Understanding K-space by Real-World Examples

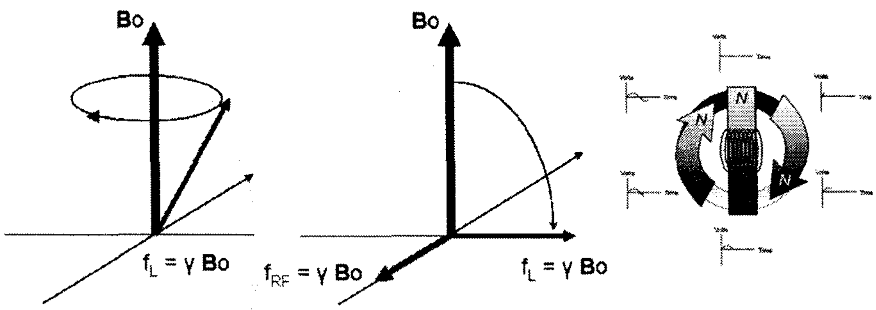
Jeffrey L. Duerk, Ph.D.
Sreenath Narayan, Candice Bookwalter, MS
University Hospitals of Cleveland

Introduction

Magnetic resonance imaging has multiple advantages over many other cross-sectional /tomographic imaging modalities. First, it provides a simple way to adjust the scan plane into any arbitrary orientation, thereby providing unparalleled flexibility in localizing the slice of interest. Second, MRI provides unrivaled soft tissue contrast. While T1, T2 and proton density contrast are the most widely utilized contrast mechanisms, there are a variety of other parameters like diffusion (ADC), perfusion, magnetization transfer, magnetic susceptibility and blood flow are routinely used to provide insight into the actual physiology. Third, MRI does not use ionizing radiation to generate the signal used to reconstruct the image. Fourth, while largely considered a "high-tech"imaging modality, virtually all of the core components like the magnet, the RF transmitter, the RF receiver, and the reconstruction algorithm have been available for decades (to centuries). However, there is one aspect of MRI that remains a challenge for many to understand or appreciate- specifically, the concept of collecting data in K-space and reconstructing an a representation of the object in "image"space is one of the most challenging concepts to more complete understanding of MRI. This lecture will provide a graphical and simulation methodology to explain the basis of K-space, what information of the image is encoded in different parts of K-space and finally, how this all comes together to create the image.

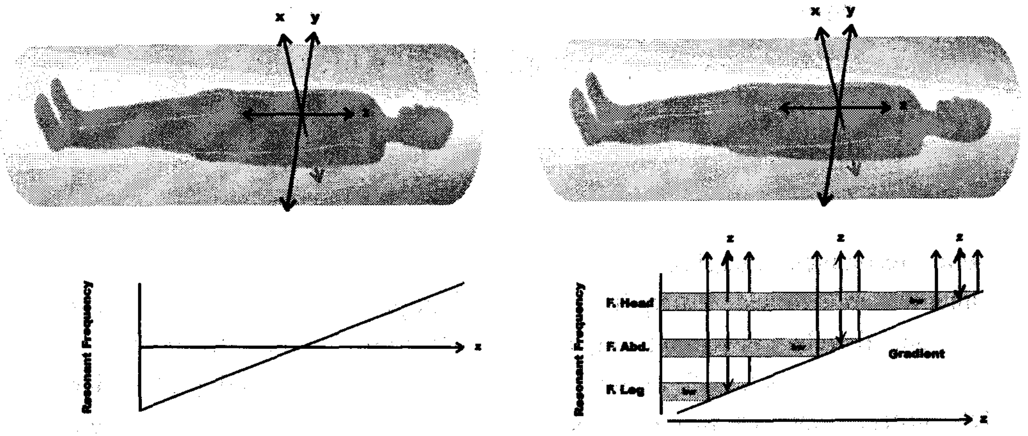
Frequency Encoding

In MRI, one often hears about a variety of frequencies: frequency of the RF pulse, frequency encoding, etc. It is well appreciated that protons precess about a magnetic field as shown below. It also determines what frequency of RF energy must be applied in order to tip the spins into the transverse plane and generate a signal, also as shown below.



