Complex envelope of sound field and its application
음장의 복소 포락과 응용

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ABSTRACT

Acoustic holography allows us to predict spatial pressure distribution on any surface of interest from measured hologram. It is noteworthy that the data size is so huge that it takes long time to calculate pressure field. Moreover the reconstructed pressure field is frequently too complicated to get what we want to know. One possible candidate is complex envelope. Complex envelope in time domain is well known and widely used in various engineering field. We have attempted to extend this method to space domain, so that we can have rather simple spatial pressure picture that provides information we need, for example, where sound sources are. First we start with the simplest case. We examine the complex envelope of a plane wave on both space and wave number domain. Then we extend to monopole case. Holographic reconstructed sound field on the monopole is processed according to what we propose. We demonstrate how this method provides better picture for analyzing the sound field.

1. Introduction

We can predict sound field that we want to know or can not afford to measure directly, for example, source plane by using acoustic holography[1~3]. We can use the holographic reconstructed results with various purposes. Especially, in a standpoint of noise control, acoustic holography is mainly used for finding out where noise sources are. Although beam forming method also can show the location of sound sources[4,5], it does not express sound pressure field. Therefore, it is not possible to observe characteristics of pressure field and magnitude of sound sources.

On the other hand, acoustic holography gives us information of pressure field which is made by sound sources. However, there are some problems on processing and analyzing reconstructed sound field; the former has to do with long calculation time, and the latter means that we usually have too much information than it is necessary.

In this paper we define the problems related with the analysis problem in acoustic holography[6] and introduce complex envelope analysis as a solution method. We attempt to explain fundamentals of the method with 1-D case for simplicity. Based on the concept, we extend it into spatial pressure field. Moreover, we briefly introduce results of acoustic holography implemented by complex envelope on a monopole source.

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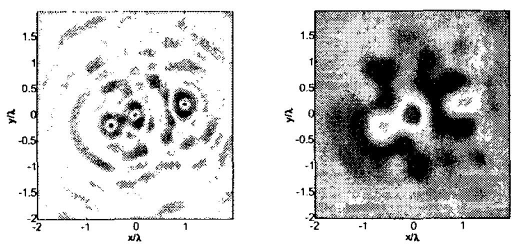
2. Problem definition

When we have interests on where the sources are and how large pressure level they make, we do not need the full information of pressure field. Moreover, the holographic reconstructed picture is often too complicated to observe characteristics of the pressure field. Fig.1.(a) shows a simple example of original sound field, which is composed of three monopoles and looks complicated by spatial variation of waves. Fig.1.(b) is simplified sound field by using complex envelope analysis. With comparing Fig.1(a) and (b), we can not only find the exact locations of the sources, but also recognize type of sources made up of monopole.

In addition, prediction process of acoustic holography is made by data on both time and space. By huge size of data



(a) (b)

Fig.1 Pressure fields, (a) An original complicated field, (b) Simplified field by using complex envelope on time and space, it takes long calculation time to obtain final reconstructed prediction field. If we can reduce the data size, we can achieve short calculation time on process of acoustic holography.

3. Solution method

By using envelope signal, we can observe tendency which contains signal power fluctuation of given narrow band signal and reduce data points enough for representing slowly varying components as well. We introduced and applied complex envelope analysis to make complicated sound field simple and calculation time short. Complex envelope is a kind of envelope, which can be defined variously according to their purposes. [7]

3.1 Definition and types of envelope

Envelope can be classified into three types with respect to their mathematical expression. Although the represented forms are different from others, they came from same analytic signal obtained by Hilbert transform. Firstly, there is (natural) envelope defined by Rice[8]. This envelope is absolute value of analytic signal. Second one is pre-envelope which is defined by Dugundji[9]. The envelope is analytic signal itself. The last one is complex envelope, which is an envelope with complex value.

Among those envelopes, if we need mathematical calculation with the envelope signal and are interested in envelope signal itself, complex envelope is most useful concept.

3.2 Complex envelope detection method

By using complex envelope analysis, we can pick out rather slowly varying components (envelope) from given signal as depicted in Fig.2 We examined the procedure for obtaining envelope in detail with a simple case.

