순간 변위시험 (Slug Test)을 이용한 연직차수벽의 투수계수 산정시 수정된 Linear Curve Fitting 방법의 적용

Use of the Modified Linear Curve Fitting Method in Analyzing Slug Tests to Evaluate Hydraulic Conductivity of Vertical Cutoff Walls

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SYNOPSIS : 연직차수벽은 오염지역이나 폐기물 매립장에서 오염된 지하수 (때로는 오염된 수증기) 흐름을 차단하거나 또는 오염지역의 정화처리시 효율을 높이기 위하여 외부로부터 지하수 흐름을 막기 위해 설치된다. 이 논문에서는 연직차수벽의 가장 중요한 설계요소인 차수벽체의 현장 투수성을 평가하는 방법들중 가장 보편적으로 사용되는 단공식 순간 변위시험 (slug test)을 소개한다. 연직차수벽에서 실행된 단공식 순간 변위시험 결과를 해석하기 위해서, 연직차수벽의 압축성과 기하학적인 특성을 고려한 수정된 linear curve fitting 방법을 제안하고 그 적용성을 case study를 통해 평가한다. 기존의 대수층 투수계수 산정에 이용된 curve fitting 방법들에 비하여 수정된 linear curve fitting 방법은 보다 정확한 연직차수벽체의 투수성을 평가하도록 한다.

Key words : Vertical cutoff wall, Slug test, Curve fitting, Hydraulic conductivity

1. Introduction

Vertical cutoff walls are frequently used at contaminated sites to restrict lateral spreading of groundwater. One of the most important aspects of installing vertical cutoff walls is verification of low hydraulic conductivity. The design maximum hydraulic conductivity varies depending on application, but is typically either 1×10^{-8} or 1×10^{-9} m/s.

Hydraulic conductivity of the cutoff wall can be evaluated with laboratory or in situ tests. In situ methods offer the potential advantage of testing a larger and more representative volume of the cutoff wall (Daniel and Choi 1999). A slug test, one of the well known in situ tests, can be performed using a single well installed in a vertical cutoff wall. An instantaneous change in the water level is produced by the sudden introduction or removal of a known volume of water. The rate of water rise or fall in the well is measured until the water level returns to the static condition. Hydraulic conductivity is computed from the test results. Slug tests may be one of the best options to measure in situ hydraulic conductivity in formations of low hydraulic conductivity. Constant-rate pumping tests are not practical in this type of formation because it is difficult to maintain a very low pumping rate. One of the most important advantages of slug tests is the fact that no water is actually removed from the well, which is very important at the sites where groundwater contamination is concerned.

2. Slug Test Configuration in Vertical Cutoff Wall

Figure 1(a) shows a plan view of a vertical cutoff wall with width W and slug test system. The radius of the well intake section, i.e., outside of the filter pack, and the inside radius of the well casing are denoted as r_w and r_c, respectively. Deviation of the well from the center of a wall to the center of an eccentric well is denoted as D_v, or non-dimensionally as $2D_v/(W-2r_w)$. The model is fully three-dimensional, but considers a symmetric condition in the y direction. The effect and application of interface boundaries between the cutoff wall and natural soil formation is studied by Choi (2002). A simple model is proposed to consider only the cutoff wall along with a constant head boundary condition on the interface with the natural soil formation. In comparing this simple model to the comprehensive model that considers both of the cutoff wall and natural soil formation, if the ratio of hydraulic conductivity of the natural soil formation to hydraulic conductivity of the cutoff wall, i.e., k_{soil}/k_{wall}, is greater than 10, the simple model gives good agreement with the comprehensive model (Choi 2002). In the simple model, a filter cake on the interface between the cutoff wall and natural soil formation is not considered.

