한국해양공학회 2006년도 추계학술대회 국제워크샵 pp. 113~120

The Spectrally Accurate Method Applied to Wave-Current Interaction as a Freak Wave Generation Mechanism

Hong Gun Sung[a], Key Yong Hong[b], Jo Hyun Kyoung[c] and Sa Young Hong[d]

Maritime and Ocean Engineering Research Institute, KORDI, Daejeon, Korea

[a]hgsung@moeri.re.kr, [b]keyhong@moeri.re.kr, [c]johyun@moeri.re.kr, [d]sayhong@moeri.re.kr

Keywords: Freak Wave, Wave-Current Interaction, Spectral Element Method, Arbitrary Lagrangian-Eulerian (ALE)

Abstract. In this paper, generation mechanisms of ocean freak waves are briefly introduced in the context of wave-current interaction phenomena. As an accurate and efficient numerical tool, the spectral element method is presented with general features and specific treatment for the wave-current interaction problem. The present model of the fluid motion is based on the Navier-Stokes equations incorporating a velocity-pressure formulation. In order to deal with the free surface motion, an Arbitrary Lagrangian-Eulerian (ALE) description is adopted. As an intermediate stage of development, solution procedure and characteristic aspects of the present modeling and numerical method features are addressed in detail, and numerical results for wave-current interaction is left as further study.

Introduction

Among wave phenomena in ocean environments, the freak waves, which are also called rogue or giant waves, are unexpectedly and surprisingly large-amplitude waves. Due to the very rare occurrence of freak waves, they have been disregarded in assessing the safety of marine vessels. According to the recent survey, several observations tell that freak waves are appearing in arbitrary depth of ocean and can potentially do considerable damage to ocean-going vessels ([1]). The cause of freak waves is attributed to spatial focusing of water waves, the wave-current interaction, and modulational instability. However, because of inherent complexity and lack of accumulated data, it is believed that generation mechanism of freak waves is not manifested completely ([1], [2]).

Currently, we, an ocean engineering research team of MOERI/KORDI, have been developing an accurate numerical modeling and efficient methodology for wave-current interaction, which is known as one of freak wave generation mechanism. In this paper, we describe the general aspects of wave-current interaction phenomenon, and the present modeling and numerical method.

Wave-Current Interaction

As noted in [1] and [3], it is believed that variable currents can lead to the formation of freak waves and wave-current interaction is a major mechanism of the freak wave phenomenon in deep water. This is the primary motivation of the present study.

Most classical theoretical approaches to the wave-current interaction problem such as [4] and [5] have utilized the averaged-Lagrangian concept or averaged equation of fluid motion. However, those kinds of theory seem to be pertinent to weakly nonlinear waves and so we believe that freak waves need a new analysis framework for better understanding and precise prediction. It is mainly attributed to the fact that we suppose freak waves belong to very extreme and so highly nonlinear phenomena.

Wolf & Prandle ([6]) summarized various mechanisms for wave-current interaction. To name a few, Doppler shift, wave steepening on an opposing current, and modulation of absolute frequency are to be noted in the context of freak waves. Among theoretical works, Stoker and Peregrine ([7]) suggested a current-modified nonlinear Schrödinger equation model, applicability of which we doubt because of strong nonlinearity of freak waves.

On the other hand, there have been several efforts to devise prediction method for wave-current interaction. For uniform current, the boundary element method can be used for the potential fluid model as shown in [8]. Non-uniform current, which may vary horizontally and/or vertically, moreover, which we believe one gateway to freak waves, however, does not allow one to use the classical potential theory. Thus we need to have a suitable fluid model of wave interaction with non-uniform current for its better understanding and prediction of wave-current. As described in [9], because the wave field will not be irrotational, we can use the Euler equation model or the Navier-Stokes (N-S) equations. Though we can use a stream function approach in two dimensions, we assume that the Euler or N-S equations are to be preferred for generality and versatility. Another reason is that many useful and robust theories and numerical methods have been developed for the Euler and N-S equations.

