eq}} = -b^{-1}\left(\sum_{i=1}^{n-1} \hat{c_i}\overline{x^{(i)}} - x_d^{(n)} + \sum_{i=1}^{n} a_i \hat{x_i}\right) \quad \text{.....................(13)}$$

Moreover, the upper bound of system uncertainties and external disturbance is variational. To obtain a smallest control quantity to compensate these uncertainties another fuzzy inference mechanism is used [6] [7].

R^j: If S is A_1^j, $\dot{s}$ is A_2^j, Then $K_N = \alpha^j$.

Then the fuzzy output to K_N can be denoted as

$$K_N = \frac{\sum_{j=1}^{k} \omega^j \alpha^j}{\sum_{j=1}^{k} \omega^j} = \alpha W^T \quad \text{..(14)}$$

Where

$$\alpha = [\alpha^1 \ \alpha^2 \ \Lambda \ \alpha^k], \quad W = \frac{[\omega^1 \ \omega^2 \ \Lambda \ \omega^k]}{\sum_{j=1}^{k} \omega^j}.$$

Assume there exists a idealization $\widehat{K_N}$ which achieves minimum control effort can be written in the following from;

$\widehat{K_N} = \hat{\alpha} W^T$ ....(15)

And use $\tilde{\alpha}$ to denote the error between α and the idealization vector $\hat{\alpha}$.

$\tilde{\alpha} = \alpha - \hat{\alpha}$ ....(16)

Choose the Lyapunov function candidate is

$$V = \frac{1}{2}\left(S^2 + \frac{1}{\gamma}\tilde{\alpha}\alpha^T\right) \quad \text{..(17)}$$

Differentiate V with respect to time and substitute equation (5) (7) (8) (13) (14)

$$\begin{aligned}
\dot{V} &= s\dot{s} + \frac{b}{\gamma}\tilde{a}\dot{\tilde{a}}^T \\
&= s\left(\sum_{i=1}^{n-1} \hat{c_i}\overline{x^{(i)}} - x_d^{(n)} + \sum_{i=1}^{n} a_i\hat{x_i} + b\widehat{u_{eq}} - bK_s s \atop - bK_n sgn(s) + f + e\right) + \frac{1}{\gamma}\tilde{a}\dot{\tilde{a}}^T \\
&= s\left(\sum_{i=1}^{n-1} \hat{c_i}\overline{x^{(i)}} - x_d^{(n)} + \sum_{i=1}^{n} a_i\hat{x_i} + b\widehat{u_{eq}} - bK_s s \atop - b\widehat{K_N} sgn(s) + f + e\right) \\
&\quad + bS\widehat{K_N} sgn(s) - bsK_N sgn(s) + \frac{1}{\gamma}\tilde{a}\dot{\tilde{a}}^T \\
&< \eta S^2 - \left(b|S|W^T - \frac{1}{\gamma}\dot{\tilde{a}}\right)\tilde{a}
\end{aligned}$$

....(18)

Defined the adaptive law as

$\dot{a}^T = \dot{\tilde{a}}^T = \gamma b|S|W^T$ ....(19)

Then

$\dot{V} < -\eta S^2 < 0$ ....(20)

From equation (20) it is obviously that the parameter to the output membership is keep changing until S equals to 0. Then the minimum bound of uncertainties can be obtained.

4. SIMULATION

By transforming, equation (2) can be turned into the following equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 59.3 & 0.074 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + f \quad \text{..................(21)}$$

In the simulation we will compare with a conventional sliding mode controller and a fuzzy sliding-mode controller. The sliding surface is defined as $S = x_3 + 20x_2 + 100x_1$. And the sampling time is 50ms. the K_S value in both simulation is 20, K_N in conventional sliding mode is 2. To smooth the chattering we defined

$$sgn(S) = \frac{S}{|S| + \delta} \quad \text{...(22)}$$

Where δ is a positive constant (0.1 here). The simulation result is shown in figure 6 and 7.

In the fuzzy sliding mode, two fuzzy inference mechanisms are used, which are with conventional triangular shapes. In order to reduce the calculation works, we try to use as less rules as possible. Figure 8~9 show the simulation results to the fuzzysliding mode.