A vertical cross section is illustrated in Fig. 1(b). The cutoff wall is assumed to be keyed into a less permeable layer (aquitard) in the z direction. The depth of a vertical cutoff wall is denoted as L, and L_t indicates the distance from the water table to the top of the well intake section. The effective length of the well intake section (L_w) represents the length of filter pack by assuming no impediment to flow through filter pack to the well screen because the filter pack is usually more permeable than the formation being tested. The value of s, which is defined in Fig. 1(b), represents the relative vertical position of the well intake section in a vertical cutoff wall and ranges from zero to infinite. If the well intake section is located at the mid-depth of the wall, s is unity.

The well intake section boundary condition adopted in this paper assumes no vertical flow through the bottom of the well intake section due to the existence of a bottom seal and does not model the domain below the well intake section (Choi 2002). Parametric studies Choi (2002) show that if the direct flow through the bottom of the well intake section is stopped due to bottom sealing, and the well aspect ratio (L_w/r_w) of the well intake section is 10 or more, this well intake section boundary condition is applicable.

A rising-head slug test is initiated by withdrawing water from the well, which causes an initial hydraulic head drop, H_0, in the casing. This initial hydraulic head drop recovers with time. The head drop, which is denoted as $H(t)$ at time t, finally goes to zero at the end of the test. A normalized head drop of $H(t)/H_0$ is used.

3. Conventional Linear Curve Fitting Method

Simplified approaches that assume a steady state flow to or out of a well have been developed based on the Thiem equation by neglecting the effect of compressibility of the formation. The Hvorslev method (1951) and the Bouwer and Rice method (Bouwer 1989, Bouwer and Rice 1976) are in this category. In the simplified approach, $H(t)$ in the well is assumed to decay exponentially with time and is expressed as the following equation by assuming that the rate of decay of head in the well at any time is proportional to the head in the well at that time (Dax,1987):

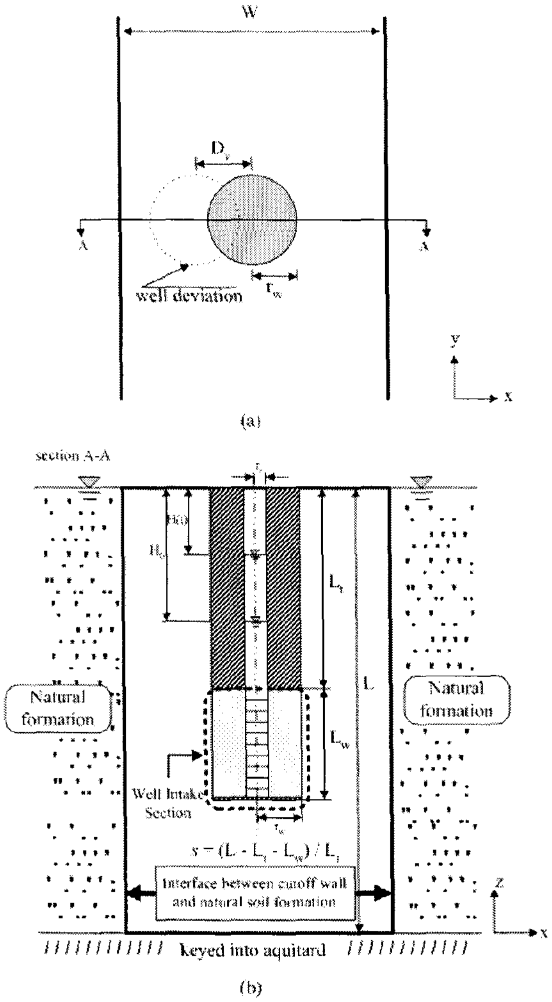


Fig. 1. Slug test configuration in vertical cutoff wall: (a) Plan view; (b) Vertical cross section A-A

$$H(t) = H_0 e^{-\lambda t} \qquad (1)$$

where is a positive constant and can be expressed as the following equation taking into account geometric conditions in a slug test:

$$\lambda = \frac{2L_w kt}{r_c^2 \ln\left(\frac{R_e}{r_w}\right)} \qquad (2)$$

where the effective radius, R_e, is defined as the equivalent radial distance over which head change is dissipated. In both the Hvorslev method and the Bouwer and Rice method, the R_e is assumed to depend only on the geometry of the flow system. Substituting Eq. (2) into (1), hydraulic conductivity can be written as follows:

$$k = -\frac{r_c^2 \ln\left(\frac{R_e}{r_w}\right)}{2L_w} \frac{1}{t} \ln\left(\frac{H(t)}{H_0}\right) \qquad (3)$$

Once the geometry of a well system is determined, hydraulic conductivity can be calculated from Eq. (3) and from the slope of slug test data plotted on the $\ln(H(t)/H_0)$ vs. t graph.

