샘플치-데이터 퍼지 시스템의 안정도 분석

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Stability Analysis of Sampled-Data Fuzzy System

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Abstract - This paper addresses the problem of stability analysis and control synthesis of a digital fuzzy control systems. The authors shows that the stability properties (in the Lyapunov sense) of a digital fuzzy control system can be deduced from the stability properties of the its approximate discretization in the sufficiently small sampling time.

1. Introduction

In recent years, the study of digital control systems has received growing attention [1-6]. The digital control systems are a class of hybrid dynamical systems consisting of a family of continuous- (or discrete-) time subsystems. Traditional analysis and design tools for continuous-time or discrete-time systems are unable to be directly used in the digital control systems. One possibility to address the digital control is to develop a discrete-time model for the controlled, continuous-time plant and then pursue a digital controller based on the discretized model. However, it is difficult to design the digital fuzzy controller because exact discrete-time models of continuous-time processes are typically impossible to compute.

There have been some researches focusing on the digital controller [1-6] for Takagi-Sugeno (T-S) fuzzy systems based on their approximate discrete-time models. Although a great deal of effort has been made on digital control such as [1-6], there still exists some matters that must be worked out. It is a very important factor to preserve the stability in the digital controller, but the previous methods [1-3] do not only assure the stability of the digital fuzzy cotrol systems but also their approximately discretized model. At this point, the results [4-6] provided that sufficient conditions to stabilize the approximate discrete-time model of the sampled-data fuzzy system. However, it is only shown that the digitally controlled fuzzy system is asymptotically stable under the assumption that there exists no approximation error.

In this paper, we studies the problem of a stability analysis and a control synthesis of digital fuzzy control systems based on the approximate discrete-time model. It is shown that the stability properties of a digital fuzzy control system can be deduced from the stability properties of the its approximate discretization in the sufficiently small sampling time.

The rest of this paper is organized as follows: Section 2 briefly reviews the T-S fuzzy system. In Section 3, the stability analysis of the digital fuzzy control system is included. Finally, Section 5 concludes this paper with some discussions.

2. Sampled-Data Fuzzy Systems

Consider the system described by the following T-S fuzzy model:

$$\dot{x}(t) = \sum_{i=1}^{r}\theta_i(z(t))(A_ix(t) + B_iu(t)) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$, r is the number of model rules, $z(t) = [z_1(t)\cdots z_p(t)]^T$ is the premise variable vector that is a function of states $x(t)$, and $\theta_i(z(t))$, $i \in \mathrm{I}_R(=\{1,2,\cdots,r\})$ is the normalized weight for each rule, that is $\theta_i(z(t)) \geq 0$ and $\sum_{i=1}^{r}\theta_i(z(t)) = 1$.

The digital fuzzy controller takes the following form:

$$u(t) = \sum_{i=1}^{r}\theta_i(z(kT))K_ix(kT) \tag{2}$$

for $t \in [kT, kT+T)$, $k \in \mathbb{Z}_{\geq 0}$. Substituting (1) into (2), we obtain the closed-loop system, which is expressed as

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(t))\theta_j(z(kT))(A_ix(t) + B_iK_jx(kT)) \tag{3}$$

for $t \in [kT, kT+T)$, $k \in \mathbb{Z}_{\geq 0}$. A mixture of the continuous-time and discrete-time signals occurs in the above system (2). It makes traditional analysis tools for a homogeneous signal system unable to be directly used. It is found in [1-7] that the approximate discrete-time model of (3) takes the following form:

$$x(kT+T) \cong \sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(kT))\theta_j(z(kT))(G_i + H_iK_j)x(kT) \tag{4}$$

where $G_i = e^{A_iT}$ and $H_i = (G_i - I)A_i^{-1}B_i$.

3. Stability Analysis

In this section, we show that the digital fuzzy controller (2) stabilizing the approximate discrete-time fuzzy model (4) would also stabilize the sampled-data fuzzy system (3) in the sufficiently sampling time.

Now, we show that the digital fuzzy control system (3) is also asymptotically stable in the sufficiently small T if the approximate discrete-time model (4) is asymptotically stable. For simplicity, we redefine (1), (3) and (4) as

$$\dot{x}(t) \triangleq f(x(t), u(t)) \tag{5}$$

$$\dot{x}(t) \triangleq f(x(t), x(kT)) \tag{6}$$

$$x(kT+T) \triangleq F(x(kT)) \quad (7)$$

respectively.