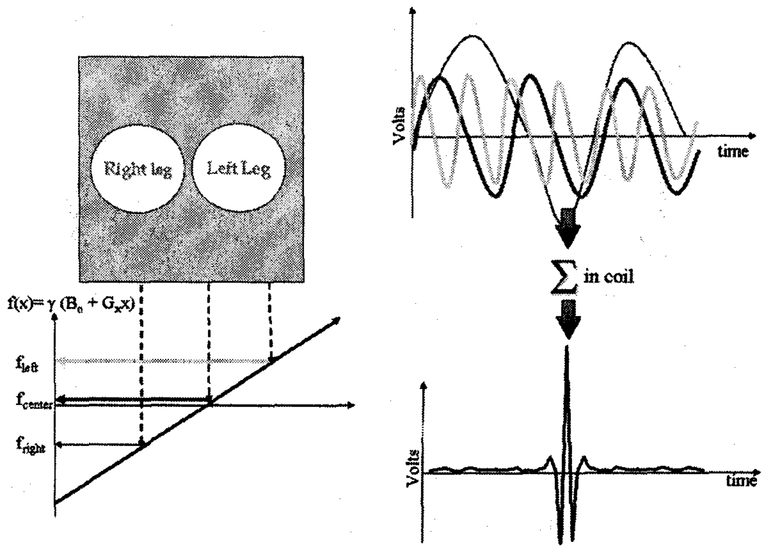
This precession, or rotation, about the static magnetic field, also determines the frequency of the signal that is induced in the receiver coil. Therefore, the frequency of the RF pulse needed to tip the magnetization, and the frequency of the signal that is induced in the coil is equal to the frequency that the spins are processing about the field.

To achieve slice selection, the pulse sequence creates a linear variation in the magnetic field. This creates a linear variation in the precessional frequency of the spins, and hence a variation in the frequencies that must be applied in order to tip the spins over. In order to tip the spins from a thin "slice"of tissue along the slice select axis requires a range of frequencies that are centered at the frequency of the spins at the center of the slice. This is shown below.

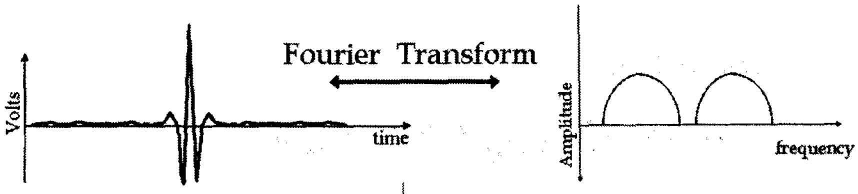


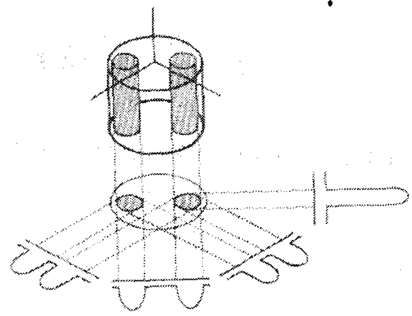
Hence, the difference in resonant frequency that exists during the application of a gradient can be used to differentiate spins. Here (above) spins along the slice select axis (z) are at different resonant frequencies, and because they are, we can apply RF pulses at different frequencies to select a slice at the legs, the abdomen, or the head, for example. The RF pulse at the frequency that the leg spins are precession, only effects spins at the legs; the same is true for RF pulses at other locations.

Once spins from the selected slice are tipped into the transverse plane, the task is to differentiate them along the remaining spatial axes. Imagine that we continue to consider the situation of exciting the spins in the legs as shown above. The rest of the pulse sequence simply encodes (or distinguishes) spins in the X and Y directions so that axial slices are obtained. One way to do this, again, is to apply another magnetic field gradient. Imagine that the gradient is applied from the left side of the patient to the left. Under these conditions, there is a one to one relationship between the frequency the spins are precessing and their position on the left to right side of the patient. This is shown graphically above. The coil then adds up (or integrates) all of the different voltages at the different frequencies.



A method is needed that can take this induced voltage, or time dependent signal, and determine the different frequencies present. If this can be done, one would know what spatial locations have magnetization that will contribute to the image. By encoding the object from multiple positions, one can generate enough information to "back project" the information and create an image. In fact, Paul Lauterbur was a recipient of the 2003 Nobel Prize for this idea (shown below).