In the Hvorslev method, the empirical values of $\ln(R_e/r_w)$ are defined according to geometric conditions. For the geometry type G that Hvorslev denoted for a partially penetrating well (Hvorslev, 1951), $\ln(R_e/r_w)$ can be written as the following equation with the assumption of isotropic conditions:

$$\ln\left(\frac{R_e}{r_w}\right)=\ln\left(\frac{1}{2}\frac{L_w}{r_w}+\sqrt{1+\left(\frac{1}{2}\frac{L_w}{r_w}\right)^2}\right) \qquad (4)$$

In the Bouwer and Rice method (1976), the value of R_e was originally determined with an electrical resistance network analog and expressed in terms of $\ln(R_e/r_w)$ as follows:

$$\ln\left(\frac{R_e}{r_w}\right)=\left\{\frac{1.1}{\ln\left[\frac{L_w}{r_w}\left(\frac{\frac{L}{L_w}+s}{s+1}\right)\right]}+\frac{A+B\ln\left[\frac{L_w}{r_w}\left(\frac{s}{s+1}\right)\left(\frac{L}{L_w}-1\right)\right]}{\frac{L_w}{r_w}}\right\}^{-1} \qquad (5)$$

where s = $[L-L_t+L_w]/L_t$ and the parameters A and B depend on the well aspect ratio (L_w/r_w) and are provided by Bouwer and Rice (1976).

Because the conventional linear curve fitting methods neglect the compressibility of the soil skeleton, the theoretical representation of $\ln(H(t)/H_0)$ vs. t is linear. Actual slug test data for highly compressible materials do not show a straight line on the $\ln(H(t)/H_0)$ vs. $t$graph, but exhibit a significant upward curvature. Therefore, hydraulic conductivity evaluated by Eq. (3) depends on how one fits a straight line to curving data.

4. Modified Linear Curve Fitting Method

4.1 Application for Aquifer

To take into account the compressibility of backfill materials, slug test data for an aquifer can be plotted on a graph of $\ln(H(t)/H_0)$ vs. β_p. β_p is denoted as the dimensionless time factor ($\beta_p = kL_w t/r_c^2$). The error of the linear curve fitting methods can be expressed by the ratio R that is defined as $k_{\text{best-fitting}}/k_{\text{real}}$. The $k_{\text{best-fitting}}$ represents the calculated hydraulic conductivity using the linear curve fitting method for a given geometry of a well. The k_{real} represents an actual hydraulic conductivity that is used for synthesizing slug test data using the implicit finite difference model Slug_3D (Choi 2002). Slug_3D has been developed to analyze a slug test in aquifer and/or cutoff walls. The numerical program is able to consider compressible materials, boundaries between a vertical cutoff wall and surrounding formation. The definition of β_p and Eq. (3) can be used to define the value of R for the aquifer case as follows:

$$R = \frac{k_{best-fitting}}{k_{real}} = -\frac{\ln\left(\frac{R_e}{r_w}\right)}{2}\frac{\ln\left(\frac{H(t)}{H_0}\right)}{\beta_p} = -1.152\ln\left(\frac{R_e}{r_w}\right)\frac{\log\left(\frac{H(t)}{H_0}\right)}{\beta_p} \tag{6}$$

One problem with this approach relates to the effective radius (R_e). Neglecting the compressibility of the soil skeleton causes the effective radius to be dependent only on the geometry. Chirlin (1989) introduced a way of modifying the linear curve fitting method to account for compressible materials and a changing R_e. Chirlin assumed that Cooper's solution (Cooper et al. 1967, Papadopulos et al. 1973) characterized real response. The Hvorlsev method was employed for fitting data with the basic time lag formula. Chirlin calculated the modified effective radius, R_e', that satisfies R=1 in Eq. (6), in other words, $k_{\text{best-fitting}} = k_{\text{real}}$, and used the values when estimating hydraulic conductivity with the Hvorslev method instead of the original values of R_e that Hvorslev recommended. The R_e' is dependent on compressibility of soil skeleton as well as on well geometry.

