Acoustical Property Evaluation of Multi-layered Material Using the Standing Wave Method

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Abstract

The acoustical properties of multi-layered treatments of materials used in applications, such as automotive liners, generally cannot be directly measured by a one-time test in a standing wave duct. Therefore, we have to consider predicting them by the four-pole transfer matrix method. This method requires performing TCM or TLM for measuring the transfer matrix of each layer and calculating the total transfer matrix of the whole multi-layered material. The final predicted absorption ratios and transmission losses of the multi-layered treatments strongly depend on the measured transfer matrix of each layer. All these functions have been included in a new designed acoustical software.

1. Introduction

During the development of acoustical measurement software, the philosophy of such software was providing easy way to users in measuring acoustic characteristics of mono-layered acoustical materials. But recently more and more multi-layered materials are applied in noise and vibration control areas. These materials, which include glass wool, polymeric fibrous materials, and various types of foams, alone or with viscoelastic materials, may be found in automotive linings, in seats, under carpets, in cavity interiors, etc. In order to optimize acoustical characteristics or to meet the demands of structural design, acoustic characteristics of multi-layered materials should be measured in a simple way just like the way in measuring mono-layered acoustical materials. To meet this demand, a new designed acoustical software which can predict the acoustical characteristics of multi-layered materials by finding the transfer matrix of the whole system has be been developed.

There are 2 methods to get the transfer matrix in calculating the acoustical properties of multi-layered materials: the two-cavity method (TCM), and the two-load method (TLM). The characteristic of each method and applied requirements are listed in Table 1. In this paper, both of the 2 methods will be introduced, and you can also find a glancing tutorial of how to use acoustical software to predict acoustical characteristics.

Method / Microphone	Two-cavity method (TCM)	Two-load method (TLM)
2 microphones	Absorption ratio, Acoustic impedance, Characteristic impedance, Propagation constant, Transmission loss	Absorption ratio, Transmission loss
4 microphones	×	Absorption ratio, Transmission loss

Table 1. Characteristic of each method (× means no corresponding method)

2. Basic theory

2.1. Standing Wave Theory

Sound in an appropriately designed standing wave duct may be always assumed as being stationary plane waves with zero mean flow speed propagating in air. If a coordinate system is chosen so that this plane wave propagates along the x axis, the complex acoustic pressure $p(x,t)$ and the associated particle velocity $v(x,t)$ of the medium are,

$$p(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)} \tag{1}$$

$$v(x,t) = \frac{1}{Z_0}\left[Ae^{j(\omega t - kx)} - Be^{j(\omega t + kx)}\right] \tag{2}$$

Where t is time, A and B are amplitudes of the incident and reflected waves, ω is the angular frequency, k is the wave number ($k=\omega/c$), Z_0 is the air characteristic impedance at 20°C ($Z_0=\rho c$), and the ρ and c are the air density and sound speed in the air, respectively. In standing wave ducts, the transfer matrix $[T]$ can be used to relate the sound pressures and normal particle velocities on the two surfaces of a material sample, or a treatment of multilayer materials, i.e.

$$\begin{Bmatrix} p_u \\ v_u \end{Bmatrix} = [T]\begin{Bmatrix} p_d \\ v_d \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{Bmatrix} p_d \\ v_d \end{Bmatrix} \tag{3}$$

Where, p_u, v_u are the sound pressure and normal particle velocity of the medium in the upstream side, p_d, v_d are the sound pressure and normal particle velocity of the medium in the downstream side, and T_{11}, T_{12}, T_{21}, and T_{22} are the elements in the transfer matrix. The four-pole transfer matrix conception is the basis for the predictions in this paper.

