Analysis on Decomposition Models of Univariate Hydrologic Time Series for Multi-Scale Approach

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ABSTRACTS

Empirical mode decomposition (EMD) is applied to analyze time series characterized with nonlinearity and nonstationarity. This decomposition could be utilized to construct finite and small number intrinsic mode functions (IMF) that describe complicated time series, while admitting the Hilbert transformation properties. EMD has the capability of being adaptive, capture local characteristics, and applicable to nonlinear and nonstationary processes. Unlike discrete wavelet transform (DWT), IMF eliminates spurious harmonics and retains meaningful instantaneous frequencies. Examples based on data representing natural phenomena are given to demonstrate highlight the power of this method in contrast and comparison of other ones. A presentation of the energy-frequency-time distribution of these signals found to be more informative and intuitive when based on Hilbert transformation.

Keywords : Nonstationary time series, Wavelet transform analysis, Hilbert spectral analysis, Intrinsic mode decomposition

1. Introduction

Traditionally, Fourier spectral analysis has been extensively used to examine the global energy frequency distribution. The many crucial restrictions associated with Fourier spectral analysis limit its applicability to linear, periodic, and stationary time series (*Titchmarsh*, 1948; *Huang et al.*, 1998). In addition, both nonlinearity and nonstationarity may induce spurious harmonic components.

Adjustable window Fourier spectral analysis is proposed in form of wavelet analysis to accommodate the nonstationarity of the time series. Yet, wavelet analysis constitutes a rigid framework owing to the selection of the mother wavelet that will be used to analyze all the data across all the scales and times. Wavelet analysis is still the best available non-stationary data analysis and it enjoys the capability of capturing gradual interwave frequency modulation and it fails in capturing intrawave frequency modulation. Therefore, and due to the non-adaptive mechanisms of wavelet analysis, physically meaningful interpretation to nonlinear phenomena based on wavelet analysis is not will advocated.

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Many other miscellaneous methods have been utilized to decompose time series into components. Largely these methods have been designed to modify the global representation of the global Fourier analysis (*Brockwell and Davis*, 1991). *Huang et al*, (1998) summarized the necessary conditions required to represent the non-linear and nonstationary time series as having complete, orthogonal, local, and adaptive.

In this manuscript, empirical mode decomposition (EMD) will generate intrinsic mode functions (IMF) based on direct extraction of the energy associated with various intrinsic time scales (Huang, et al., 1998). The extracted components admit well-behaved Hilbert transforms and their associated instantaneous frequencies capture linear and nonlinear behaviors ad dictated by data.

Climatic time series are usually of finite duration, nonstationary, and intrinsically nonlinear. EMD is best suited to capture across-scale interactions in climatic time series. In this manuscript, we apply EMD to Nino1.2 time series and derive new insights into the physical attributes of the phenomena across scale. Some limitations of the method will be discussed. We follow two steps of analysis. First, obtain the intrinsic mode function components. Second, apply wavelet transform to the decomposed EMF and construct the energy frequency time distribution.

2. Background

Empirical Mode Decomposition

Empirical mode decomposition algorithm has been recently introduced for processing of non-linear and non-stationary signals (*Huang et al*, 2006). EMD defines the empirically based data-driven method to extract physically meaningful representation of data from nonlinear and nonstationary processes. EMD ability to extract the intrinsic features stems from its adaptive capacity which enables one to determine the instantaneous frequency of a signal. Hilbert-Huang Transformation (HHT) is the process of extracting the empirical mode decomposition and performing Hilbert spectral analysis. EMD has been devoted to decompose signals into intrinsic mode functions (IMF). The resulting component will have equal number of extremes and troughs and can have a variable amplitude and frequency as function of time.

The first step for obtaining IMF is to connect both all minim and maxima by cubic spline. The first IMF is obtained by subtracting the mean of the two cubic functions from the original signal. The sifting process is the repetitive extraction of IMF, till residuum constitutes monotonic function (*Huang et al*, 2006). Once the IMFs are computed when could resort to discrete wavelet transformation or Hilbert transformation to determine the associated instantaneous frequency. EMD was shown to have a great potential for real-time decomposition and processing of climatic signals.

3. Application

Synthetic Time Series

In this section we synthesize a time series where we show nonlinear interaction between the signals. Let our signal be $X = \sum_{i=1}^{2} \sin(w_i) + \sin(w_3 + \sin(w_4)) + \varepsilon$. The challenge for IMF is decompose the signal that is associated with noise into its original intrinsic signals.

Figure 1 shows the synthesized signal and its building principal components.

