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LMI를 이용한 불확실한 시스템의 슬라이딩 모드 관측기 설계

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Sliding Mode Observer for Uncertain Systems with Mismatched Uncertainties: An LMI Approach

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Abstract - This paper considers a method to design sliding mode observers for a class of uncertain systems using Linear Matrix Inequalities(LMI). In an LMI-based sliding mode observer design method for a class of uncertain systems the switching surface is set to be the difference between the observer and system output. In terms of LMIs, a necessary and sufficient condition is derived for the existence of a sliding-mode observer guaranteeing a stable sliding motion on the switching surface. The gain matrices of the sliding-mode observer are characterized using the solution of the LMI existence condition. The results are illustrated by an example.

1. Introduction

Sliding mode observers differ from linear Luenberger observers in that there is a non-linear discontinuous term injected into the observer depending on the output estimation error. These observers are more robust than Luenberger observers, as the discontinuous term enables the observer to reject disturbances, and also a class of mismatch between system and observer. The discontinuous term is designed to drive the trajectories of the observer so that the state estimation error vector is forced onto and subsequently remains on a surface in the error space. This motion is referred to as the sliding mode[1]. In most cases, the sliding surface is set to be the difference between the observer and system output which is therefore forced to be zero. When a sliding mode is achieved the system will experience a reduced order motion which is insensitive to a class of system/plant mismatch. Utkin designed a simple observer, with only the discontinuous term being feedback through an appropriate gain. Walcott and Zak designed an observer which also has the output error being fed back linearly and used a Lyapunov approach to prove stability. The method in [1] invariably requires a symbolic manipulation package to solve the synthesis problem which is formulated. Their method described in [2] utilized both linear and discontinuous output error injection. A method for computing the gain associated with the linear output error injection term is presented. The solution is explicit, but does not exploit all the degree of freedom.

Several authors have proposed sliding mode observer design methods[1-10]. The method of Walcott and Zak[1] requires a symbolic manipulation package to solve the design problem. Edwards and Spurgeron[2] proposed a canonical form for sliding mode observer design and they give a numerically tractable method for computing the gain matrices of sliding mode observers and the state transformation matrix to obtain the canonical form. Tan and Edwards[3] proposed an LMI-based sliding mode observer design method based on [2]. Because both methods [2] used state transformation they require not only finding state transformation matrices but also changing co-ordinates to obtain the canonical form, and therefore they are indirect and more or less complex. Considering these facts, an LMI-based sliding mode observer design methods proposed which does not require state transformation. As in the previous method [3] the switching surface is set to be the difference between the observer and system output. All the methods given in [8-10] and the references therein do not guarantee that the sliding mode dynamics is completely invariant to mismatched uncertainties.

Using LMIs a necessary and sufficient condition is derived for the existence of a sliding mode observer guaranteeing a stable sliding motion on the switching surface that is insensitive to matched uncertainties. In terms of the solution of the LMI existence condition, explicit formulas of the gain matrices of the sliding mode observer are derived [6]. Because the approach is based on LMIs, it offers degrees of freedom which can be used to improve the design [3]. The approach does not require a change of system model in to canonical form as in the previous methods[2], thus the approach is direct and has advantages in computation aspect The new invariance condition is stated in terms of simple linear matrix inequalities. The feasibility of the condition can be easily tested and a feasible controller that guarantees the invariance can be determined efficiently via LMI optimization, thus one can easily design sliding surfaces by using proposed method.

2. Main results

2-1 Preliminaries

Consider the following dynamical equation:

$$\dot{x}(t) = A_i x(t) + B_i u(t) + A_d x(t) + D\xi(t,x,u) \quad (1)$$
$$y(t) = Cx(t)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control, $y(t) \in R^p$ is the output, and the following assumptions are satisfied:

1. $A \in R^{n\times n}$, $B \in R^{n\times m}$, $A_d \in R^{n\times n}$, $C \in R^{p\times n}$ and $D \in R^{n\times q}$ are constant matrices.
2. Matrices C and D are full rank and $p \geq q$.
3. The function $\xi(t,x,u)$ is unknown but bounded as $\| \xi(t,x,u) \| \leq r_1 \| u \| + \beta(t,y)$ where r_1 is a known scalar and $\beta : R^+ \times R^p \to R^+$ is a known function.

Utilizing this assumption, Consider an observer of the form

$$\dot{\hat{x}}(t) = (A + A_d)\hat{x}(t) + B_i u(t) - L(\hat{y}(t) - y(t)) + G_n \nu \quad (2)$$
$$\hat{y}(t) = C\hat{x}(t)$$

where $L \in R^{n\times p}$, $G_n \in R^{n\times p}$ are the gain matrix and a design parameter satisfying $rack(CH) = p$. The discontinuous vector ν is defined by

$$\nu = -\rho(t,x,u) \| F \| \frac{e_y}{\| e_y \|} \quad (3)$$

where $e_y = \hat{y} - y$ and F is symmetric positive definite. The matrix F will be formally defined in section 2-2 The scalar function $\rho : R^+ \times R^p \times R^m \to R^+$ satisfies $\rho(t,x,u) \geq r_1 \| u \| + \beta(t,y) + \gamma_0$ where γ_0 is a positive scalar .If the state estimation error $e_y = \hat{y} - y$, then it is straightforward to show from equation (1) and (2) that

$$\dot{e}(t) = (A_0 - LC)e(t) + G_n \nu - D\xi(t,x,u) \quad (4)$$

where $\rho : R^+ \times R^p \times R^m \to R^+$. In [5] show that necessary and sufficient conditions for existence of a stable sliding motion on $S = \{ e \in R^n : e_y = 0 \}$ that is independent of ξ are

1. $Rank(CD) = q$.
2. invariant zeros of (A, D, C) lie in the open LHP.