In the current study, Chirlin's approach was extended to considera partially penetrating well in the vertical cutoff wall. Because the effective radius is replaced by R_e', there will be no difference in the calculated hydraulic conductivity for either the Hvorslev method or the Bouwer and Rice method. The modified $\ln(R_e'/r_w)$ is derived from Eq. (6) by letting R=1 as follows:

$$\ln\left(\frac{R_e'}{r_w}\right) = -\frac{2\beta_p}{\ln\left(\frac{H(t)}{H_0}\right)} = -\frac{0.868}{\log\left(\frac{H(t)}{H_0}\right)/\beta_p} \tag{7}$$

The value of $\ln(R_e'/r_w)$ can be obtained for any fitting method. The ratio of $\log(H(t)/H_0)/\beta_p$ is determined from the slope of a fitting line at given fitting points. The Hvorslev's basic time lag formula that fits a straight line between $H(t)/H_0$=1 and 0.37 is adopted in this paper. This is equivalent to fitting a straight line through curved data at the point of $H(t)/H_0$=1 and 0.37. Other fitting points were considered, but no more convenient or accurate points were identified. With the basic time lag formula, the value of $\ln(R_e'/r_w)$ reduces as follows for these fitting points:

$$\ln\left(\frac{R_e'}{r_w}\right) = 2\beta_{p\_0.37} \tag{8}$$

where $\beta_{p\_0.37}$ represents β_p that is in correspondence with $H(t)/H_0$ = 0.37. Because $\beta_{p\_0.37}$ is a function of the dimensionless compressibility parameter $\alpha_p(=S_s L_w r_w^2/r_c^2)$, the value of $\ln(R_e'/r_w)$ also depends on α_p.

The values of $\ln(R_e'/r_w)$ are plotted in Fig. 2 for four different well aspect ratios, L_w/r_w=5, 10, 15 and 20. One can evaluate hydraulic conductivity in an aquifer with the aid of the Hvorslev's basic time lag formula and the values of $\ln(R_e'/r_w)$ from Fig. 2. Considering the compressibility of the soil skeleton represented by α_p, the hydraulic conductivity of an aquifer is calculated by substituting $\ln(R_e'/r_w)$ for $\ln(R_e/r_w)$ in Eq. (3). However, the values of $\ln(R_e'/r_w)$ from Fig. 2 are for the aquifer case and require modification to be applicable to vertical cutoff walls.

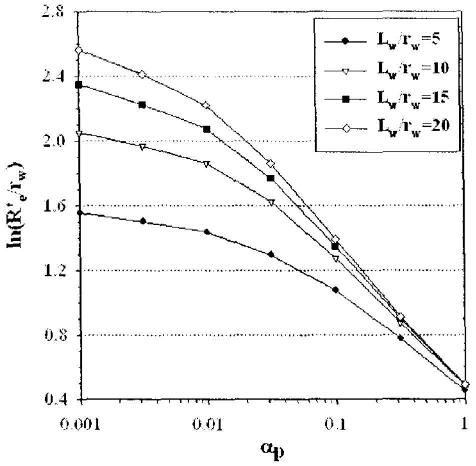


Fig. 2. Values of $\ln(R_e'/r_w)$ for aquifer case (L/L_w=11 and s=1)