Let us consider a signal with a characteristic frequency f_c and time-varying amplitude, $A(t)$ which is $A(t) = A\cos(2\pi f_m t)$. A is constant amplitude and f_m is a modulating frequency. Then, we can write the signal as

$$x(t) = A(t)\cos(2\pi f_c t) = \frac{A}{2}\{\cos[2\pi(f_c + f_m)t] + \cos[2\pi(f_c - f_m)t]\}. \quad (1)$$

Equation (1) shows that $x(t)$ is composed of two

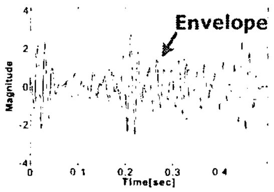


Fig.2 Original fast varying signal and its envelope

sinusoidal signals whose frequency difference is $2f_m$. This is occurred by convolving the modulating frequency (f_m) with characteristic frequency (f_c) in frequency domain.

Envelope of this signal is determined by the frequency difference, in other words, bandwidth of banded signal. Therefore, we need frequency shift and low pass filtering of the signal for removing high frequency components. It is frequency shift in frequency domain to multiply given signal by $e^{j2\pi f t}$ in time domain, which is called complex modulation[10]. Let us multiply $e^{-j2\pi f_c t}$ with characteristic frequency component. Then we obtain modulated signal,

$$\tilde{x}_m(t) = x(t)e^{-j2\pi f_c t} = \frac{A}{4}\left\{e^{j2\pi(f_c+f_m)t} + e^{-j2\pi(f_c+f_m)t} + e^{j2\pi(f_c-f_m)t} + e^{-j2\pi(f_c-f_m)t}\right\}e^{-j2\pi f_c t}. \quad (2)$$

The tilde over signals indicates that they are complex value. We can get frequency spectrum as

$$\tilde{X}_m(f) = \frac{A}{4}\{\delta(f - f_m) + \delta(f + f_m) + \delta(f + (2f_c + f_m)) + \delta(f + (2f_c - f_m))\} \quad (3)$$

by Fourier transform. The first two terms represent envelope signal and the other two terms indicate fast varying components as we can see in Fig.3. Capital letter means that the quantity is in frequency domain.

Finally we can get the envelope signal by low pass filtering:

$$x_m^L(t) = A\cos(2\pi f_m t), \quad (4)$$

where superscript L indicates that the quantity is passed low pass filter. From the result, we can get exact envelope signal of given signal in equation (4).

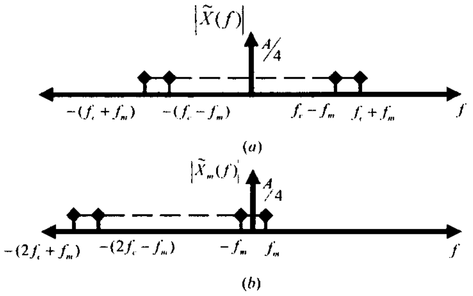


Fig.3 Frequency spectrum of (a) original signal and (b) modulated signal by complex modulation (f_c is characteristic frequency, f_m is modulating frequency, and A is constant amplitude)

4. Complex envelope on space domain

We extended 1-D complex envelope analysis as explained in previous section to 2-D spatial domain for utilizing the advantages of complex envelope signal. We considered plane wave firstly as a simple case in space domain. Furthermore, we devised complex envelope of monopole because the monopole is a basic singular sound source by whose superposition any arbitrary wave field can be described in linear system.[11]

4.1 Plane wave

It is needless to say that the plane wave is just simple extension of 1-D case. Therefore, we can verify validity of the methods and observe phenomena in extended domain.

With comparing 1-D case in mathematical description, the plane wave is the same with it except that independent variables are two: x and y. Let us consider a plane wave with characteristic wavenumber is k_c and modulating wavenumber is k_m. Then, we can write modulated plane wave is written as

$$p(x,y) = A(x,y)\cdot\cos(k_{cx}x + k_{cy}y + \varphi)$$
$$= \frac{1}{2}\left\{\tilde{A}(x,y)e^{j(k_{cx}x+k_{cy}y)} + \tilde{A}^*(x,y)e^{-j(k_{cx}x+k_{cy}y)}\right\}, \quad (5)$$

where $\tilde{A}(x,y)$ is a complex envelope including initial phase φ and asterisk indicates complex conjugate, $k_c = \sqrt{k_{cx}^2 + k_{cy}^2}$, and $k_m = \sqrt{k_{mx}^2 + k_{my}^2}$. From the mathematical form, we can imagine that complex envelope of plane wave is obtained by the same procedure with 1-D case. Fig.4 shows the signal described in equation (5) and

