동적 유한요소해석에서의 반무한 경계조건의 실행
Implementation of semi-infinite boundary condition for dynamic finite element analysis

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개 요 : 실제 지반은 경계가 없는 무한상태로 존재하기 때문에 지반구조물의 동적거동을 유한요소법을 이용하여 해석할 시 모델의 영역을 성립하는 것은 특별한 고려가 필요하다. 유한요소법에서의 동적해석은 파동의 전달을 포함하기 때문에 모델의 경계에서 인공적인 경계조건이 필요하다. 인공적인 경계조건은 유한요소내의 지반상태를 무한상태로 변형시킬 수 있어야 하며, 경계에 도달하는 응력 파동을 모델내로 반사시키지 않고 흡수 할 수 있어야 한다. 본 논문에서는 간단한 점·탄성 반무한 불연속 요소를 이용하여 지반구조물의 동적해석을 수행하는 방법을 보여준다. 반무한 요소의 실행은 OpenSees라는 유한요소 해석프로그램을 이용하여 수행되었으며, 예를 통하여 불연속 요소가 경계에 도달하는 응력 파동을 충분히 흡수하여 유한요소 모델을 반무한 상태로 전환 시킬 수 있다는 것을 보여준다.

주 요 어 : 반무한 요소, 동적 유한 요소 해석, OpenSees, 점·탄성

1. Introduction

Many problems dealing with the dynamic response of geotechnical structure involve the wave propagation in semi-infinite domains. In that case, the numerical solution usually requires the introduction of an artificial boundary in order to render the domain finite and this artificial boundary must absorb the stress waves arriving at the boundary in order to simulate the physical fact that the energy decreases as the wave travels outer domain. This phenomenon is usually referred as radiation condition or geometric attenuation and it is distinguished from material damping in which elastic energy is actually dissipated by viscous, hysteretic, or other mechanism (Kramer 1996). Thus, it is required that the incident waves do not reflect back into the numerical domain at the boundary and that the incident waves be transmitted freely through the boundary for the case of dynamic excitation. This observation leads directly to the idea of determining the dynamic response of the interior region from a finite model consisting of the interior region subjected to a boundary condition, which ensures that all energy arriving at the boundary is absorbed.

As one approach for seeking numerical solutions to this problem, the viscous boundary conditions have been formulated by Lysmer and Kuhlemeyer (1969), which damp out the spurious reflections. It has been shown that the level of absorption for traveling waves is in overall satisfactory for a wide range of incident angles and is efficient enough for practical purposes (Castellani 1974). In consequent research White et al. (1977) suggested a particular choice of parameters for the dashpot and showed that the amount of reflections can be reduced further.

In this paper, the visco-elastic infinite element described above is reviewed and implemented in the recently developed OpenSees finite element code (OpenSees 2006) in which the Open System for Earthquake Engineering Simulation (OpenSees) is a finite element software framework for simulating the seismic response of structural and geotechnical systems developed by Pacific Earthquake Engineering Research (PEER). The effectiveness of the dashpot to mitigate reflections is addressed via comparison of example models created on unbounded and bounded domains. The procedure to create the model geometry and to generate the mesh for OpenSees is coded in Tcl/Tk script language and it is used to create the current models. Details on Tcl/Tk script is found in Flynt (2003).

2. Viscous boundary condition

One of the simplest absorbing boundary elements is the one originally proposed by Lysmer & Kuhlemeyer (1969) and usually referred as the classical viscous boundary condition. Figure 1 shows this boundary element for the case of an angle of incidence Θ = 90.

