TS 퍼지 상태 추정에 관한 강인 칼만 필터

노선영*, 주영훈**, 박진배*
*연세대학교 전기전자공학과, ** 군산대학교 전자정보공학부

Robust Kalman filtering for the TS Fuzzy State Estimation

Sun Young Noh*, Young Hoon Joo**, Jin Bae Park*
*Yonsei University, **Kunsan National University

Abstract - In this paper, the Takagi-Sugeno (TS) fuzzy state estimation scheme, which is suggested for a steady state estimator using standard Kalman filter theory with uncertainties. In that case, the steady state with uncertain can be represented by the TS fuzzy model structure, which is further rearranged to give a set of uncertain linear model using standard Kalman filter theory. And then the unknown uncertainty is regarded as an additive process noise. To optimize fuzzy system, we utilize the genetic algorithm. The steady state solutions can be found for proposed linear model then the linear combination is used to derive a global model. The proposed state estimator is demonstrated on a truck-trailer.

1. Introduction

The design of a Kalman filter relies on having an exact dynamic model of the system under consideration in order to provide optimal performance when the design contains relatively small modeling errors [1]. However, most dynamical systems in the world have severe nonliterary. In order to design of optimal filters, many recent works have dealt with the problem of robust filter design for all admissible uncertainties by minimizing the upper bound on the variance of estimation error. The uncertainty is parameterized in terms of a norm-bounded parameter matrix [2, 3]. Conventionally, the extended Kalman filter has been proposed for state estimation by using a linearization of the nonlinear systems around the present estimate through an application of linear filter theory [4]. However, few works have studied the estimation problem for nonlinear uncertain systems.

In this paper, the uncertain nonlinear system is represented by T-S fuzzy model structure, which is further rearranged to give a set of uncertain linear systems. The unknown uncertainty is regarded as an additive process noise and then the time-varying variance of the overall process noise is calculated. This model is designed for a local linear state space model using standard Kalman filter theory. Finally, the proposed state estimator is demonstrate on a trcuk-trailer.

2. Fuzzy State Estimation

2.1 Preliminaries and problem formulation

Nonlinear systems can be approximated as locally linear systems in much the same way that nonlinear functions can be approximated as piecewise linear functions. Consider a nonlinear dynamical system of the form.

$$x(k+1)=f(x(k),u(k)) \tag{1}$$

This uncertain nonlinear system can be approximated or represented by the T-S fuzzy model, which is composed of a set of fuzzy inference rules. Nonlinear dynamical system with uncertainties can be represented by fuzzy linear models of the form.

$$\begin{aligned} &IF \quad z_1(k)\ is\ F_{1i}\ \text{and} \ \dots \ \text{and}\ z_n(k)\ is\ F_{ij} \\ &THEN\, x(k+1)=(A_i+\Delta A_i)x(k)+B_iu(k)+Gw(k) \\ &y(k)=C_ix(k)+v(k), \qquad i=1,2,3,\dots,L \end{aligned} \tag{2}$$

where F_{ij} is the fuzzy set, L is the number of fuzzy rules, $y(k)$ is measured output, A_i and B_i are known constant matrices, the process noise $w(k)$ is white with PSD S_w, the measurement noise $v(k)$ is white with PSD S_v, and the process noise and measurement noise are uncorrelated, $\Delta A_i(k)\Delta A_i^T(k)\le 1$ is uncertain time-varying matrix, and $z_1(k),\cdots,z_n(k)$ are premise variables.

Now we define L discrete time signals $x(k)$ and $y(k)$. The final output of the fuzzy system is inferred as follows:

$$x(k+1)=\sum_{i=1}^{L}\mu_i(z(k))\left[(A_i+\Delta A_i)x(k)+B_iu(k)+G_iw(k)\right] \tag{3}$$

$$y(k)=\sum_{i=1}^{L}\mu_i(z(k))\left[C_ix(k)+v(k)\right]$$

where

$$A_i(k)=\sum_{i=1}^{L}\mu_i(z(k))A_i$$

$$\Delta A(k)=\sum_{i=1}^{L}\mu_i(z(k))\Delta A_i$$

$$w_i(z(k))=\prod_{j=1}^{n}F_{ij}(z_j(k))$$

$F_{ij}(z_j(k))$ is the grade of membership of $z_j(k)$ in F_{ij}, and

$$\mu_i(z(k))=\frac{w_i(z(k))}{\sum_{i=1}^{L}w_i(z(k))}$$

with $w_i(z(k))\ge 0$ for all k, we get the following form:

$$\sum_{i=1}^{L}\mu_i(z(k))=1 \tag{4}$$

From these definitions, we define L discrete time signals and it can be defined as

$$x_i(k)=\mu_i(z(k))x(k), \qquad x(k)=\sum_{i=1}^{L}x_i(k) \tag{5}$$

2.2 Filtering using the fuzzy model

In this section we modify the Kalman filter for the system given by using (3) and (5). Suppose we are given the linear discrete time system of the form.