2.2. Prediction of primary acoustical properties of multi-layered treatments

For multi-layered treatments, after obtaining the transfer matrix of each layer by performing TCM or TLM, a total transfer matrix describing the complete system can then be expressed by multiplying each transfer matrix of the n layers sequentially, i.e.

$$[T] = [T_1][T_2][T_3]\cdots\cdots[T_n] \tag{4}$$

Using the four elements T_{11}, T_{12}, T_{21}, T_{22} of the total transfer matrix, one can calculate the absorption ratio and the transmission loss directly as,

$$\alpha = 1 - \left|\frac{T_{11} - Z_{uc}T_{21}}{T_{11} + Z_{uc}T_{21}}\right|^2 \tag{5}$$

$$TL = 20\log_{10}\left[\frac{1}{2}\sqrt{\frac{Z_{uc}}{Z_{dc}}}\left|T_{11} + \frac{T_{12}}{Z_{dc}} + Z_{uc}T_{21} + \frac{Z_{uc}}{Z_{dc}}T_{22}\right|\right] \tag{6}$$

Where, α is the absorption ratio of a material sample backed by a rigid wall and TL is the transmission loss.

2.3. Two-load method (TLM)

This method, as its name indicates, makes use of two loads in the measuring procedure. During the measuring procedure, one can use the four-microphone test one time (the four-microphone TLM) or the two–microphone test two times (the two-microphone TLM) for each load. Theoretically, there is no difference between the two methods when employed under a stationary sound source.

For an isentropic plane sound wave propagation across an acoustic element, the complex pressure amplitudes of the positively and negatively traveling waves, i.e., p_u^+, p_u^- in the upstream section and p_d^+, p_d^- in the downstream section, can be related by a pressure transfer matrix $[t]$,

$$\begin{Bmatrix} p_u^+ \\ p_u^- \end{Bmatrix} = [t]\begin{Bmatrix} p_d^+ \\ p_d^- \end{Bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}\begin{Bmatrix} p_d^+ \\ p_d^- \end{Bmatrix} \tag{7}$$

After some derivations, the elements in the transfer matrix $[T]$, which represent a material sample in a standing wave duct, may be expressed as,

$$\begin{Bmatrix} T_{11} \\ T_{12} \\ T_{21} \\ T_{22} \end{Bmatrix} = 2\begin{bmatrix} 1 & \frac{1}{Z_{dc}} & Z_{uc} & \frac{Z_{uc}}{Z_{dc}} \\ 1 & -\frac{1}{Z_{dc}} & Z_{uc} & -\frac{Z_{uc}}{Z_{dc}} \\ 1 & \frac{1}{Z_{dc}} & -Z_{uc} & -\frac{Z_{uc}}{Z_{dc}} \\ 1 & -\frac{1}{Z_{dc}} & -Z_{uc} & \frac{Z_{uc}}{Z_{dc}} \end{bmatrix}^{-1}\begin{Bmatrix} t_{11} \\ t_{12} \\ t_{21} \\ t_{22} \end{Bmatrix} \tag{8}$$

Where, Z_{uc}, Z_{dc} denote the medium characteristic impedances in the upstream and downstream sections in the standing wave duct, $Z_{uc}=Z_{dc}=Z_0$.

To calculate t_{11}, t_{12}, t_{21}, and t_{22}, we defined the incident transmission coefficient T_{in}, the reflected transmission coefficient T_{re} and the reflection coefficient R as

$$T_{in} = \frac{p_u^+}{p_d^+}, \quad T_{re} = \frac{p_u^-}{p_d^-}, \quad R = \frac{p_d^-}{p_d^+}, \tag{9}$$

Based on the measured data from the two cases (a) and (b), T_{in}, T_{re} and R can be carried out easily, thus, t_{11}, t_{12}, t_{21} and t_{22} are:

$$t_{11} = \frac{R_b T_{ina} - R_a T_{inb}}{R_b - R_a},$$
$$t_{12} = \frac{T_{inb} - T_{ina}}{R_b - R_a} \qquad (10a,b)$$

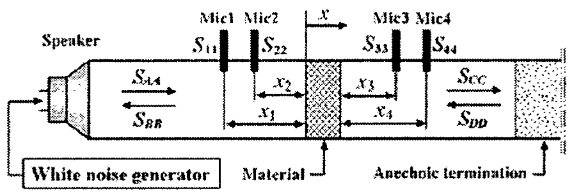

Fig. 1 Experimental setup for evaluating the acoustical properties of a material sample in a standing wave duct with the two-load method (TLM in case (a)).

$$t_{21} = R_a R_b \frac{T_{rea} - T_{reb}}{R_b - R_a},$$
$$t_{22} = \frac{R_b T_{reb} - R_a T_{rea}}{R_b - R_a} \qquad (10c,d)$$

Where the subscripts a and b represent the cases (a) and (b), respectively. The transfer matrix $[T]$ may be determined by substituting T_{in}, T_{re} and R into Eq. (8).

