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비국소조건을 가지는 충격 준선형퍼지적분미분방정식에 대한 제어가능성

Controllability for the Impulsive Semilinear Fuzzy Integrodifferential Equations with Nonlocal Conditions

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Abstract

In this paper. we study the controllability for the impulsive semilinear fuzzy integrodifferential equations with nonlocal condition in E_N by using the concept of fuzzy number whose values are normal, convex, upper semicontinuous and compactly supported interval in E_N.

Key Words : controllability, impulsive, fuzzy number, integrodifferential equation, nonlocal

1. Introduction

Many authors have studied several concepts of fuzzy systems. Kaleva ([3]) studied the existence and uniqueness of solution for the fuzzy differential equation on E^n where E^n is normal, convex, upper semicontinuous and compactly supported fuzzy sets in R^n. Seikkala ([6]) proved the existence and uniqueness of fuzzy solution for the following equation:

$$\dot{x}(t) = f(t, x(t)),\ x(0) = x_0,$$

where f is a continuous mapping from $R^+ \times R$ into R and x_0 is a fuzzy number in E^1. Kwun and Park ([4]) proved the existence of fuzzy optimal control for the nonlinear fuzzy differential system with nonlocal initial condition in E_N^1 using by Kuhn-Tucker theorems. Balasubramaniam and Muralisankar([1]) investigated the existence and uniqueness of fuzzy solutions for the semilinear fuzzy integrodifferential equation with nonlocal initial condition. Recently, Park, Park and Kwun[7] are proved the sufficient condition of exact controllability for semilinear fuzzy integrodifferential equations with nonlocal conditions.

In this paper, we consider the following impulsive semilinear fuzzy integrodifferential equations with nonlocal condition:

$$\frac{dx(t)}{dt} = A\left[x(t) + \int_0^t G(t-s)x(s)ds\right] + f(t,x) + u(t),\ t \in I = [0, T],\ t \neq t_k \quad (1)$$

$$x(0) + g(x) = x_0 \in E_N, (2)$$

$\Delta x(t_k) = I_k(x(t_k)), k = 1, 2, \ldots, m,$ (3)
where $A : I \to E_N$ is a fuzzy coefficient, E_N is the set of all upper semicontinuous convex normal fuzzy numbers with bounded α-level intervals, $f : I \times E_N \to E_N$ is a nonlinear continuous function, $G(t)$ is $n \times n$ continuous matrix such that $\frac{dG(t)x}{dt}$ is continuous for $x \in E_N$ and $t \in I$ with $|G(t)| \le k$, $k > 0$, $u : I \to E_N$ is control function and $g : I \times E_N \to E_N$ is a nonlinear continuous function, $I_k \in C(E_N, E_N)$ are bounded functions, $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, where $x(t_k^+)$ and $x(t_k^-)$ represent the left and right limit of $x(t)$ at $t = t_k$, respectively. Control function $u : I \to E_N$.

In this paper, we find the sufficient conditions of nonlocal controllability for the control system (1)-(3).

2. Preliminaries

We denote the suprimum metric d_∞ on E^n and the suprimum metric H_1 on $C(I : E^n)$.

Definition 1. Let $a, b \in E^n$.

$d_\infty(a, b) = \sup\{d_H([a]^\alpha, [b]^\alpha) : \alpha \in (0, 1]\}$

where d_H is the Hausdorff distance.

Definition 2. Let $x, y \in C(I : E^n)$

$H_1(x, y) = \sup\{d_\infty(x(t), y(t)) : t \in I\}$.

Let I be a real interval. A mapping $x : I \to E_N$ is called a fuzzy process. We denote

$[x(t)]^\alpha = [x_l^\alpha(t), x_r^\alpha(t)], \; t \in I, 0 < \alpha \le 1.$

The derivative $x'(t)$ of a fuzzy process x is defined by

$[x'(t)]^\alpha = [(x_l^\alpha)'(t), (x_r^\alpha)'(t)], \; 0 < \alpha \le 1.$

provided that is equation defines a fuzzy $x'(t) \in E_N$.