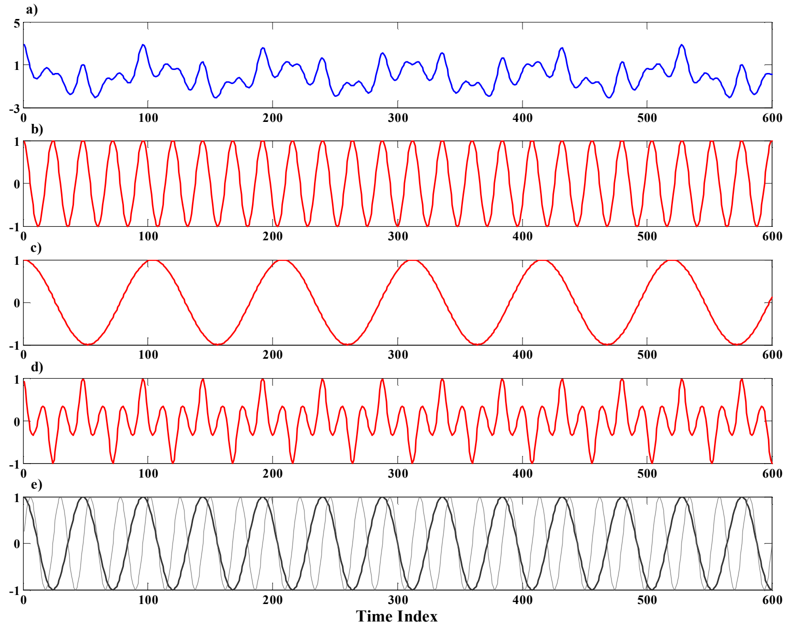

Figure 1: Synthetic signal, (a) is the aggregated signal, (b), (c), and (e) are the original signal and (d) is nonlinear combination of signals in (e).

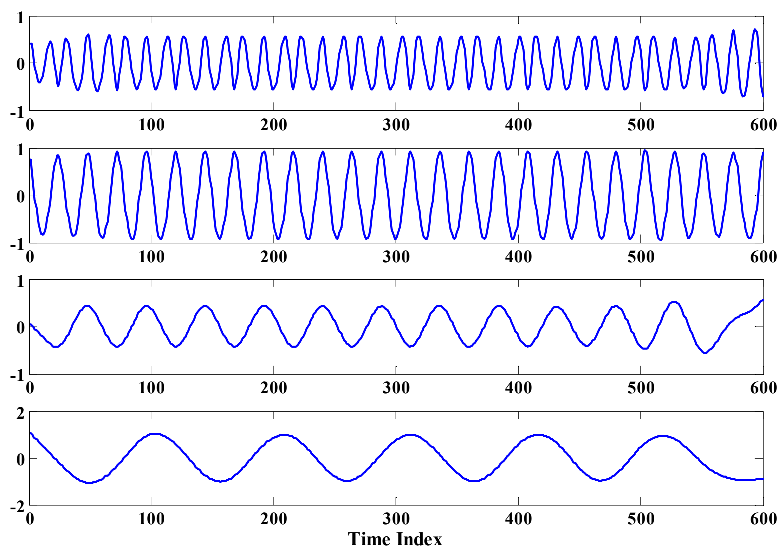

Figure 2: IMFs derived from the synthetic time series.

IMF was carried out to decompose the signal. The sifting process in IMF has stopped after generating the four components shown in Figure 2. This figure shows the constituent major signals reconstructed adequately well. Let's notice that because one fits a cubic spline for the peaks and troughs of the original signal, the fitted spline might be subjected to edge effect. However, the major components are reclaimed and the nonlinear interaction didn't hinder the ability of IMF to extract the intrinsic modes of the signal.

For comparison purposes, we decompose the signal using DWT. The nonlinear interaction in the signal will be challenging for DWT formulation that solely depends on linear correlation between the signal and the mother wavelet function at different scales. The performance of DWT on this synthetic signal is shown in Figure 3. Once could notice that the original intrinsic components are not adequately reconstructed. The two signals that are linearly added to make the synthetic time are extracted fairly well. However, the nonlinear components are not extracted. One also could notice that there are spurious signals with spurious frequencies are extracted due to the contamination with noise. The DWT transformation is biased to fit the shape of the mother wavelet the constituent signals.

Figure 4 further illustrates the original frequencies and the frequencies as picked by IMF and DWT. One could confidently conclude that IMF has managed to capture the major frequencies. This shows the ability of IMF to perform well when the intra-wave variability is present. The spurious frequency picked by DWT transformation is considered to be an average between the two nonlinearly combined signals.

IMF could lend themselves to model the nonlinear and nonstationary climatic time series owing to the distinctive features of adaptability.
Here we used Nino3 seas surface temperature to show the IMF capabilities in capturing across scale modulations. Figure 5 shows the one-year to 2-8-years cross wavelet spectrum as derived from IMF and DWT. The nonlinear interaction between the extracted components is very evident in the Figure x. It is shown that there is high energy signature (i.e., high variance) around 1880 and also 1985-2000. The phase of the interaction is different in the two cases reflecting the nonlinearity in the system. The modulation across scales could be quantified using IMF.