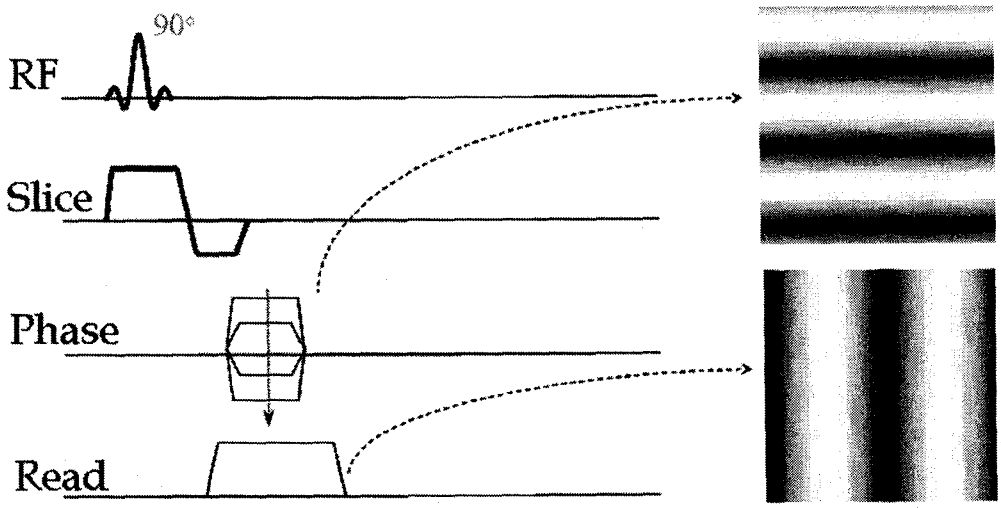


Lauterbur, 1971, Nature

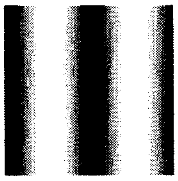
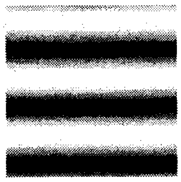
This is the first way in which MR imaging was performed, and is clearly the easiest way to conceptually present the basis for how MR imaging can be performed. Unfortunately, today, this method of spatial encoding is rarely used. Instead, a process of frequency and phase encoding is performed. This is described below.

2D Fourier Encoding and K-space

The most common way for MR imaging to be performed today is a process called two dimensional Fourier Transform encoding using frequency encoding and phase encoding gradients. I am sure all of you have heard of these. Essentially, during the pulse sequence, gradients along the two image domain axes are turned on at different times. These gradient waveforms essentially impose a sinewave of amplitude across the object, as shown below.



Hence, while the coil still integrates all of the signal, graphically the signal being summed is related to the object being imaged weighted by the applied gradient waveforms sinusoidal variation, as shown below.

$s(k_x,k_y)=\iint$  X  X  dx dy

Which is described mathematically as:

$$s(t_x,t_y)=\iint \rho(x,y)e^{-i2\pi\gamma(G_x t_x X+G_y t_y Y)}\,dx\,dy$$

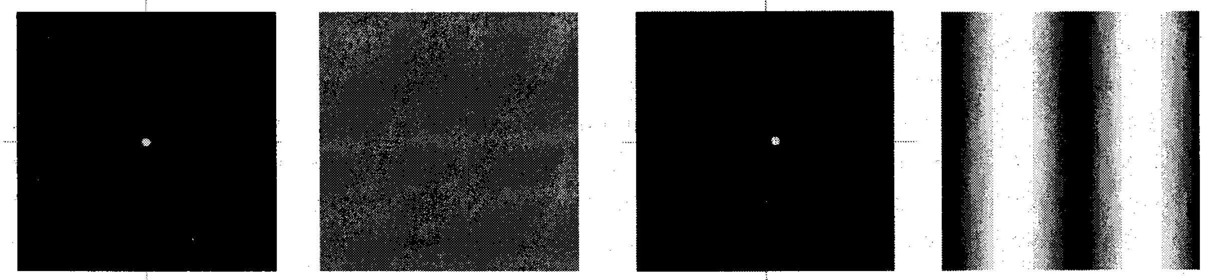
$$s(k_x,k_y)=\iint \rho(x,y)e^{-i2\pi\gamma(K_x X+K_y Y)}\,dx\,dy$$

$$K_x=\gamma G_x t_x \rightarrow MHz/T \bullet \mathrm{T/cm} \bullet \mathrm{sec} = \mathrm{Hz/cm}$$

The key right now, is don't panic! What this tells us is that the signal we detect is exactly equal to the two dimensional Fourier Transform of the object.