According to our literature survey, we could find some trials to utilize the N-S equations for the free surface or wave-current interaction such as [10] ~ [14], but all of them dealt with very simple problems and so we need to extend those methods for the context of freak wave generation.

In the present paper, for solving the governing equation of fluid motion, we require the so-called spectral accuracy by using the spectral element, which is regarded as a high-order finite element method utilizing carefully selected spectral nodes and basis functions for guaranteeing the high-order accuracy as described in [15] and [16]. An alternative interpretation of the spectral element method is a generalization of the global spectral method ([17], [18]) and multi-domain spectral method ([19]) by incorporating the element concept as in the finite element concept.

We note that the spectral accuracy for the free surface waves could be found in [20], and [21], but the convergence of this method is limited up to about 80% of the highest wave, mainly because the corresponding method uses the Taylor's series expansion around the mean water level.

Conclusively, motivated by high-order accuracy of the spectral element method (SEM) and the need to resolve non-uniform currents, we are developing SEM-based wave-current interaction N-S equation model.

Theories vs. Numerical Methods

Theoretical analysis by using such as the nonlinear Schrödinger equation, the Davey-Stewartson system, the extended Dysthe equation, etc. plays an important role in understanding freak wave phenomena ([1]). However, we believe that from the quantitative viewpoint weakly nonlinear models seem to be replaced by the fully nonlinear models of extreme waves. As stated in [1], most of the works related with fully nonlinear fluid models deal with the potential fluid. Hence, they are not able to predict the wave interaction with non-uniform currents as indicated above. We have to resort to other type of fluid model, such as the Euler and N-S equation model.

As for Euler or N-S equations, very few solutions exist even within the linear regime and none in the fully nonlinear regime. Thus we try to build an effective numerical method to solve the N-S equations for viscous free surface flows which closely resemble wave-current interaction as a freak wave generation mechanism.

Formulation

As mentioned above, the present fluid model is the viscous free surface flow, which is depicted in Figure 1. For simplicity of explanation, a definition sketch for two dimensions is shown. The current can be generated by imposing a suitable inflow and outflow condition on the later side of fluid domain, and required wave field is also obtained by specifying the physical wave-maker or any theoretical velocity profile on one lateral side. Computationally, the current flow field may be developed in advance and then the waves generated afterwards for stable solution. The bottom effect is taken into account, and the deep water condition can also be realizable by utilizing a pertinent far-field condition with the use of an approximate theoretical solution.

We assume the fluid to be incompressible, and the flow to be Newtonian. We thus have the N-S equations for the divergence-free velocity vector. In order to describe the free surface motion in this paper, we utilize the Arbitrary Lagrangian-Eulerian (ALE) framework. The corresponding set of governing equations read:

$$\frac{\partial \vec{u}}{\partial t} + N(\vec{u},\vec{w}) = -\frac{1}{\rho}\nabla p + \nu L(\vec{u}) + \vec{f} \; in \; \Omega(t) \tag{1}$$

$$\nabla \cdot \vec{u} = 0 \; in \; \Omega(t) \tag{2}$$

with $\vec{u}(\vec{x},t)$ the velocity field, $\vec{w}(\vec{x},t)$ the ALE mesh velocity vector, $p(\vec{x},t)$ the pressure field, ρ the density of the fluid, ν the kinematic viscosity of the fluid, and $\vec{f}$ the body force vector per unit mass. The underlying Cartesian coordinates is denoted by $\vec{x}$ and the time coordinate by t. The fluid domain in the current time instant, t is $\Omega(t)$. We also define as $N(\vec{u},\vec{w}) = (\vec{u}-\vec{w})\cdot\nabla\vec{u}$, and $L(\vec{u}) = \nabla\cdot\left[\nabla\vec{u} + (\nabla\vec{u})^T\right]$. The boundary surface of the fluid domain consists of the Dirichelt-type boundary, Γ_D and the free surface Γ_f. The former includes side walls and sea bottom for pure physical situations, and vertical truncation boundary and the truncated sea floor for some numerical reasons. We assume that these two boundaries are disjoint except the intersection points between the vertical truncation boundaries and the free surface. In addition, the whole boundary is the sum of the Dirichlet-type boundary and the free surface, i.e. $\Gamma = \Gamma_D \cup \Gamma_f$.

