A VIRTUAL BOUNDARY METHOD FOR SIMULATION OF FLOW OVER SWIMMING STRINGS

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In the present study, we propose a virtual boundary method for simulation of massive inextensible flexible strings immersed in viscous fluid flow. The fluid motion is governed by the Navier-Stokes equations and a momentum forcing is added in order to bring the fluid to move at the same velocity with the immersed surface. A massive inextensible flexible string model is described by another set of equations with an additional momentum forcing which is a result of the fluid viscosity and the pressure difference across the string. The momentum forcing is calculated by a feedback loop. Simulations of several numerical examples are carried out, inlcuding a hanging string which starts moving under gravity without ambient fluid, a string swimming within a uniform flow and a uniform flow over two side-by side strings. The numerical results agree well with the theoretical analysis and previous experimental observations. Preliminary results of a swimming elongated fishlike body will also be presented.

Key Words: Virtual Boundary Method, Fluid/Structure Interaction, Inextensible Flexible String

1. Introduction

In nature, many phenomena involve interactions between the flexible bodies and their surrounding viscous fluids, such as a swimming fish or flapping flags. The intrinsic dynamics is complicate and not well understood. A flexible string can be regarded as a one-dimesional flag model. Many similarities can be found between the flapping string and swimming fish, although the different wake speed results in drag force for the former and in propulsion force for the latter.

A literature survey reveals that several methods have been developed for simulating fluid/flexible-body interactions. The distributed-Lagrange-multiplier/fictitious domain method originally proposed by Glowinski et al.[1], which was based on the finite element method, has recently been applied to simulate flow over 2D flexible plate[2,3,4]. Another one is Peskin's immersed boundary method[5], which has recently been used to simulate a flapping string immersed in 2D soap film[6, 7], in order to compare with the experiment carried out by Zhang et al.[8].

The virtual boundary method was proposed independently by Goldstein et al.[9] and was then developed by Saiki & Biringen [10] and Lee[11]. It shares the same spirit of Pesking's immersed boundary method, while it uses a feedback forcing to bring the fluid to rest at solid body surfaces. In the present study, the virtual boundary method is extended to the fluid/flexible-body interaction problems.

2. Numerical Methods

2.1 Numerical model

The Navier-Stokes equations for unsteady viscous fluid flow are

$$\rho_f \frac{D\boldsymbol{u}}{Dt} = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \boldsymbol{f}_{ib} \quad (1)$$

$$\nabla \cdot \boldsymbol{u} = 0 \quad (2)$$

where $\boldsymbol{u}$ denotes the velocity, p denotes the pressure, ρ_f denotes the fluid density, μ denotes the dynamic viscosity, and $\boldsymbol{f}_{ib}$ denotes the force exerted by the immersed structure on the fluid.

For an inextensible flexible string, the governing equations are

$$\rho_s \frac{\partial^2 \boldsymbol{X}_{ib}}{\partial t^2} = \frac{\partial}{\partial s}\left(\sigma \frac{\partial \boldsymbol{X}_{ib}}{\partial s}\right) - \frac{\partial^2}{\partial s^2}\left(\gamma \frac{\partial^2 \boldsymbol{X}_{ib}}{\partial s^2}\right) + \rho_s \boldsymbol{g} - \boldsymbol{F}_{ib} \quad (3)$$

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$$\frac{\partial \boldsymbol{X}_{ib}}{\partial s}\cdot\frac{\partial \boldsymbol{X}_{ib}}{\partial s}=1 \qquad (4)$$

where X_{ib} denotes the string position, ρ_s denotes the line density of string, σ denotes the stretching coefficient, γ denotes the bending coefficient, g denotes the gravity force, and F_{ib} denotes the force exerted by fluid on string. Eq.(4) is the constraint of the inextensible condition, which bahaves like the incompressible condition for fluid motion. Using Eq.(4) we can obtain the poisson equation for σ

$$\frac{\partial \boldsymbol{X}}{\partial s}\cdot\frac{\partial^2}{\partial s^2}\left(\sigma\frac{\partial \boldsymbol{X}}{\partial s}\right)=\frac{1}{2}\frac{\partial^2}{\partial t^2}\left(\frac{\partial \boldsymbol{X}}{\partial s}\cdot\frac{\partial \boldsymbol{X}}{\partial s}\right)-\frac{\partial^2 \boldsymbol{X}}{\partial s\partial t}\cdot\frac{\partial^2 \boldsymbol{X}}{\partial s\partial t}+\frac{\partial \boldsymbol{X}}{\partial s}\cdot\frac{\partial^3}{\partial s^3}\left(\gamma\frac{\partial^2 \boldsymbol{X}}{\partial s^2}\right) \qquad (5)$$

where the first term on the right hand side is left for the numerical purpose.