2.2. Two-cavity method (TCM)

The two-cavity method, though originally designed for calculating the propagation constant and the characteristic impedance of a single-layer noise control material, has generally been used to determine a material's transfer matrix. The measurement apparatuses for TCM are shown in Fig. 2.

$$Z_c = \pm\sqrt{\frac{Z_u Z_u'(Z_d - Z_d') - Z_d Z_d'(Z_u - Z_u')}{(Z_d - Z_d') - (Z_u - Z_u')}} \qquad (11)$$

Where, Z_c is the characteristic impedance Z_u or Z_u' is the acoustic impedance of the sample with thickness x_3 backed by an air layer with depth x_4 or x_4', which is sandwiched between the sample and a movable piston. As viewed from the front surface of the sample, Z_d or Z_d' is the acoustic impedance of a closed tube with air depth x_4 or x_4'. Here x_4 and x_4', the depths of the air layers, should be set to different values (nonzero) for the two iterations of the test. In terms of x_4 or x_4', Z_u or Z_u' can be calculated by,

$$Z_u = jZ_0 \frac{-H \sin kx_2 + \sin kx_1}{H \cos kx_2 + \cos kx_1} \qquad (12)$$

$$Z_d = -jZ_0 \cot(kx_4) \qquad (13)$$

Where H is the transfer function between microphones 1 and 2, and k, Z_0, x_1, and x_2 have the same meanings as in Eqs. (1), (2) and (3).

The propagation constant of the sample material is also a complex value, and may be expressed as,

$$\gamma = \alpha_m + j\beta_m = \frac{1}{2x_3}\ln\left(\frac{Z_u + Z_c}{Z_u - Z_c}\cdot\frac{Z_d - Z_c}{Z_d + Z_c}\right) \qquad (14)$$

Where γ is the propagation constant of the sample material with a real part α_m (attenuation constant) and an imaginary part β_m (phase constant). Furthermore, the transfer matrix representing this homogeneous and isotropic material sample can be computed like this:

$$[T] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos k_m x_3 & jZ_c \sin k_m x_3 \\ j \sin k_m x_3 / Z_c & \cos k_m x_3 \end{bmatrix} \qquad (15)$$

Where k_m is the wave number in the acoustic material, $k_m = \gamma / j$. Therefore, the absorption ratio, as well as the transmission loss, may be easily calculated from this transfer matrix, and expressions will be given in the following text.

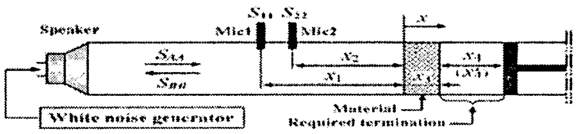

Fig. 2 Experimental setup for evaluation of the absorptive characteristics of a material sample in a standing wave duct with the two-cavity method (TCM)

3. Introduction of the new designed acoustical software

Among the all complex functions , we will only concern the Prediction part, through which we could know how to get the transfer matrix to predict the acoustical properties of multi-layered materials.

Once we enter the prediction function, a property sheet with three pages involved in prediction function will be shown up. They may be applied to calculate transfer matrix, predict a

Table 2. Material properties of the tested samples

Sample Symbol	Material Ingredient	Thickness (mm)	Density (g/m^2)	Flow Resistance (ralys/m)
UF	Urethane Foam	30	540	2.742×10^3
PET1	Poly Ethylene Terephthalate	50	1700	5.251×10^3
PET2	Poly Ethylene Terephthalate	50	2600	9.516×10^3
GW	Glass Wool	10	1400	5.405×10^4
RUB	Rubber	1.5	3600	$+\infty$

multi-layer material configuration and perform material optimization. See Fig. 3.