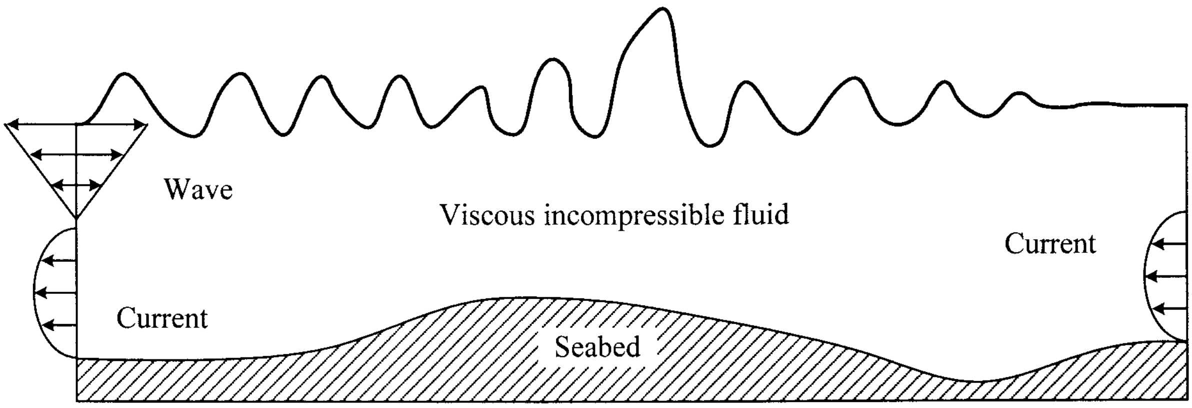

Figure 1. Modeling of wave-current interaction in a numerical basin

The associated boundary conditions are usually the Dirichlet-type as follows.

$$\vec{u} = \vec{u}_D \; in \; \Gamma_D \tag{3}$$

where the vector, $\vec{n}$ is the outward unit normal vector, and the right hand side of the above equation, $\vec{u}_D$ is given in advance.

The free surface motion can be described by applying the dynamic and kinematic compatibility conditions. The dynamic condition of the free surface, Γ_f ensures the continuity of the pressure by equating the fluid stress vector with the external traction:

$$\vec{s}\cdot\nu\left[\nabla\vec{u} + (\nabla\vec{u})^T\right]\cdot\vec{n} = T_s^{ext}/\rho \; in \; \Gamma_f \tag{4}$$

$$\vec{m}\cdot\nu\left[\nabla\vec{u} + (\nabla\vec{u})^T\right]\cdot\vec{n} = T_m^{ext}/\rho \; in \; \Gamma_f \tag{5}$$

$$-p + \vec{n}\cdot\nu\left[\nabla\vec{u} + (\nabla\vec{u})^T\right]\cdot\vec{n} = T_n^{ext}/\rho + \gamma\kappa/\rho \quad in \; \Gamma_f \tag{6}$$

where γ and κ denote the surface tension force coefficient and normal curvature, respectively. The vectors in the above, $\vec{s}$ and $\vec{m}$ are an orthogonal vector pair lying on the tangent plane, and they are related to the unit normal vector by $\vec{n} = \vec{s}\times\vec{m}$. Hence, the vector pair, $(\vec{s},\vec{m},\vec{n})$ constitutes the orthonormal curvilinear coordinates, and so the relation of (4)~(6) is equivalent to the following.