In Lysmer's formulation, the complete absorption of incidence wave is accomplished by entering normal and tangential stresses of the form

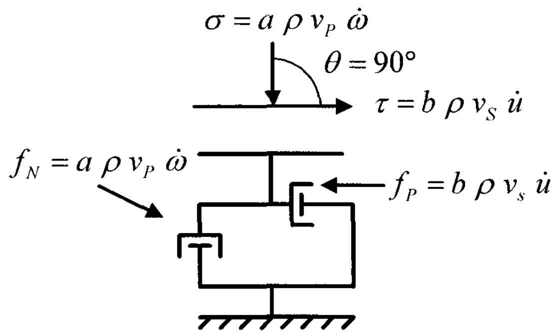

Figure 1. Simple infinite visco-elastic dashpot proposed by Lysmer & Kuhlemeyer (1969).

$$\sigma = a\,\rho\,V_P\,\dot{\omega} \tag{1}$$

$$\tau = b\,\rho\,V_S\,\dot{u} \tag{2}$$

where σ and τ are the normal and shear stresses in the boundary, respectively; $\dot{\omega}$ and $\dot{u}$ are the normal and tangential velocities, respectively; ρ is the mass density; V_p and V_s are the volumetric P-wave and distortional S-wave velocities of soil, respectively, and defined by

$$V_S = \sqrt{\frac{G}{\rho}} \tag{3}$$

$$V_P = \frac{1}{s} V_S \tag{4}$$

in which G is the shear modulus and s is a coefficient defined in terms of Poisson's ratio μ as

$$s^2 = \frac{1-2\mu}{2(1-\mu)} \tag{5}$$

and a and b are dimensionless parameters. The parameters a and b are chosen to minimize the reflected energy corresponding to an incident plane wave reaching the boundary at a given angle of incidence. The parameter selection is considered separately for an incident longitudinal wave, an incident transverse wave, and a surface wave. Lysmer & Kuhlemeyer (1969) have shown that the viscous boundary defined with a=b=1 is very effective if the angle of incidence is greater than 30 degree with absorbing 98% of P-waves and 95% of S-waves. As shown in Figure 1, if the angle of incidence angle is perpendicular to the plane, the stress that can be absorbed by the dashpot is equivalent to the stress produced by the incident wave both in the parallel and normal directions.

White et al. (1977) proposed the following a and b values to maximize the absorption efficiency at the boundary:

$$a = \frac{8}{15\pi}(5 + 2s - 2s^2) \tag{6}$$

$$b = \frac{8}{15\pi}(3 + 2s) \tag{7}$$

where s is given in Eq. (5). Thus, the viscous boundary parameters depend only on poisson's ratio. It was shown that using these a and b parameters the absorption efficiency is slightly increased for both P- and S-waves.

Even though the simple viscous boundary condition presented here yields spurious reflections in certain conditions such as in the case of a sharp incident angle, it has been proven to be satisfactory and useful for most practical problems.

3. Numerical scheme in OpenSees

The advantage of using viscous boundaries to represent the absorbing condition is that it can be simply coded in any numerical tool. In this particular work, the existing OpenSees Zero-Length element was used to include the artificial boundary condition (Mazzoni, et al. 2005). The Zero-Length element is represented by two nodes defined with the same geometric location (line 5 and 6 in Table 1). The nodes are connected through multiple UniaxialMaterial (Simo & Hughes 1998) objects that represent the elemental force-deformation relationship. Table 1 shows a simple example where a Zero-length element is used. The viscous damping coefficients corresponding to the parallel and normal direction to boundary in which

$$C_P = a\,\rho\,V_S \tag{8}$$

$$C_N = b\,\rho\,V_P \tag{9}$$

are defined in line 1 and line 2. These coefficients are used to create the material object in line 3 and 4. Then, the artificial element is defined using the material object and the orientation.

Table 1. Example of Zero-Length element construction in OpenSees.

```
1. set DampP 93
2. set DampN 173
3. uniaxialMaterial Elastic   1  0  $DampP
4. uniaxialMaterial Elastic   2  0  $DampN
5. node 1 16.0 0.0
6. node 2 16.0 0.0
7. element zeroLength 1  1  2  -mat 8  10 -dir 1 2
```

This element absorbs the incident elastic energy through the force-displacement relationship shown in Figure 1. The degree of absorption is governed by C_P and C_N which are defined in terms of the material properties given to the interior region.

It's a little complicated to formulate this boundary element within a soil domain in OpenSees since the "so called preprocessing" procedure to generate the mesh for a plane problem is not defined in OpenSees. Thus, a Tcl routine, which creates the input file for OpenSees, was developed by the author and it is found in http://www.ce.washington.edu/~geotech/opensees.