The fuzzy integral

$$\int_a^b x(t)\,dt, \qquad a, b \in I$$

is defined by

$$\left[\int_a^b x(t)\,dt\right]^\alpha = \left[\int_a^b x_l^\alpha(t)\,dt, \int_a^b x_r^\alpha(t)\,dt\right]$$

provided that the Lebesgue integrals on the right exist.

Definition 3. The fuzzy process $x : I \to E_N$ is a solution of equations (1)-(2) without the inhomogeneous term if and only if

$$(\dot{x}_l^\alpha)(t) = \min\{A_l^\alpha(t)[x_j^\alpha(t) + \int_0^t G(t-s) x_j^\alpha(s)ds], \; i, j = l, r\},$$

$$(\dot{x}_r^\alpha)(t) = \max\{A_r^\alpha(t)[x_j^\alpha(t) + \int_0^t G(t-s) x_j^\alpha(s)ds], \; i, j = l, r\},$$

and

$(x_l^\alpha)(0) = x_{0l}^\alpha - g_l^\alpha(x),$

$(x_r^\alpha)(0) = x_{0r}^\alpha - g_r^\alpha(x).$

3. Existence and Uniqueness of Fuzzy Solutions

In order to define the solution of (1)-(3), we shall consider the space $\Omega = \{x : I \to E_N | x_k \in C(I_k, E_N), J_k = (t_k, t_{k+1}], \quad k = 0, \ldots, m,$ and there exist $x(t_k^-)$ and $x(t_k^+)$ $(k = 1, \ldots, m)$, with $x(t_k^-) = x(t_k)\}$.

(H1) The nonlinear function $g : I \times E_N \to E_N$ is a continuous function and satisfies the inequary

$d_H([g(x)]^\alpha, [g(y)]^\alpha) \le c_1 d_H([x]^\alpha, [y]^\alpha)$

for all $x(\cdot), y(\cdot) \in E_N$, c_1 is a finite positive constant.

(H2) The inhomogeneous term $f : I \times E_N \to E_N$ is a continuous function and satisfies a global Lipschitz condition

$$d_H([f(s, x(s))]^\alpha, [f(s, y(s))]^\alpha \le c_2 d_H([x(s)]^\alpha, [y(s)]^\alpha),$$

for all $x(\cdot), y(\cdot) \in E_N$, and a finite positive constant $c_2 > 0$.

(H3) $S(t)$ is a fuzzy number satisfying for $y \in E_N, S'(t)y \in C^1(I : E_N) \cap C(I : E_N)$ the equation

$$\frac{d}{dt} S(t)y = A[S(t)y + \int_0^t G(t-s)S(s)yds]$$

$$= S(t)Ay + \int_0^t S(t-s)AG(s)yds, \quad t \in I,$$

such that $[S(t)]^\alpha = [S_l^\alpha(t), S_r^\alpha(t)]$, $s(0) = I$ and $S_i^\alpha(t)(i = l, r)$ is continuous. That is, there exists a constant $c > 0$ such that $|S_i^\alpha(t)| \le c$ for all $t \in I$.

(H4) There exists $d_k, k = 1, \ldots, m$, such that

$$d_H([I_k(x(t_k^-))]^\alpha, [I_k(y(t_k^-))]^\alpha) \le d_k d_H([x(t)]^\alpha, [y(t)]^\alpha),$$

where $\sum_{k=1}^m d_k = \bar{d}$.

(H5) $c(c_1 + C - 2T + \bar{d}) < 1$.

Lemma 1. If x is an integral solution of (1)-(3), then x is given by

$$x(t) = S(t)(x_0 - g(x)) + \int_0^t S(t-s) \times f(s, x(s))ds + \sum_{0 < t_k < t} S(t - t_k) \times I_k(x(t_k^-)) \quad \text{for} \quad t \in I. \quad (4)$$

Theorem 1. Let $T > 0$, and hypotheses (H1)-(H5) hold. Then for every $x_0 \in E_N$, the fuzzy initial value problem (1)-(3) without control function has a unique solution $x \in \Omega' = \Omega \cap C(I : E_N)$.