$$\nu\left[\nabla\vec{u} + (\nabla\vec{u})^T\right]\cdot\vec{n} = \left(T_s^{ext}/\rho\right)\vec{s} + \left(T_s^{ext}/\rho\right)\vec{m} + \left(p + T_n^{ext}/\rho + \gamma\kappa/\rho\right)\vec{n} \quad in \; \Gamma_f \tag{7}$$

The external traction or stress vector, $(T_s^{ext}, T_m^{ext}, T_n^{ext})$ is specified by the additional modeling of external force. For example, the external normal traction, T_n^{ext} may include the surface tension effect, and tangential tractions, T_s^{ext}, T_m^{ext} may include the shear stresses due to atmospheric forcing. In the present study, the external traction vector is set to zero, i.e. $(T_s^{ext}, T_m^{ext}, T_n^{ext}) = \vec{0}$ in fact, though the formulation retain it for generality. Instead, it is noted that the effect of atmospheric forcing on the freak waves occurrence is referred to [1].

With the ALE methodology, the following relation enforces the fictitious fluid particle on the free surface to stay on it.

$$\vec{u}\cdot\vec{n} = \vec{w}\cdot\vec{n} \quad in\ \Gamma_f \tag{8}$$

The motion of the mesh nodes is actually arbitrary, and in this paper it is determined by using the Laplace equation following Robertson et al. ([11]) and Karniadakis & Sherwin ([15]):

$$\nabla^2 w = 0 \tag{9}$$

$$\frac{d\vec{X}}{dt} = \vec{w} \tag{10}$$

with $\vec{X}$ denoting the node position vector. We note that some studies such as [13] and [14] utilized the elastic motion description for mesh nodes.

Numerical Method: High-Order Splitting Scheme for ALE N-S Equations

A high-order splitting method is chosen to solve the ALE N-S equations following ([11]).

$$\hat{\vec{u}} - \sum_{q=0}^{J_i-1}\alpha_q \vec{u}^{n-q} = \delta t\left[-\sum_{q=0}^{J_i-1}\beta_q N\left(\vec{u}^{n-q}, \vec{w}^{n-q}\right) + f^{n+1}\right] \quad in\ \Omega^n \tag{11}$$

$$\hat{\hat{\vec{u}}} - \hat{\vec{u}} = -\frac{\delta t}{\rho}\nabla p^{n+1} \quad in\ \Omega^n \tag{12}$$

$$\gamma_0 \vec{u}^{n+1} - \hat{\hat{\vec{u}}} = \nu \delta t L\left(\vec{u}^{n+1}\right) \quad in\ \Omega^n \tag{13}$$

In the above, the superscript refers to time level index, and J_i the order of the time integration, and γ_0, α_q and β_q the coefficients for stiffly stable time integration ([11], [15]).

We can get the so-called pressure Poisson equation by assuming $\nabla\cdot\hat{\hat{\vec{u}}} = 0$, as usual splitting scheme.

$$\nabla^2 p^{n+1} = \frac{\rho}{\delta t}\nabla\cdot\left(\hat{\vec{u}}\right) \quad in\ \Omega^n \tag{14}$$

Compatible pressure boundary condition is written as follows.

$$\frac{\partial p^{n+1}}{\partial n} = -\rho\left[\frac{\partial \vec{u}^{n+1}}{\partial t} + \sum_{q=0}^{J_i-1}\beta_q N\left(\vec{u}^{n-q}, \vec{w}^{n-q}\right) + \nu\sum_{q=0}^{J_i-1}\beta_q\left(\nabla\times\vec{\omega}^{n-q}\right) - f^{n+1}\right]\cdot\vec{n}^n \quad in\ \Gamma_D^n \tag{15}$$

$$p^{n+1} = \vec{n}^n \cdot \nu\left[\nabla\vec{u}^n + (\nabla\vec{u}^n)^T\right]\cdot\vec{n}^n - (T_n^{ext})^n/\rho - \gamma\kappa^n/\rho \quad in\ \Gamma_f^n \tag{16}$$

In the above, the diffusion term are extrapolated in time on the boundary in order to avoid coupling between the pressure and the velocity field through the pressure boundary condition. The boundary acceleration term $(\partial\vec{u}^{n+1}/\partial t)\cdot\vec{n}^n$ can be evaluated because on the Dirichlet and Neumann boundary we know the values of velocity and its normal component, respectively. The body force term is also trivial.