Note that Eqs.(3) and (4) are writen in Lagrangian form, while Eqs.(1) and (2) are writen in Eulerian form. The transformation between these two forms are realized by Dirac delta function. Here the interaction force F_{ib} is simply given by feedback law

$$\boldsymbol{F}_{ib}=\alpha\int(\boldsymbol{U}-\boldsymbol{U}_{b})dt+\beta(\boldsymbol{U}-\boldsymbol{U}_{b}) \qquad (6)$$

where α and β are large negative coefficients, $U_{ib}=\partial X_{ib}/\partial t$, and U is the fluid velocity interpolated at the string position

$$\boldsymbol{U}(s,t)=\int_{\Omega}\boldsymbol{u}(\boldsymbol{x},t)\delta(\boldsymbol{X}(s,t)-\boldsymbol{x})d\boldsymbol{x} \qquad (7)$$

where δ is Dirac delta function. Then f_{ib} in Eq.(1) is given by

$$\boldsymbol{f}_{ib}(\boldsymbol{x},t)=\int_{\Gamma}\boldsymbol{F}_{ib}(s,t)\delta(\boldsymbol{x}-\boldsymbol{X}(s,t))ds \qquad (8)$$

2.2 Computational scheme

The N-S equations are solved by the fractional step method on a staggered Cartesian grid. The velocity components and momentum forcings are defined on the staggered grid, whereas the pressure are applied at the centers of cells. Fully implicit time advancement is employed, where the Crank-Nicholson scheme is used for the discretization ofthe diffusion and convection terms. Decoupling of the velocity and pressure is achieved by block LU decomposition in conjunction with approximate factorization. The pressure Poisson equation is solved by iteration using the multigrid method. The pressure is then used to correct the velocity field to satisfy the continuity equation. No iteration is needed to solve the velocity field. Details regarding the N-S solver can be found in Kim et al. [12].

To solve the 1D inextensible string model, Eq.(5) is first solved on a staggered grid to obtain σ. Then Eq.(3) is solved to update the position. It is found that the inextensiblity is well satisfied without any iteration. More details will be explained in the presentation.

The interaction force is simply calculated in an explicit form.

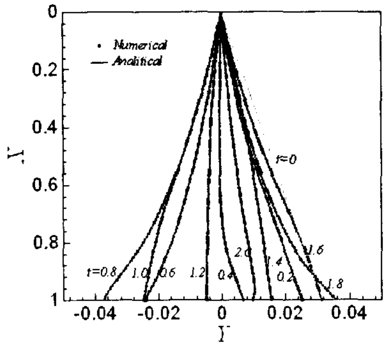

Fig.1 A hanging string starts moving under gravity without ambient fluid.

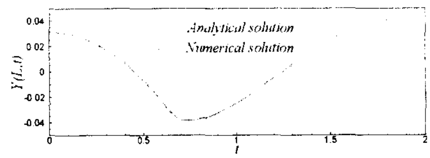

Fig.2 Time history of position of the free end of string

3. Numerical Results

3.1 A hanging string without ambient fluid

For small amplitude, Eqs.(3) and (4) can be linearized and the analytical solution is derived by using Bessel functions. Figure 1 shows that a hanging string starts moving under gravity without ambient fluid. The initial position is

$$Y(s,0)=ks \qquad (9)$$

In the present case, we use $k=0.01\pi$. The numerical results agree well with the analytical solution, as shown in Fig.1. Figure 2 shows a time history of the position of the free end of string. We can see the agreement between numerical simulation and analytical solution is excellent.

3.2 A hanging string immersed in uniform flow

In Zhang et al.'s experiment[8], they found that for a hanging string immersed in flowing soap film, there are two distinct stable states: the stretched-straight state and the self-sustained flapping state, depending on its initial situation. In the present study, simulations of uniform flow over a hanging string are carried out. Figure 3 shows the instantaneous vorticity contours of flow over a hanging string with two different initial conditions at $Re=200$. For $k=0.1\pi$(Fig.3a), votices are shedding alternatively due to the flapping string, while for $k=0.001\pi$(Fig.3b), the flapping amplitude is decaying and no vortex is shedding.

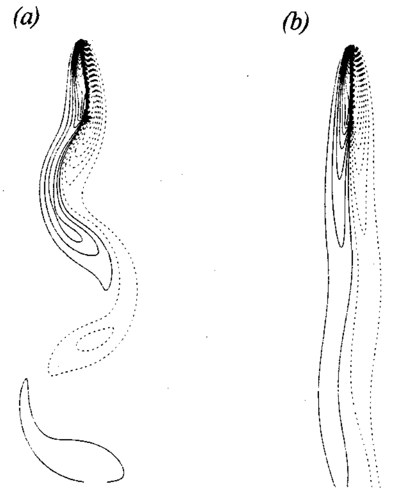

Fig. 3 Instantaneous vorticity contours of flow over a hanging string at $Re=200$: (a) $k=0.1\pi$; (b) $k=0.001\pi$.