The first page on the above property sheet is "Transfer Matrix" (see Fig. 4), which including three parts, "Prediction Setting", "Two Cavity Method" and "Two Load Method", respectively. The first part is for setting the system's parameters, and the other two parts are for transfer matrix calculation.

After setting the necessary parameters, we can get the transfer matrix (see Fig.5) by performing a certain sets of process; consequently, we can finally get the predicted value of acoustical properties of multi-layered materials.

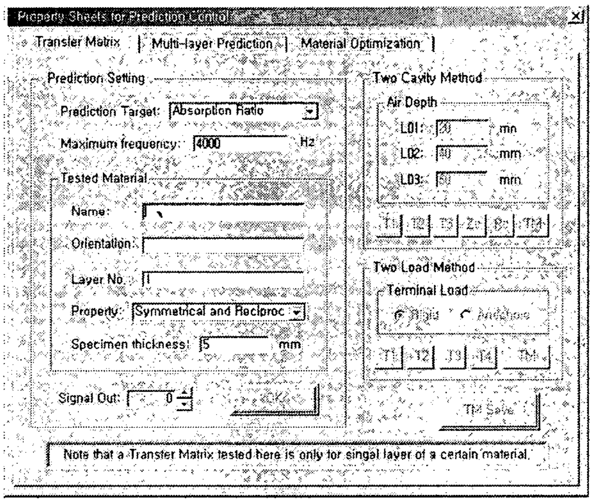

Fig.3 Property sheet for prediction

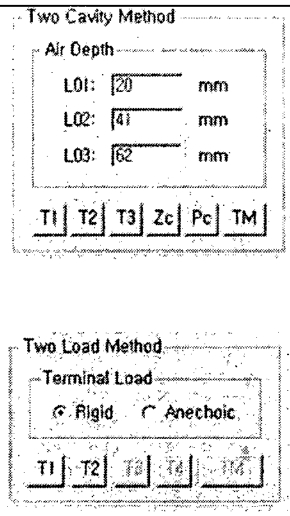

Fig.4 Property page of Transfer Matrix

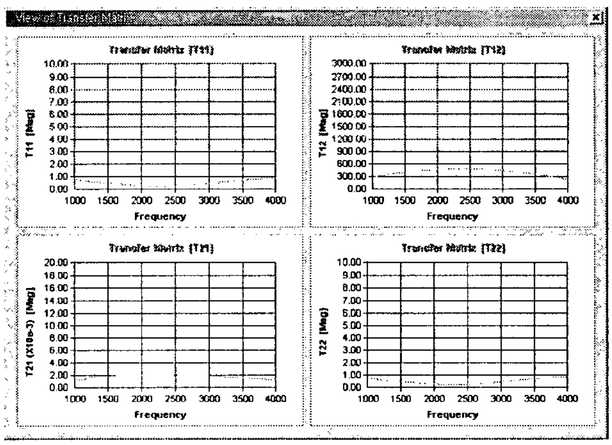

Fig.5 Final Result of Transfer Matrix

4. Experimental procedure

Several types of experimental apparatuses were needed in this research. A typical experimental setup was used for the four-microphone measurements shown in Fig.6; other measurements were easily made by changing the sample holder and required termination, depending on the measurement method used in the experiment (see Figs. 1 and 2). As seen in the standing wave duct system shown in Fig.6, a loudspeaker driven by a wave file in a digital computer at the left end was used to generate broadband white noise as the sound source over the frequency range of 500Hz to 6000Hz. The test sample was mounted in the sample holder. Acoustical software will process the data gathered and converted by A/D converter to calculate the final result required.