For the viscous correction part, we obtain the following Helmholtz equation by rearranging the equation (14).

$$\left(\nu L - \frac{\gamma_0}{\delta t}\right)\vec{u}^{n+1} = \nu\hat{\hat{\vec{u}}} \quad in\ \Omega^n \tag{17}$$

An appropriate set of boundary conditions reads as following.

$$\vec{u}^{n+1} = (\vec{u}_D)^{n+1} \ in\ \Gamma_D^n \tag{18}$$

$$\nu[\nabla\vec{u}^{n+1} + (\nabla\vec{u}^{n+1})^T]\cdot\vec{n}^n = [(T_s^{ext})^n/\rho]\vec{s}^n + [(T_s^{ext})^n/\rho]\vec{m}^n + [p^{n+1} + (T_n^{ext})^n/\rho + \gamma\kappa^n/\rho]\vec{n}^n \ in\ \Gamma_f^n \tag{19}$$

We note the form of the alternate boundary conditions and corresponding formulations of the viscous terms in the context of weak form statement are demonstrated in [22]. For instance, if we take the form of the diffusion term as $\nu\nabla\cdot\nabla\vec{u}$, then we will have the resulting boundary term as $\nu\vec{u}\cdot\vec{n}$ after applying the divergence theorem. So, the boundary condition on the velocity field that we can impose as a natural consequence of the weak form is directly related to the form of the diffusion term that we take in the original N-S equations.

The motion of the mesh nodes at the new time instant can be determined as following.

$$\nabla^2\vec{w}^{n+1} = 0 \quad in\,\Omega^n \tag{20}$$

$$\vec{w}^{n+1}\cdot\vec{n}^n = (\vec{u}\cdot\vec{n})^n \quad in\,\Gamma_f^n \tag{21}$$

$$\vec{w}^{n+1}\cdot\vec{s}^n = \vec{w}^{n+1}\cdot\vec{m}^n = 0 \quad in\,\Gamma_f^n \tag{22}$$

$$\vec{w}^{n+1} = \vec{0} \quad in\,\Gamma_D^n \tag{23}$$

A similar form of equations for two dimensions can be found in [12]. The boundary conditions, (21)~(23) state that the mesh nodes move in the normal direction of the velocity vector on the free surface, and they are stationary on the Dirichlet-type boundary. Thus, on the free surface we have the following.

$$\vec{w}^{n+1} = (\vec{u}\cdot\vec{n})^n\,\vec{n}^n \quad in\,\Gamma_f^n \tag{24}$$

After solving the above equation, we can update the mesh nodes by the following equation.

$$\gamma_0\vec{X}^{n+1} - \sum_{q=0}^{J_i-1}\beta_q\vec{X}^{n-q} = \delta t\vec{w}^{n+1} \tag{24}$$

Numerical Method: Weak Forms

In this section, we present weak forms needed to apply the spectral element method. For the Poisson equation, (14)~(16), we rewrite the equation as follows for simplicity.

$$\nabla^2 p = \frac{\rho}{\delta t}\nabla\cdot\left(\vec{u}\right) \quad in\,\Omega \tag{25}$$

$$\frac{\partial p}{\partial n} = \overline{p}_{H,n} \quad in\,\Gamma_D \tag{26}$$

$$p = \overline{p}_D \quad in\,\Gamma_f \tag{27}$$

where the superscripts are omitted and the left hand sides of the equations (15) and (16) are abbreviated by introducing new auxiliary variables, $\overline{p}_{H,n}$ and $\overline{p}_D$. Here, in the first place, we have to construct the boundary transformation subject to the Dirichlet condition:

$$p_D = \overline{p}_D \; in\,\Gamma_f, \tag{28}$$

the solution of which, p_D is a known non-homogeneous contribution that satisfies the boundary condition (27).