4.2 Application for Vertical Cutoff Wall

In order to consider the vertical cutoff wall boundary condition, a reduction factor, f, is introduced. The reduction factor equals the ratio of $\ln(R_e'/r_w)_{\text{cutoff wall}}$ to $\ln(R_e/r_w)_{\text{aquifer}}$. To compensate for a faster head recovery due to close boundaries in the vertical cutoff wall, the value of $\ln(R_e'/r_w)$ has to be reduced by f in calculating hydraulic conductivity by substituting $\{f\times\ln(R_e'/r_w)\}$ for $\ln(R_e/r_w)$ in Eq. (3). The reduction factors, which range from about 0.4 to 1.0, are plotted in Fig. 3 for the case of $2D_v/(W-2r_w)$=0, 0.4, and 0.8, and L_w/r_w=10 and 15 for a typical geometric condition of L/L_w=11 and s=1 are selected. Graphs of the reduction factor for other geometric combinations can be found in Choi (2002).

The procedure to be followed in using the modified linear curve fitting method for vertical cutoff walls is as follows:

(1) Step 1: Determine geometrical parameters- length of well screen, L_w radius of well screen, r_w inside radius of well casing, r_c width of cutoff wall, W eccentricity of well from center of cutoff wall, D_v.

(2) Step 2: Determine specific storage, S_s, based on experience or laboratory oedometer tests in the appropriate stress range

$$S_s = \rho_w g\left(n\bar{\beta} + \bar{\alpha}\right) \qquad (9)$$

where ρ_w=bulk mass density of water; g=acceleration due to gravity; n=porosity; $\bar{\alpha}$ = compressibility of soil skeleton measured by 1-D compression test, that is, oedometer test; and $\bar{\beta}$ = compressibility of water (= 4.410^{-10} Pa^{-1}). Therefore, the dimension of S_s is the inverse of length (e.g., cm^{-1} or m^{-1}). In Eq. (9), $\bar{\alpha}$ represents the ratio of volumetric strain of the soil to the change in effective stress and is identical to the coefficient of volume compressibility or m_v in soil mechanics literature. $\bar{\beta}$ is so small compared to $\bar{\alpha}$ that it is usually ignored for soils.