$$\begin{aligned} x(k+1)=&\ Ax(k)+\mu(z(k))\Delta Ax(k)+\mu(z(k))Bu(k) \\ &+\mu(z(k))Gw(k) \\ y(k)=&\ Cx(k)+\mu(z(k))v(k) \end{aligned} \tag{6}$$

where the scalar $h(z(k))\in(0,1)$ and because the uncertain time-varying matrix is $\Delta A_i(k)\Delta A_i^T(k)\le 1$, we assume that the uncertainty matrix is an additive process noise.

Then, the state of the system can be rewritten as

$$x(k+1)=Ax(k)+\mu(z(k))Bu(k)+\mu(z(k))\bar{w}(k) \tag{7}$$

where, $\bar{w}(k)=(\Delta A+w(k))$ is the overall process noise with unknown uncertain matrix; thus we can treat the uncertain linear discrete system by adjusting this process noise variance.

We combine the Kalman filter for the local systems given in (7) to obtain a state estimator for the TS fuzzy model given in (2). The state $\hat{x}$ of the system can be derived by assuming a recursive estimator of the form. The predicted state is represented as

$$\hat{x}^+(k)=(I-K(k)C(k))\hat{x}^-(k)+K(k)v(k) \tag{8}$$

$$\hat{x}^-(k+1)=A\hat{x}^+ +h(z(k))Bu(k) \tag{9}$$

where "-"superscript is to indicate a quantity before the measurement is taken into account, and "+" superscript to indicate a quantity after the measurement is taken into account.

Because of the modified state $\hat{x}^+(k)$, the measurement residual is defined as

$$v(k)=y(k)-\hat{x}^+(k) \tag{10}$$

where $v(k)$ is the measurement residual, and $K(k)$ is a Kalman gain whose matrices are to be determined.

The unknown process noise with uncertainty $\bar{w}$ are inferred by a double-input single-output fuzzy system, for which the jth fuzzy IF-THEN rule is represented by

$$IF\ x_1\ is\ A_{1i}\ \text{and}\ x_2\ is\ A_{2i},\ THEN\ y_j\ is\ \bar{w}_j \tag{11}$$

where two premise variables x_1 and x_2 are the measurement residual $v(k)$ and change rate $\dot{v}(k)$, respectively, consequent variable y_j is the overall process noise, and A_{ij} are fuzzy set. It has the Gaussian membership function with center $\bar{x}_{ij}$ and standard deviation $\bar{\sigma}_{ij}$

$$f(x_i;\bar{\sigma}_{ij};\bar{x}_{ij}) = \exp\left[-\frac{1}{2}\left(\frac{x_i-\bar{x}_{ij}}{\sigma_{ij}}\right)^2\right] \quad (12)$$

In this paper, the GA methods will be applied to optimize the parameters and the structure of the system, using the product-sum inference method, singleton fuzzifier, center average defuzzifier, and Gaussian membership function. That is, the defuzzified output of the fuzzy model based on the overall process noise with unknown uncertainty is given by

$$\bar{w}_k = \frac{\sum_{j=1}^{L} w_j A(x_{1j}) \times A(x_{2j})}{\sum_{j=1}^{L} A(x_{1j}) \times A(x_{2j})} \quad (13)$$

According to the approximation theorem by the GA, the overall proccss noise is optimized.

We define the estimator error $\tilde{x}$ and its covariance with P as,

$$\tilde{x} = x - \hat{x}, \qquad P = \left(\tilde{x}\tilde{x}^T\right) \quad (14)$$

The steady state Kalman filter presented can be used to estimate the states of each of the L dynamic systems given in (7). This will give us L local steady state estimates. Due to the estimated term $\bar{w}$, the covariance matrix of $P^-(k+1)$ becomes

$$P^-(k+1) = A_i\left(P_i^-(k) - K_i(k)C_iP_i^-(k)\right)A_i^T + G_i\bar{w}_iG_i^T \quad (15)$$

When the unknown uncertainty is employed, the conventional Kalman filter has to be modified.