5. Discussion and prediction

In order to obtain exact multilayer predictions of absorption ratios and transmission losses, the accuracy of the transfer matrix of each layer in the multilayer system was first examined. Aside from some deviations in the frequencies below 1300Hz, TCM and TLM are adequate methods for evaluating the absorption ratio and transmission loss,

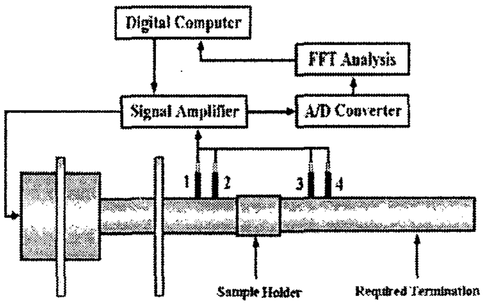

Fig.6 Experimental Apparatus (Acoustic Duct of the SCIEN Co.) for a four-microphone measurement

respectively. As seen from the experiments described above, in terms of accuracy, it is not easy to determine whether the TCM or TLM is more suitable for measuring or further predicting absorption ratios or transmission losses

Accordingly, we measured all the transfer matrices of the material samples from Table 2 as the elements were ready for further multilayer prediction. It should be noted that, for an in-situ sample with either single- or multi-layered materials, if its thickness is in the measurement range of the standing wave-duct system and the sound in the downstream section is loud enough. The above hybrid prediction procedures also apply to a sample with a thickness that exceeds the measurement range of the standing wave duct, which have to be cut into pieces and treated as a multilayer sample consisting of the same materials. During the tests for the element (each layer) transfer matrices, it was found that TLM was more cumbersome to apply than the TCM because of its complex calibration and measurement procedures. As to the number of microphones in the experiments, the two-microphone configuration was convenient for determining the absorption ratio, but usually cannot provide reasonable estimates of transmission loss, and the four-microphone configuration is the essential choice for determining transmission loss.

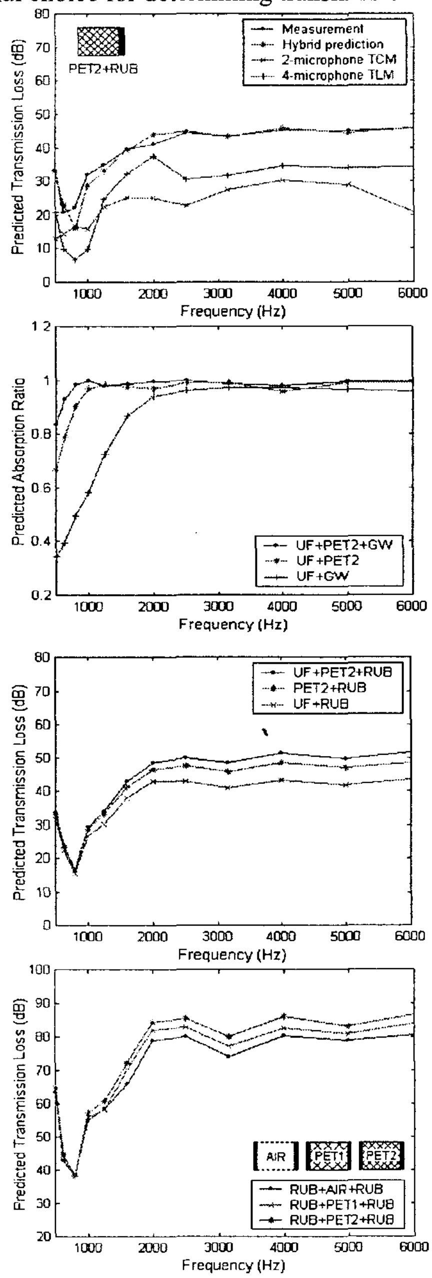

Fig.6 Result of the prediction of multi-layered materials

6. Conclusion

This paper introduced a new designed acoustical software, and its functions for evaluating the acoustical properties of noise control materials using standing wave-duct systems. Instead of measuring the real sample materials, the newly proposed experimental multilayer prediction (EHMP) was performed to evaluate absorption and transmission characteristics of a set of multilayered treatments consisting of different materials by directly taking the corresponding element transfer matrix from the ETMD. It can be concluded that the EHMP method is accurate and credible enough for the prediction of acoustical properties of multi-layered noise control configurations. Acoustical property optimizations of a multi-layered system may be obtained by adding more layers or by substituting certain layers with higher-quality material treatments. The EHMP extends the measurement scope of standing wave-duct systems and is capable of predicting and optimizing the acoustical properties of any material samples used in absorption and transmission characteristic design.

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