Lemma 1. Let $f(x,u)$ be locally Lipschitz in their arguments. The exact discrete-time model of (3) takes the following form:

$$x(kT+T) \quad = F(x(kT)) + T^2 E(x(kT)) \quad (8)$$

Proof. The solution of (3) over $t \in [kT, kT+T)$ is given by

$$\begin{aligned} x(t) \quad &= x(kT) + (t-kT)f(x(kT), x(kT)) \\ &+ \int_{kT}^{t} \{f(x(\mu), x(kT)) - f(x(kT), x(kT))\} d\mu. \end{aligned}$$

On compact sets of x, we can use the Lipschitz property of f and the Gronwall--Bellman inequality, to show that

$$\| x(t) - x(kT+\kappa\tau) \| \le \frac{1}{L_1}(e^{(t-kT)L_1} - 1) \| f(x(kT), x(kT)) \|$$

for $t \in [kT, kT+T)$, where is a Lipschitz constant L_1 of f. Hence

$$x(kT+T) \quad = x(kT) + Tf(x(kT), x(kT)) + T^2 e_1(x(kT))$$

Also the equation (4) can be expressed as

$$F(x(kT)) \quad = x(kT) + Tf(x(kT), x(kT)) + T^2 e_2(x(kT)) \quad (9)$$

Therefore, using $E(x(kT)) \triangleq e_1(x(kT)) - e_2(x(kT))$, we can represent the exact discrete-time model of (6) as (8). ∎

Theorem 1. The zero equilibrium $x_{eq} = [0]_{n\times 1}$ of (6) is asymptotically stable in the sufficiently small T if the zero equilibrium $x_{eq} = [0]_{n\times 1}$ of the approximate discrete-time model (4) is asymptotically stable.

Proof. The approximate discrete-time model (4) of (3) is assumed to be asymptotically stable and therefore there are positive definite matrices P and Z such that

$$\| x(kT+T) \|_P^2 - \| x(kT) \|_P^2 = -Z \quad (10)$$

where $\| x \|_P = \sqrt{x^T P x}$. Now choose the Lyapunov function $V(x(kT)) = \| x(kT) \|_P^2$. Then, the first forward difference of V evaluated along the solutions of (4) yields

$$\begin{aligned} \Delta V &= V(x(kT+T)) - V(x(kT)) \\ &= \left\| F(x(kT)) + T^2 E(x(kT)) \right\|_P^2 - \| x(kT) \|_P^2 \quad \text{(By Lemma 1)} \end{aligned}$$

It follows from the Cauchy-Schwarz inequality that, for any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$,

$$\begin{aligned} \Delta V &\le (1+\varepsilon_1)\| F(x(kT)) \|_P^2 + T^4\left(1+\frac{1}{\varepsilon_1}\right)\| E(x(kT)) \|_P^2 - \| x(kT) \|_P^2 \\ &= (1+\varepsilon_1)\| x(kT+T) \|_P^2 - (1+\varepsilon_1)\| x(kT) \|_P^2 \\ &\quad + T^4\left(1+\frac{1}{\varepsilon_1}\right)\| E(x(kT)) \|_P^2 + \varepsilon_1 \| x(kT) \|_P^2 \\ &= -(1+\varepsilon_1)Z + T^4\left(1+\frac{1}{\varepsilon_1}\right)\| E(x(kT)) \|_P^2 + \varepsilon_1 \| x(kT) \|_P^2 \end{aligned}$$

Note that, as T becomes smaller, so does constant ε_1. Hence, ΔV will be negative definite if T is sufficiently small. Assume that $\| E(x(kT) \| \le \nu_2$, where ν_2 is constant. It is shown that

$$\begin{aligned} \| x(t) \| &\le \| F(x(kT)) \| + T^2 \| E(x(kT) \| \\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r} \nu_{1ij} \| x(kT) \| + T^2 \nu_2 \end{aligned} \quad (11)$$

where $\nu_{1ij} = e^{\|A_i\|T} + T e^{\|A_i\|T} \| B_i F_j \|$ is independent of k. Therefore, $x(t)$ converges to the origin simultaneously wi th $x(kT)$ in the sufficiently small T. ∎

3. Conclusions

In this paper, we have examined that a digital controller that stabilize approximate discrete-time fuzzy model would also st abilize the resulting digital fuzzy system in the sufficiently s mall control update time. From the obtained result, we know that digital fuzzy controller can be simply designed based on the approximate discrete-time fuzzy model.

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