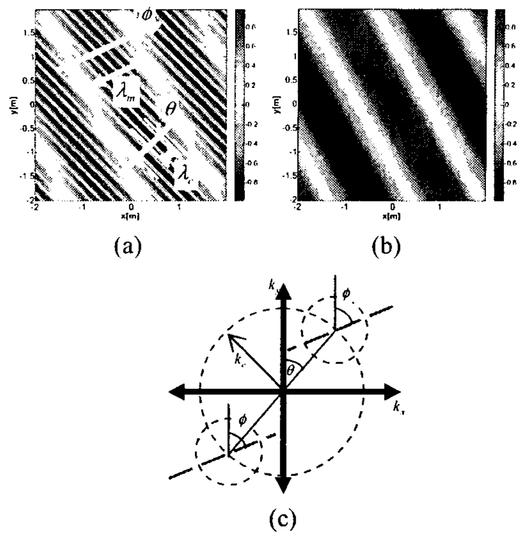


Fig.4 Pressure field and wavenumber spectrum of plane wave (a) Original pressure field (b) Modulated pressure field (envelope signal) (c) Wavenumber spectrum (k_c is characteristic wavenumber, θ is an angle of characteristic wavenumber, and ϕ is an angle of modulating wavenumber)

its wave number spectrum.

4.2 Monopole

Although monopole source is not simple expansion of 1-D case, modulation of spatially slowly varying components and detection method of the envelope are the same with plane wave case. Mathematical expression of modulated monopole can be written as

$$p(x,y) = \cos(k_m\sqrt{x^2+y^2})\cdot\frac{\exp[j(k_c\sqrt{x^2+y^2})]}{\sqrt{x^2+y^2}}. \quad (6)$$

The former term is envelope signal with modulating wavenumber k_m and the latter is a monopole sound source with characteristic wavenumber k_c.

Comparing with plane wave case, envelope pressure filed of monopole source always composed of additive slowly varying components and $1/r$. Therefore, if we consider the envelope of monopole both two terms together, the only remaining term is fast varying propagating wave, $\exp\left[j(k_c\sqrt{x^2+y^2})\right]$. That means the procedure for detecting envelope signal is the same with plane wave case. Fig.5 shows us the original pressure field and obtained envelope pressure filed by complex modulation. Fig.5.(d)

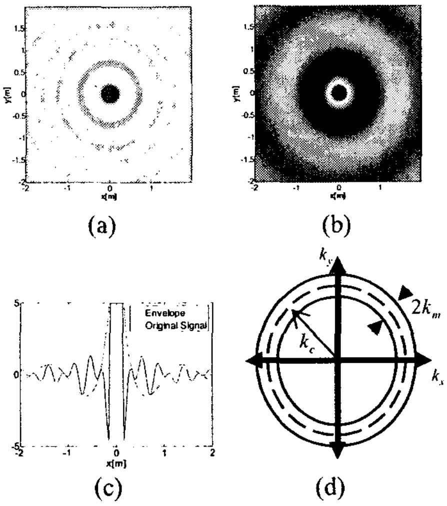
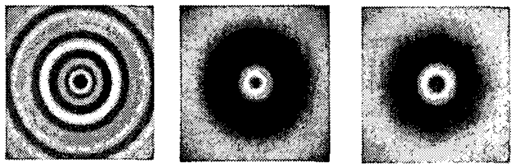


Fig.5 Pressure field and wavenumber spectrum of a monopole source (a) Original pressure field (b) Modulated pressure field (envelope signal) (c) Section plane of original and envelope pressure filed at y=0 (d) Wavenumber spectrum (k_c is characteristic wavenumber, k_m is modulating wavenumber)

shows wavenumber spectrum of the modulated monopole. We can observe feature of modulated monopole signal, which behaves like a banded signal similar with 1-D case, because most signal power is concentrated in radiation circle with respect to given wavenumber.

5. Discussions

We stated briefly the results and effects of complex envelope on acoustic holography to ascertain the possibility of implementation. For obtaining complex envelope of holographic reconstructed pressure field, we apply complex envelope before propagation process of acoustic holography to get the merits on analysis and processing time. In this paper, however, we focused on the analysis problem of acoustic holography as stated. As a result, we can get a quite similar complex envelope with true envelope signal as depicted in Fig.6.



(a) (b) (c)

Fig.6 Pressure fields (a) Original pressure field (b) True envelope field (c) Holographic reconstructed envelope field

6. Conclusions

We introduced complex envelope analysis to improve holographic reconstructed sound field for clear and easy analysis. We verified the method with simple cases, plane wave and monopole, and confirmed the effect of complex envelope of pressure field. Furthermore, it is verified that the proposed method can be applied to the holographic reconstructed pressure field successfully.

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