4. Implementation Vicous boundary infinite model

To examine the effectiveness of the viscous boundary and to demonstrate that it can be easily included in OpenSees, two models were used to analyze a semi-infinite domain. The response of each model using fixed and viscous boundaries, respectively, is compared. The model properties are given in Table 2.

Table 2. Material properties of example models.

ρ t/m^3	Vs $m/\sec$	Vp $m/\sec$	E kN/m^2	μ	C_P $t/m^2\sec$	C_N $t/m^2\sec$
1.6	58	108	14000	0.3	93	173

As a first verification case a rectangular domain was selected as shown in Fig. 2. By forcing the nodes on the left side boundary to behave in a same way as the right nodes, the one-dimensional wave propagating condition was achieved by use of the OpenSees command "equalDOF". The fixed and transmitting boundaries were applied along the base and sides of the model. A pulse load defined using sine and rectangular loading patterns allows examine how waves reflect at the boundary. Figure 3 compares the vertical displacement at the top and middle nodes for the fixed and transmitting boundary conditions. The reflection occurs in the fixed case, but it disappears when the transmitting boundary is used.

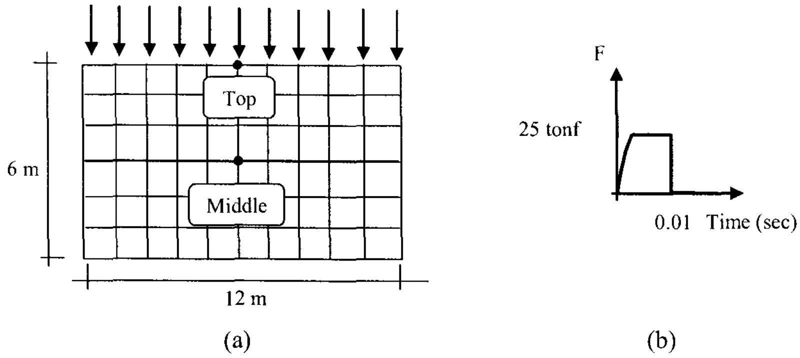

Figure 2. (a) One-dimensional model and (b) loading condition.

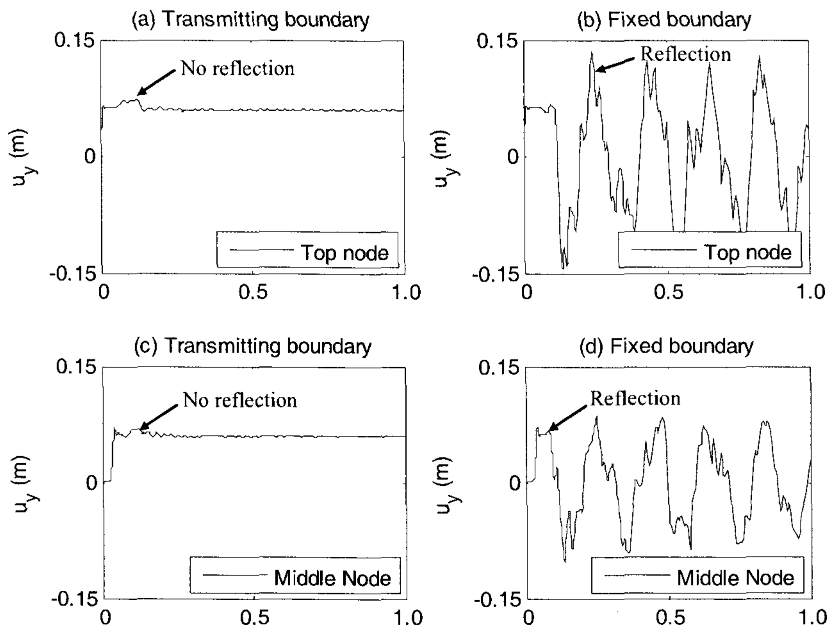

Figure 3. Vertical displacement at the top and midddle nodes for one-dimensional model : (a) top node of transmitting boundary, (b) top node of fixed boundary, (c) middle node of transmitting boundary, and (d) middle node of fixed boundary.