By the help of lifting ([15]), we can get the corresponding weak form for the homogeneous component of the pressure field as follows: Find p_H satisfying the weak statement in the below for all δp with $\delta p = 0\; in\,\Gamma$.

$$\int_\Omega \nabla\delta p\cdot\nabla p_H d\Omega = -\frac{\rho}{\delta t}\int_\Omega \delta p\nabla\cdot\left(\vec{u}\right)d\Omega + \int_\Gamma \delta p\overline{p}_{H,n}d\Gamma - \int_\Omega \nabla\delta p\cdot\nabla p_D d\Omega \tag{29}$$

Finally, the total pressure field is obtained as the sum of the Dirichlet component and the remaining homogeneous part as in the following.

$$p^{n+1} = p_H + p_D \tag{30}$$

The velocity correction part can be rewritten as

$$\left(\nu L - \frac{\gamma_0}{\delta t}\right)\vec{u} = \nu\vec{\tilde{u}} \quad in\,\Omega \tag{31}$$

$$\vec{u} = \vec{u}_D \quad in\, \Gamma_D \tag{32}$$

$$\nu\left[\nabla\vec{u} + (\nabla\vec{u})^T\right]\cdot\vec{n} = \vec{t} \quad in\, \Gamma_f \tag{33}$$

where the superscripts for time indexing are dropped and the left hand sides of the equations (18) and (19) are also abbreviated for brevity. Likewise, the velocity lifting is preceded ahead of the homogeneous part: Find the Dirichlet boundary component as

$$\vec{u}_D = \vec{u}_D \; in\, \Gamma_D\,. \tag{34}$$

The resulting weak form for homogeneous part of the corrected velocity states that for $n \geq 0$, find $\vec{u}_H$ satisfying the following for all $\delta\vec{v}$ with $\delta\vec{v} = \vec{0} \; in\, \Gamma$.

$$\begin{aligned}&\frac{1}{2}\int_\Omega \left[\nabla\delta\vec{v} + (\nabla\delta\vec{v})^T\right] : \nu\left[\nabla\delta\vec{u} + (\nabla\delta\vec{u})^T\right] d\Omega + \frac{\gamma_0}{\delta t}\int_\Omega \delta\vec{v}\cdot\vec{u}\, d\Omega \\ &= \nu\int_\Omega \delta\vec{v}\cdot\vec{\tilde{u}}\, d\Omega - \int_{\Gamma_f}\delta\vec{v}\cdot\vec{t}\, d\Gamma - \frac{1}{2}\int_\Omega \left[\nabla\delta\vec{v} + (\nabla\delta\vec{v})^T\right] : \nu\left[\nabla\delta\vec{u}_D + (\nabla\delta\vec{u}_D)^T\right] d\Omega\end{aligned} \tag{35}$$

Eventually, the total velocity corrected to the pressure projection is obtained as follows.

$$\vec{u}^{n+1} = \vec{u}_H + \vec{u}_D \tag{36}$$

The mesh node velocity vector is rearranged into the following form.

$$\nabla^2\vec{w} = 0 \quad in\, \Omega \tag{37}$$

$$\vec{w} = \vec{w}_D \quad in\, \Gamma \tag{38}$$

where the vector, $\vec{w}_D$ stands for the left hand sides of the equations (23) and (24) collectively. Similarly, we find the non-homogeneous contribution in advance as

$$\vec{w}_D = \vec{w}_D \; in\, \Gamma \tag{39}$$

by boundary transformation procedure. Then, the following weak statement is written.