(3) Step 3: Calculate the dimensionless compressibility parameter, α_p

$$\alpha_p = \frac{S_s L_w r_w^2}{r_c^2} \qquad (10)$$

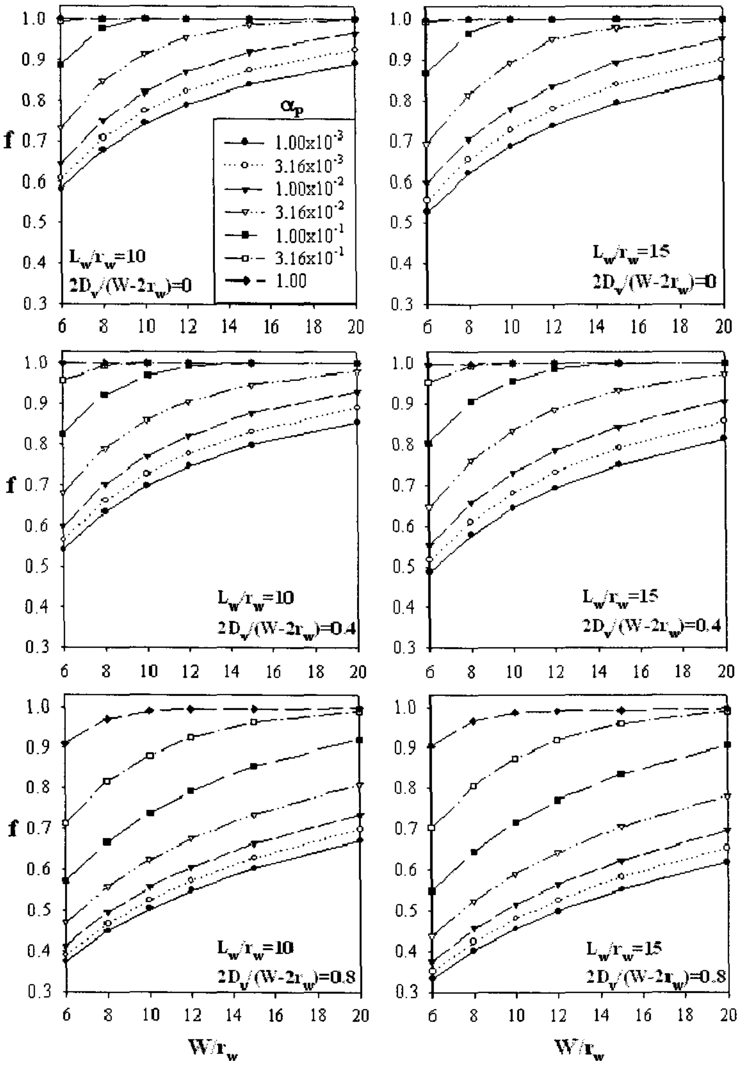


Fig. 3. Reduction factors (L/L_w=11 and s=1)

(4) Step 4: Plot slug test data as $H(t)/H_0$ versus log of time. Fit a straight line through the origin and the point at which $H(t)/H_0$= 0.37. Determine the corresponding time, $t_{0.37}$.

(5) Step 5: For α_p from Step 3, determine the value of $\ln(R_e'/r_w)$ from Fig. 2.

(6) Step 6: Determine the reduction factor, f, from Fig. 3.

(7) Step 7: Calculate hydraulic conductivity, k, from the following equation.

$$k = \frac{r_c^2\left[f \times \ln\left(\frac{R_e'}{r_w}\right)\right]}{2L_w}\frac{1}{t_{0.37}} \qquad (11)$$

One of the interesting aspects of a slug test in a vertical cutoff wall is that the reduction factor for the most compressible formation considered (α_p=1) is close to unity, which means there is little effect of cutoff wall boundaries on the result of a slug test. This is because water flowing into a well installed in a highly compressible material is derived primarily from water released from storage rather than water flowing from the formation, through the wall, and into the well. With the modified linear curve fitting method, it is necessary to estimate specific storage, i.e., compressibility of the soil skeleton. This can be done by employing other geotechnical experiments (e.g., consolidation tests) or engineering correlations.

5. Case Study

The case study described here is taken from EMCON (1995) and involves slug test response data from the slurry trench cutoff walls constructed in the early 1990's at the West Contra Costa Sanitary Landfill in Richmond, California. Test data were submitted to the state regulatory agency as part of the permitting process. The M-11/15 cutoff wall is located along the northern and western border of the landfill. The M-17/21 cutoff wall is located along the southwestern and southern side of the landfill. Two sets of slug test data are presented herein. The first data set is the 93-1 case, which was performed in the M-11/15 cutoff wall, and the other is the 94-15 case, which was performed in the M-17/21 cutoff wall. Well deviation surveys using a slope indicator were performed to evaluate eccentricity of wells at the depth of the well screen. Based on the report by EMCON (1995), the geometry of wells and cutoff walls is summarized in Table 1.