As a second verification example, a more realistic two-dimensional model was simulated. Figure 4 shows the geometry and loading condition. As in the previous example, the fixed and transmitting boundaries were used along the base and sides of the model. The response shown in Fig. 5 indicates that the transmitting boundary absorbs the incident waves satisfactory while the fixed boundary generates spurious reflections.

5. conclusions

The primary objective of this study was to demonstrate the capabilities of OpenSees to simulate a semi-infinite domain for geotechnical purposes. A simple viscous dashpot was

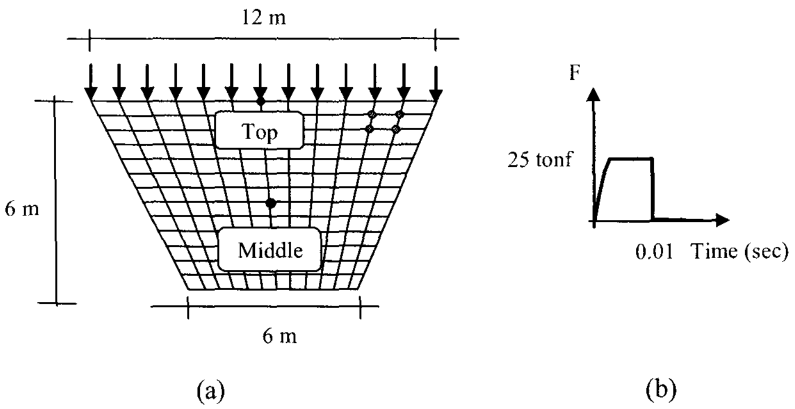

Figure 4. (a) Two-dimensional model and (b) loading condition.

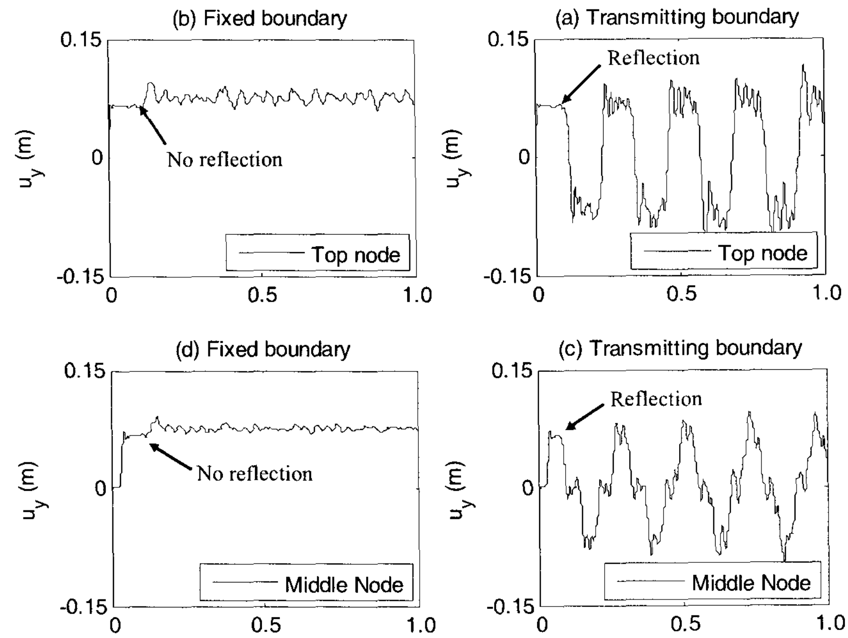

Figure 5. Vertical displacement at the top and midddle nodes for two-dimensional model : (a) top node of transmitting boundary, (b) top node of fixed boundary, (c) middle node of transmitting boundary, and (d) middle node of fixed boundary.

implemented to consider geometric attenuation. One- and two-dimensional models were simulated using OpenSees. The comparison made using fixed and transmitting boundaries shows visco-elastic transmitting element provides a method to simulate the semi-infinite domain within OpenSees program. The advantages of the viscous boundary type are that (1) it can be easily included into finite element code, OpenSees (2) it absorbs the incident wave satisfactory for most practical purposes.

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