Figure 4 shows the instantaneous vorticity contours at $Re=500$ and $Re=1000$. It can be seen that with increasing Reynolds number, the vortex size decreases and the number of vortices shedding during one flapping period increases. At $Re=200$, as shown in Fig.3a, the vortex size is comparable with the string length, and there are a positive and a negative vortex shedding alternatively. At $Re=500$(Fig.4a), the vortex size becomes smaller and there are two positive and two negative vortices shedding during one period, while at $Re=1000$ (Fig.4b), the size is smaller and the number increases to three. In Zhang et al.'s experiment[8], the Reynolds number is of the order of 10^4, so we can see many fine vortex structures shedding from the string. In the present simulations, although the Reynolds numbers are much lower, we can find the tendency with increasing Reynolds number is consistent with the experiment.

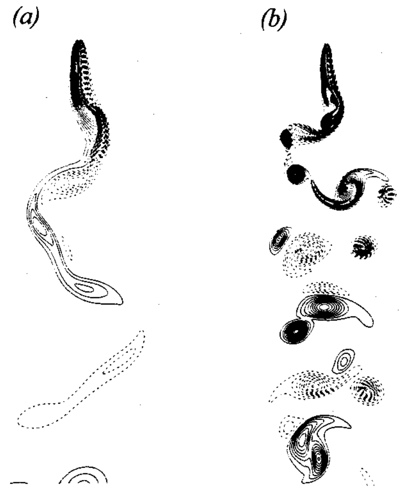

Fig.4 Instantaneous vorticity contours of flow over a hanging string at (a)$Re=500$; (b)$Re=1000$

3.3 a uniform flow over two side-by side strings

The hydrodynamically coupled interaction between two side-by-side strings are more complicated. Zhang et al.[8] found that the two strings tended to flap in phase with each other if the inter distance is sufficiently small, while as the distance increases, the strings switched to become locked in out-of-phase and flapped symmetrically about the center line.

Numerical simulations of this problem are carried out in the present study. Figure 5 shows the instantaneous vorticity contours for the inter distance of $0.1L$ at $t=17.1$ and 18.0. We can see that the two strings are locked in phase and flap parallel to each other. Figure 6 shows the instantaneous vorticity contours for the inter distance of L at $t=27.6$ and 28.5. It is interesting to find that although the initial status is parallel to each other, the two strings are finally locked out-of-phase and flap symmetrically about the center line. All these properties agree well with the experimental findings.

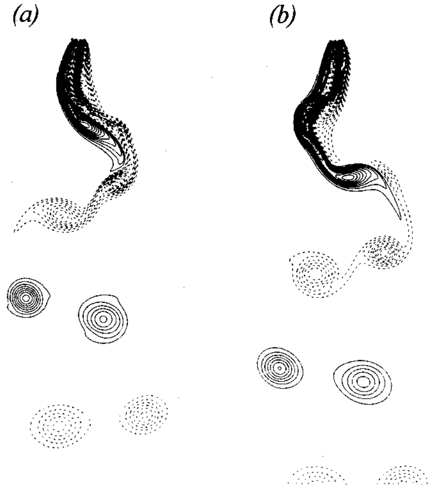

Fig.5 Instantaneous vorticity contours of flow over two side-by-side strings with inter distance of $0.1L$ at (a) $t=17.1$ and (b) $t=18.0$.

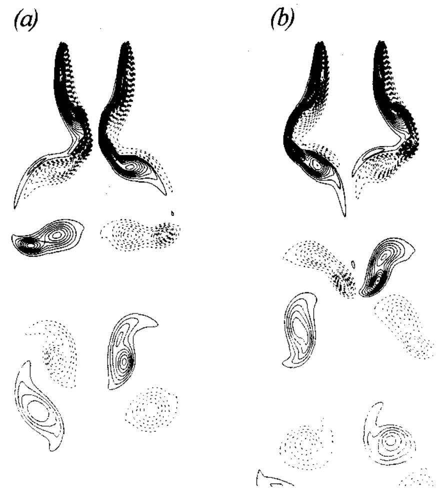

Fig.6 Instantaneous vorticity contours of flow over two side-by-side strings with inter distance of L at (a) $t=27.6$ and (b) $t=28.5$.

4. Conclusions

In the present study, we propose a virtual boundary method for simulation of flow over massive inextensible flexible strings. The fluid motion is governed by the Navier-Stokes equations and a momentum forcing is added in order to bring the fluid to move at the same velocity with the immersed surface. A massive inextensible flexible string model is described by another set of equations with an additional momentum forcing which is a result of the fluid viscosity and the pressure difference across the string. The momentum forcing is calculated by a feedback loop. Simulations of several numerical examples are carried out. A hanging string which starts moving under gravity without ambient fluid is simulated and the analytical solution of motion with small amplitude is also obtained by the perturbation technology. The agreement of the numerical and analytical solution is excellent. A hanging string swimming within a uniform flow is then studied. Two distinct stable states (stretched-straight or self-sustained flapping) are observed, agreeing with the experimental results. Simulation of a uniform flow over two side-by-side strings is also carried out for comparison with the experiments. In-phase flapping is observed at small spacing between the two strings, while anti-phase flapping is resulted at large spacing. Further simulations regarding a swimming fish is under consideration.

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