Table 1. Modeling geometry description of vertical cutoff wall

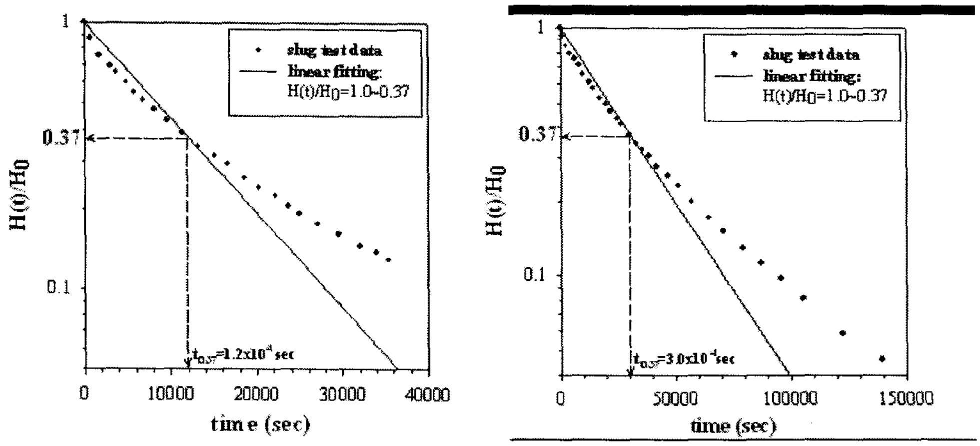
Geometry Variables		Values			
		93-1 (M-11/15 cutoff wall)		94-15 (M-17/21 cutoff wall)	
Well	r_c (m)	0.025		0.025	
	r_w (m)	0.105		0.036	
	L_w (m)	1.15		0.55	
Cutoff wall	L (m)	9.04		9.26	
	L_t (m)	4.72		4.42	
	W^{*1} (m)	0.92		0.92	
	D_v (m)	0		0.23	
Model variables		Real value	Approximation	Real value	Approximation
	L_w/r_w	11.0	10.0	15.1	15.0
	L/L_w^{*2}	8.0	11.0	16.8	11.0
	s^{*3}	0.70	1.00	0.97	1.00
	W/r_w	8.8	8.0	25.4	20.0
	$2D_v/(W-2r_w)$	0	0	0.54	0.60

Note: *1 : minimum requirement of a width of a cutoff wall (=0.92 m)

*2 : if $L/L_w > 7.0$, not significant effect (Choi 2002)

*3 : when large value of L/L_w, the effect of s is not significant (Choi 2002)

Hydraulic conductivity was calculated with modified linear curve fitting method. Test data and analysis procedures are presented in Fig. 4 and the results are compared with EMCON'S results obtained utilizing a conventional linear curve fitting method (Bouwer and Rice method) in Table 2.



(a) Case 93-1 in M-11/15 (b) Case 94-15 in M-17/21

Fig. 4. Slug test analysis with the modified linear curve fitting method

Table 2. Case study results of EMCON case

Case	Modified Curve Fitting Method				EMCON'S Original Results	
	$t_{0.37}$ [H(t)/H_0=0.37] (sec)	ln(R_e/r_w)	f	k (m/s)	S_s [lab. test] (m^{-1})	k [B-R method] (m/s)
93-1 (M-11/15)	1.2×10^4	1.15	0.98	2.6×10^{-8}	2.5×10^{-2}	3.0×10^{-8}
94-15 (M-17/21)	3.0×10^4	17.1	0.92	3.1×10^{-8}		4.0×10^{-8}

The original values of hydraulic conductivity estimated by EMCON (1995) yield higher hydraulic conductivity by 20 to 30 percent. These differences are caused primarily by the fact that the Bouwer and Rice method used in the original analysis of data ignores the compressibility of a backfill material and the vertical cutoff wall boundary. A 20 to 30 percent difference in hydraulic conductivity is not considered to be especially significant. The behavior of a slug-test system in a narrow cutoff wall backfilled with compressible material is complex. Some of the errors that arise from simplified analysis may offset one another. However, the modified linear curve fitting method described in this paper eliminates most of the significant errors, and are no more difficult to use than more conventional methods applicable to aquifer.

5. Conclusion

A practical method to evaluate hydraulic conductivity of a vertical cutoff wall was developed with the aid of a numerical program. The modified linear curve fitting method provides an easy and rigorous way to evaluate hydraulic conductivity of a compressible vertical cutoff wall.

To illustrate the application of the linear curve fitting method, a case study involving two slug tests in vertical cutoff walls at a landfill site was considered. The original values of hydraulic conductivity calculated by the project team, which were evaluated using the Bower and Rice method for aquifer with a linear regression analysis, overestimated hydraulic conductivity by 20 to 30 percent, compared to results from the modified linear curve fitting method. These differences are caused primarily by the fact that the Bouwer and Rice method ignores the compressibility of the backfill material and the vertical cutoff wall boundary. In general, an error of 20 to 30 percent is not considered especially significant for determination of hydraulic conductivity. Yet, the modified linear curve fitting method is no more difficult to use than conventional methods, are more rigorous, and do not rely on offsetting errors for accuracy.

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