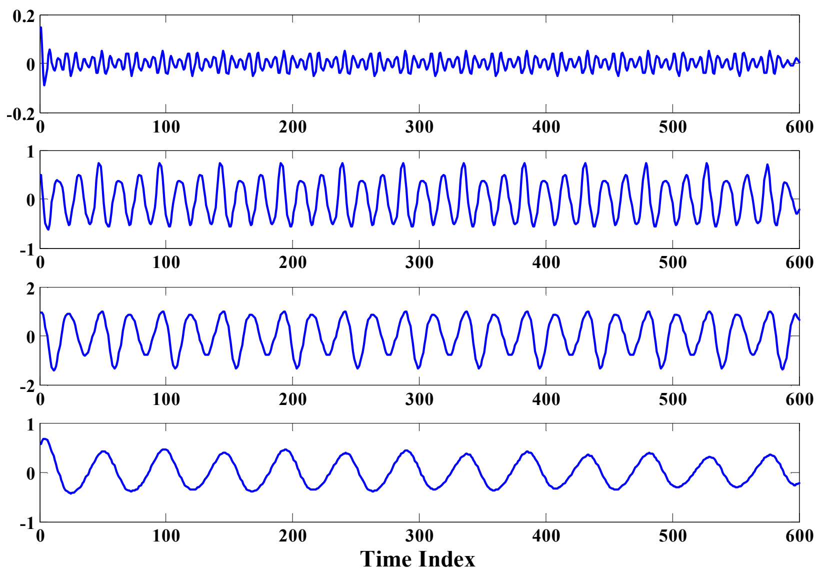

Figure 3: DWT components derived from the synthetic time series.

5. Summary and Conclusion

The comparisons between IMF and DWT were intended to be merely illustrative, it should be highly emphasized that no broader generalizations can be made about the superiority of any of the method for all classes of time series. The EMD allows the instantaneous frequency and the amplitude of the signal to be represented as functions of time. The importance of this three dimensional representation (frequency, amplitude, and time) allows accurate determination of the signal magnitude. EMD can adapt well to the local variation of the data which makes the decomposition fully account for the underlying physics of the process and not just to satisfy the devised mathematical setup. The feature of locality in EMD is crucial for nonstationarity where the time scale is not the determinant but the time of occurrence.

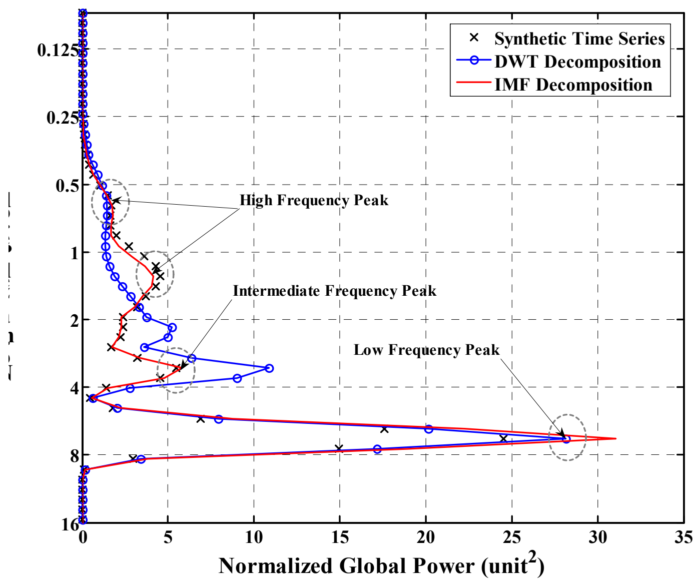

Figure 4: Global wavelet spectrum associated with the original signal, IMF, and DWT.

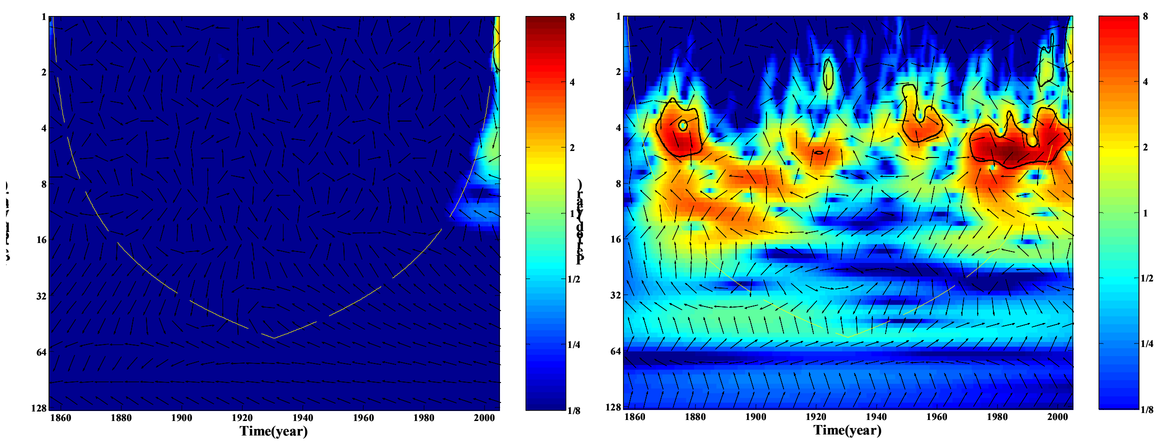

Figure 5: Cross wavelet spectrum, (a) 1-year versus 2-8-year as derived from DWT, (b) 1-year versus 2-8-year as derived from IMF.

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