$$\int_\Omega \nabla\delta\vec{w}\cdot\nabla\vec{w}_H d\Omega = -\int_\Omega \nabla\delta\vec{w}\cdot\nabla\vec{w}_D d\Omega \tag{40}$$

for all $\delta\vec{w}$ with $\delta\vec{w} = \vec{0} \; in\, \Gamma$. In the end, the required mesh velocity is given as the sum of the two components, i.e.

$$\vec{w}^{n+1} = \vec{w}_H + \vec{w}_D\,. \tag{41}$$

Numerical Method: Spectral Element Method

Due to the spectral element method, the velocity vector, $\vec{u}$ and pressure, p are expressed as in the following.

$$\vec{u}(\xi_1,\xi_2,\xi_3) = \sum_{m(p,q,r)=0}^{M} \vec{u}_{m(p,q,r)}\varphi_{m(p,q,r)}(\xi_1,\xi_2,\xi_2) \tag{42}$$

$$p(\xi_1,\xi_2,\xi_3) = \sum_{m(p,q,r)=0}^{M} \hat{p}_{m(p,q,r)}\varphi_{m(p,q,r)}(\xi_1,\xi_2,\xi_2) \tag{43}$$

The physical coordinates are related with the transformed space coordinates as

$$x_i(\xi_1,\xi_2,\xi_3) = X_i^e(\xi_1,\xi_2,\xi_3),\, i = 1,2,3 \tag{44}$$

Resulting algebraic system assumes the following form for the velocity equation (32), for instance.

$$\left[\frac{\gamma_0}{\delta t}M + \nu K\right]U = S \tag{45}$$

where M , K and S are the mass, stiffness matrices, and the source vector from the equation (33), respectively. The remaining parts are the Poisson and Laplace equations having different boundary conditions, which is fully covered in [15].

Numerical Method: Solution Procedure

The whole solution procedure of the present study for obtaining the time-dependent viscous free surface flow is described as follows.

- Start with the given B.C.: time instant index, $n = 0$
- Iterate the following loop until the specified time
  - For $n \geq 0$, calculate the first intermediate velocity, $\hat{\vec{u}}$ by using the previous solution field, $\vec{u}^{n-q} (q = 0,\ldots,J_i)$: Equation (11)
  - Compute the pressure field, p : Equation (14)~(16) or (25)~(27)
    - Lifting part: Equation (28)
    - Homogeneous part: Equation (29)
    - Add the two sub-solutions: Equation (30)
  - Evaluate the second intermediate velocity, $\hat{\hat{\vec{u}}}$: Equation (12)
  - Obtain the corrected velocity at the next time instant, $\vec{u}^{n+1}$: Equation (17)~(19) or (37)~(38)
    - Lifing part: Equation (34)
    - Homogeneous part: Equation (35)
    - Add the two sub-solutions: Equation (36)
  - Update the mesh nodes and the free surface, $\vec{X}^{n+1}$: Equation (20)~(23)
    - Lifing part: Equation (39)
    - Homogeneous part: Equation (40)
    - Add the two sub-solutions: Equation (41)

Numerical Results and Discussion

As an intermediate stage of development, numerical results for wave-current interaction is left as further study.

Summary

We briefly outlined the freak waves and its generation mechanism with the focus on the wave-current interaction phenomenon. The present model of the fluid motion is based on the Navier-Stokes equations with ALE description, and numerical method utilizes the spectral element method. The detailed methodology and numerical solution procedure are demonstrated. In near future, the capability of the present modeling and method will be validated for benchmark cases, such as solitary waves and some available wave-current experiments.

Acknowledgement

The present study is supported by the principal R&D program of KORDI: "Development of Safety Evaluation Technologies for Marine Structure in Disastrous Ocean Waves" granted by Korea Research Council of Public Science and Technology.

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