Fuzzy rules and memberships to the time-varying surface:

R_1: If $|x_1|$ is S and $|x_2|$ is S and $|x_3|$ is S, then $S_1 = \hat{x_3} + 12\hat{x_2} + 36\hat{x_1}$

R_2: If $|x_1|$ is M or $|x_2|$ is M or $|x_3|$ is M, then $S_2 = \hat{x_3} + 18\hat{x_2} + 81\hat{x_1}$

R_3: If $|x_1|$ is B or $|x_2|$ is B or $|x_3|$ is B, then $S_5 = \hat{x_3} + 20\hat{x_2} + 100\hat{x_1}$

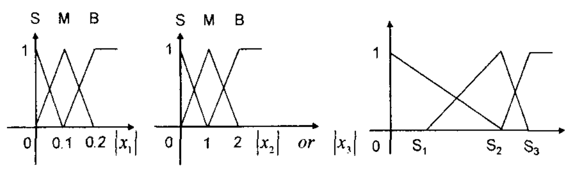

Fig.2: Memberships to S

Fuzzy rules and memberships to K_N are shown as follows where the initialization to α is [0 1 2], the constant γ is 0.2 :

R_1: If $|S|$ is S and $|\dot{S}|$ is S, then $K_N = \alpha^1$

R_2: If $|S|$ is M or $|\dot{S}|$ is M, then $K_N = \alpha^2$

R_3: If $|S|$ is B or $|\dot{S}|$ is B, then $K_N = \alpha^3$

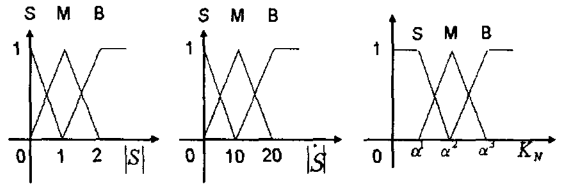

Fig.4: Memberships to K_N.

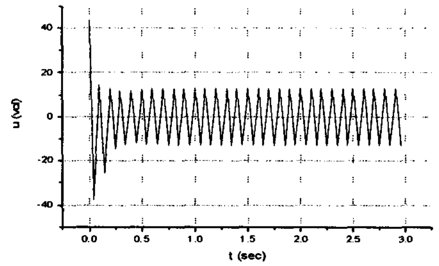

Fig.7 : Control output to conventional sliding mode.

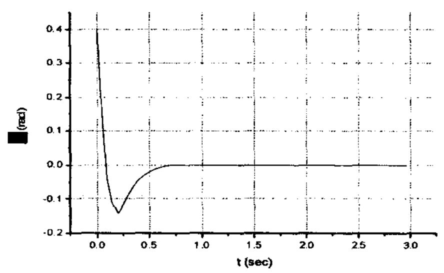

Fig.6 : Pendulum angle to conventional sliding mode.

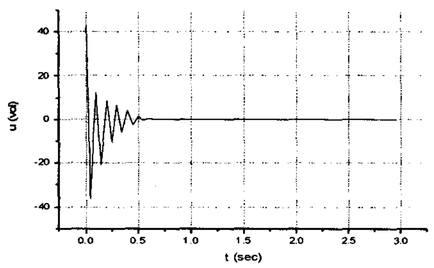

Fig.9 : Control output to fuzzy sliding mode.

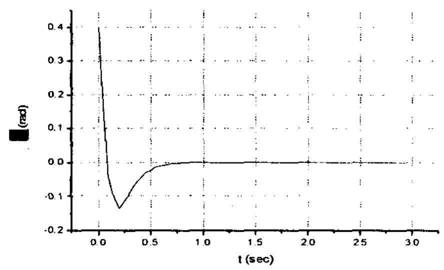

Fig.8 : Pendulum angle to fuzzy sliding mode.

5. CONCLUSIONS

In this paper, the sliding-mode control's robustness against the perturbations and model uncertainties was shown distinctly. Compare with the conventional sliding- mode, the applying of fuzzy inference mechanisms in this paper resolve the chattering phenomenon. Moreover, the use of adaptive law here provides an available algorithm to obtain the upper bound to the uncertainties. In the further research, the experimentations will be done to demonstrate the result.

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