We can find the optimal Kalman gain by using (15).

$$K_i(k) = P_i^-(k)C_i^T\left(C_iP_i^-(k)C_i^T + S_v\right)^{-1} \quad (16)$$

The predicted state is replaced by (16)

$$\hat{x}_i^+(k) = (I - K_i(k)C_i)\hat{x}_i^-(k) + K_i(k)v_i(k) \quad (17)$$

From (5), we can combine the local state estimates as

$$\hat{x}(k) = \sum_{i=1}^{L} \hat{x}_i(k) \quad (18)$$

2.3 Simulation results

In this section we consider state estimation for a discrete time model of a truck-trailer system. A noise-free representation of a truck-trailer system can be described as [6]

$$\alpha(k+1) = \alpha(k) + \frac{VT}{l}tan(u(k)),$$

$$\beta(k+1) = \beta(k) + \frac{VT}{L}sin(\alpha(k)),$$

$$N(k+1) = N(k) + VTcos(\alpha(k))\sin\left(\frac{\beta(k+1)+\beta(k)}{2}\right)$$

$$E(k+1) = E(k) + VTcos(\alpha(k))\cos\left(\frac{\beta(k+1)+\beta(k)}{2}\right) \quad (19)$$

where α is the angle of the truck, β is the angle of the trailer, N is northerly position of the rear of the trailer, and E is the easterly position of the rear of the trailer, l is the length of the truck, L is the length of the trailer, T is the sampling time, V is the constant speed of backward movement of the truck, and u is the controlled steering angle(measured counterclockwise with respect to the truck orientation). The following noisy fuzzy model, adapted from [7], can be used to represent the above system:

$IF\ z(k)\ is\ F_1$
$THEN\ x(k+1) = ((A_1 + \triangle A_1 x(k) + B_1u(k) + G_1w(k))$
$y(k) = C_1x(k) + v(k)$
$IF\ z(k)\ is\ F_2$ (20)
$THEN\ x(k+1) = ((A_2 + \triangle A_2 x(k) + B_2u(k) + G_2w(k))$
$y(k) = C_2x(k) + v(k)$

The premise variable $z(k)$ is given as

$$z(k) = \beta(k) + \frac{\alpha(k)\,VT}{2/L} \quad (21)$$

The membership function in (20) are defined as $F_1 = \{0\}$ and $F_2 = \{\pm\pi\}$ and we use following system parameters:

$$A_1 = \begin{bmatrix} 1-VT/L & 0 & 0 \\ VT/L & 1 & 0 \\ (VT)^2/(2/L) & VT & 1 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 1-VT/L & 0 & 0 \\ VT/L & 1 & 0 \\ (VT)^2/(2/L)(\pi/100) & V/(\pi/100) & 1 \end{bmatrix},$$

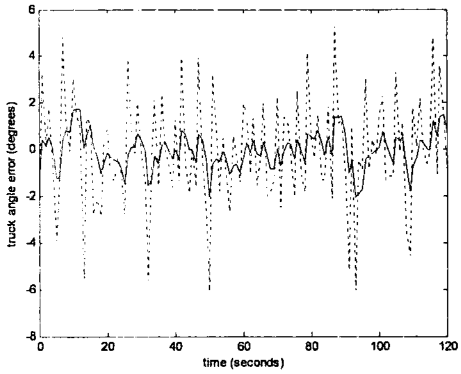
$$B_1 = B_2 = \begin{bmatrix} VT/l \\ 0 \\ 0 \end{bmatrix} \qquad C_1 = C_2 = I_{3\times3}, \quad G_1 = G_2 = I_{3\times3}$$

$\triangle A = \pm 0.3$, $l = 2.8m$, $L = 5.5m$, $V = -1m/s$ $T = 0.5s$

We will use the following matrices for the measurement noise covariance $S_v = 0.2^2$ and the overall process noise with uncertainty is following by the (11)

<Table1>The initial parameters of the GA

Parameters	Values
Maximum Generation	200
Maximum Rule Number	50
Population Size	500
Crossover Rate	0.9
Mutation Rate	0.01
λ	0.75



<Fig.1> Truck angle error(degrees)

The two local state vectors of (7) are estimated according to (17), and are then combined according to (18) to obtain the global state estimate. Figure 1 shows that the simulation results of the proposed method. The dotted lines are measurement errors and the solid lines are estimation errors. And trailer position for a typical simulation with the initial conditions $\alpha[0] = -45°$, $\beta[0] = -45°$, and $N[0] = -5m$.

3. Conclustion

The uncertain nonlinear systems via the TS fuzzy system has been presented. The steady state with uncertain is represented by the TS fuzzy model structure, which is further rearranged to give a set of uncertain linear model using standard Kalman filter theory. Then the unknown uncertainty is regarded as an additive process noise, it represented as a fuzzy system to compute the time-varying variance of the overall process noise. To optimize the employed fuzzy system, the genetic algorithm is utilized. Finally, the proposed state estimator is demonstrate on a truck-trailer.

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