4. Nonlocal controllability

In this section, we show the nonlocal controllability for the control system (1)-(3).

Definition 4. The equation (4) is nonlocal controllable if, there exists $u(t)$ such that the fuzzy solution $x(t)$ of (4) satisfies $x(T) = x^1 - g(x)$ where x^1 is target set.

Assume that the following hypotheses:

(H6) Linear system of equation(4)$(f = 0)$ is nonlocal controllable.

(H7) $c[(c_1 + \bar{d})(1 + T) + c_2(1 + T)T] < 1$.

Theorem 2. Suppose that hypotheses (H1) -(H7) are satisfied. Then the equation(4) is nonlocal controllable.

4. Example

Consider the semilinear one-dimensional heat equation on a connected domain $(0, 1)$ for a material with memory, boundary conditions $x(t, 0) = x(t, 1) = 0$ and with initial condition $x(0, z) + \sum_{k=1}^p c_k x(t_k, z) = x_0(z)$ where $x_0(z) \in E_N$. Let $x(t, z)$ be the internal energy and $f(t, x(t, z)) = \tilde{2} tx(t, z)^2$ be the external heat. $\Delta x(t_k, z) =$

$x(t_k^+, z) - x(t_k^-, z)$ is impulsive effect at $t = t_k (k = 1, \ldots, m)$. Let $A = \tilde{2}\frac{\partial^2}{\partial z^2}$, $\sum_{k=1}^{p} c_k x(t_k z) = g(x)$ $\Delta x(t_k, z) = \Delta x(t_k)$, $x(t_k^+, z) - x(t_k^-, z) = I_k(x(t_k))$ and $G(t-s) = e^{-(t-s)}$ then the balace equation becomes

$$\frac{dx(t)}{dt} = \tilde{2}[x(t) - \int_0^t e^{-(t-s)} x(s)ds] + \tilde{2}tx(t)^2 + u(t), \quad (5)$$

$$x(0) + g(x) = x_0 \quad (6)$$

$$\Delta x(t_k) = I_k(x(t_k)),\ k = 1, \ldots, m. \quad (7)$$

The α-level set of fuzzy number $\tilde{2}$ is

$$[\tilde{2}]^\alpha = [\alpha + 1, 3 - \alpha] \text{ for all } \alpha \in [0,1].$$

Then α-level set of $f(t, x(t))$ is

$$[f(t, x(t)]^\alpha = t[(\alpha+1)(x_l^\alpha(t))^2, (3-\alpha)(x^{\alpha_r}(t))^2]. \quad (8)$$

Further,

$$d_H([f(t, x(t))]^\alpha, [f(t, y(t))]^\alpha)$$
$$= d_H(t[(\alpha+1)(x_l^\alpha(t))^2, (3-\alpha)(x_r^\alpha(t))^2], t[(\alpha+1)(y_l^\alpha(t))^2, (3-\alpha)(y_r^\alpha(t))^2])$$
$$= t\max\{ (\alpha+1)|(x_l^\alpha(t))^2 - (y_l^\alpha(t))^2|, (3-\alpha)|(x_r^\alpha(t))^2 - (y_r^\alpha(t))^2| \}$$
$$\le 3T|x_r^\alpha(t) + y_r^\alpha(t)]|\max\{ |x_l^\alpha(t) - y_l^\alpha(t)|, |x_r^\alpha(t) - y_r^\alpha(t)| \} = c_2 d_H([x(t)]^\alpha, [y(t)]^\alpha)$$

where c_2 is satisfies the inequality in hypothesis (H7), and also

$$d_H([g(x)]^\alpha, [g(y)]^\alpha)$$
$$= d_H(\sum_{k=1}^{p} c_k[x(t_k)], \sum_{k=1}^{p} c_k[y(t_k)]^\alpha)$$
$$\le |\sum_{k=1}^{p} c_k| \, max_k d_H([x(t_k)]^\alpha, [y(t_k)]^\alpha)$$
$$= c_1 d_H([x(\cdot)]^\alpha, [y(\cdot)]^\alpha)$$

where c_1 is satisfies the inequality in hypothesis (H7).

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