$Rank(CH) = p$ is necessary for the existence of the unique equivalent control[6].

2-2 A design framework

Theorem 1. Consider the error dynamics (4). There exists a stable sliding motion on the switching surface $e_y = Ce = 0$ that is independent of $\xi(t,x,u)$ if and only if there exists a solution matrix P satisfying

$$P>0,\ \psi(P(A+A_d)+(A+A_d)^T P)\psi^T<0,\ \psi PD=0 \qquad (5)$$

where ψ is any full rank matrix whose columns form the basis of the null space of the matrix C. ■

Theorem 2. There exists a solution matrix P satisfying theorem 1. if and only if the following LMI condition is feasible:

$$\begin{aligned} &C^T XC+\Xi Y\Xi^T>0, \\ &C^T XC(A+A_d)+\Xi Y\Xi^T(A+A_d)-KC+*<0, \\ &X=X^T, Y=Y^T \end{aligned} \qquad (6)$$

where $*$ denotes blocks that are readily inferred by symmetry and Ξ is any full rank matrix whose columns form the basis of the null space of the matrix D^T. ■

Lemma 1[5]. Consider the error dynamics (4). There exists a stable sliding motion on the switching surface $e_y = Ce = 0$ that is independent of $\xi(t,x,u)$ if and only if the LMI condition (6) is feasible. Assume that the LMI condition (6) is feasible for (X, Y, K) and the gain matrices are given by

$$\begin{aligned} &H=(C^T XC+\Xi Y\Xi^T)^{-1}C^T \\ &L=(C^T XC+\Xi Y\Xi^T)^{-1}K,\quad F=D^T C^T Y. \end{aligned} \qquad (7)$$

3. Numerical Example

The new design method proposed in this paper will now be demonstrated by an example[8].

$$A=\begin{bmatrix}0&1&1&0\\1&2&0&0\\1&0&0&0\\0&0&1&0\end{bmatrix}, B=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, A_d=\begin{bmatrix}0&0&\frac{\sin b}{b}&0\\0&0&0&0\\0&a^2&0&0\\0&0&\frac{\sin b}{b}&0\end{bmatrix},$$

$$\xi(t,x,u)=\begin{bmatrix}0\\ \zeta\sin(2\pi t)\\0\\0\end{bmatrix},\quad D=\begin{bmatrix}0\\1\\0\\0\end{bmatrix},\quad C=\begin{bmatrix}0&1&0&1\\0&1&1&0\end{bmatrix}$$

where ζ, a, b is an uncertain but bounded time-varying variable ranging in $[-1,1]$. let $a=0.8$ and $b=0.6$.

Since a basis of the null space of given D^T can be given as $\Xi=\begin{bmatrix}1&0&0\\0&0&0\\0&1&0\\0&0&1\end{bmatrix}$. We apply Theorem 1 and 2 to the above system, and the result are as follows. LMI condition (6) has following feasible solutions,

$$X=\begin{bmatrix}425.54&-77.693&-269.337\\-77.693&33858&-33618\\-269.333&-33618&33776\end{bmatrix}, Y=\begin{bmatrix}-33346&33456\\33456&-33406\end{bmatrix},$$

$$K=\begin{bmatrix}-108.01&749.94\\6722.9&-6240.3\\7173.8&-335.36\\130.28&-6933.8\end{bmatrix}.$$

From lemma 1, The gain matrices can be taken as follows.

$$L=\begin{bmatrix}-27.178&-18.128\\72.473&-15.979\\-10.944&-12.098\\-39.383&-27.937\end{bmatrix},\quad G_n=\begin{bmatrix}0.0022127&-0.0048667\\0.0037143&0.011809\\0.0011075&-0.0024358\\0.0031766&-0.0069868\end{bmatrix},$$

$$F=[110.09\ 50.055]$$

Fig.1 shows a simulation results, It is seen that the overall error dynamic system is stable. The all of estimation error converge to zero as $t\rightarrow\infty$.

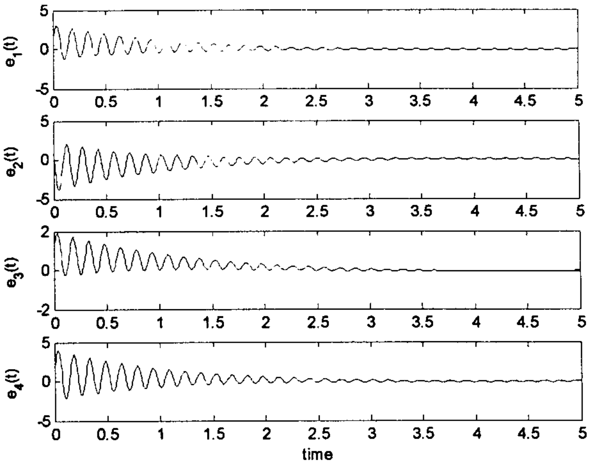

Fig 1. The estimation error for system states.

4. Conclusions

This paper considered the problem of designing sliding mode observers for a class of uncertain systems. An LMI existence condition of a sliding mode observer and was given the gain matrices of the sliding mode observer using the solution of the LMI existence condition. We have parameterized such linear switching surface in terms of the solution matrices to the LMI condition. Simulation results are included to illustrate the effectiveness of the proposed sliding mode observer.

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