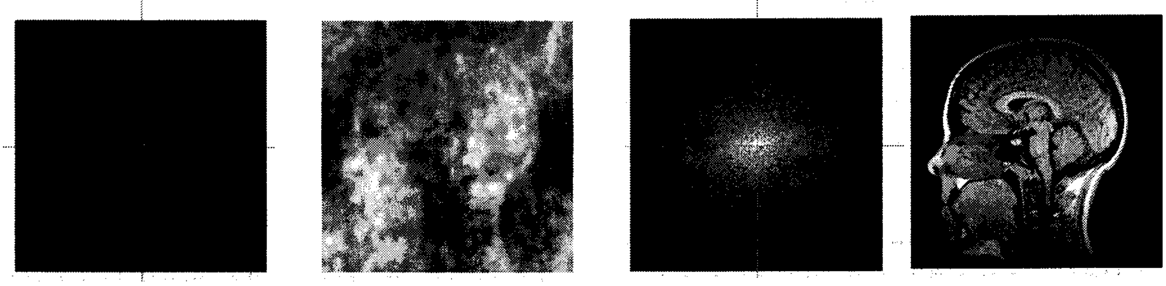
For example, a datapoint acquired at the center of K-space, corresponds to the average value in the image, as shown below. The center of K-space is the data point collected at the echo time. A few microseconds after that, the system collects another data point, lets say at Kx=2. This corresponds to two cycles across the object (below).

$K_x = K_y = 0$ | Image Space | $K_x = 2\ K_y = 0$ | Image Space

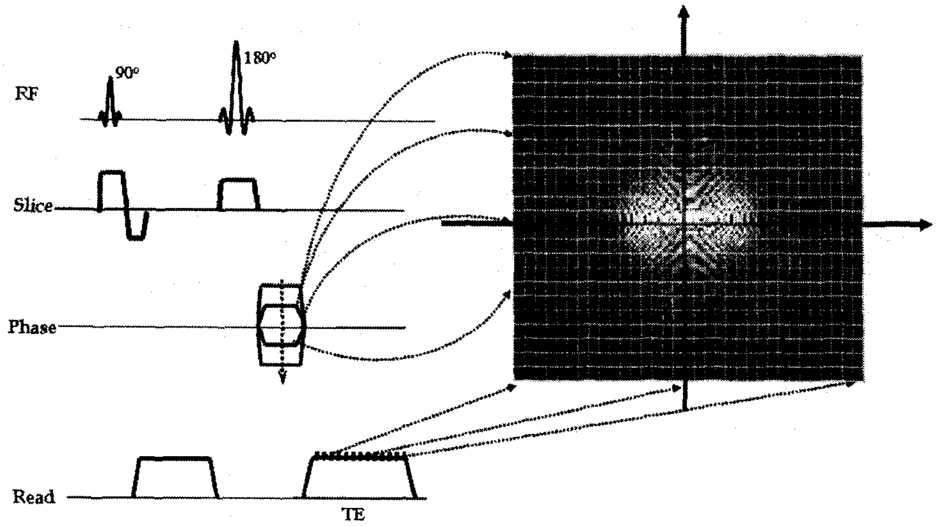


Finally, if hundreds of points (e.g., 505) are put together in K-space at random, and reconstructed, then the image begins to emerge. Finally, by sampling all of K-space, we are able to reconstruct the image, essentially by adding up all the spatial sinewaves along X and Y that are collected

505 points | Image Space | 256 x 256 points | Image Space



with each data point. Essentially, each phase encoding gradient selects another row in K-space, while each point sampled during the application of the read gradient selects another column (as shown below).



16 x 16 points

Image Space

30 x 30 points

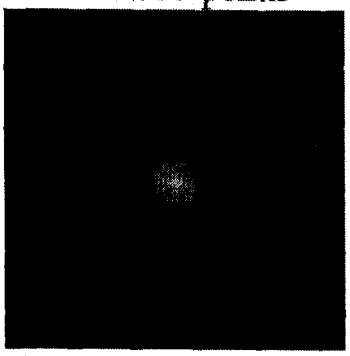
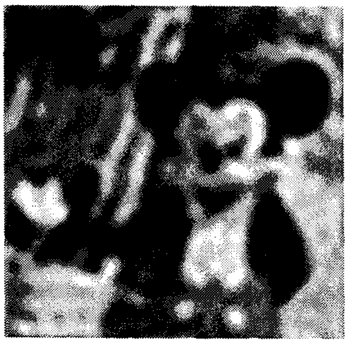


Image Space



256 x 256 points

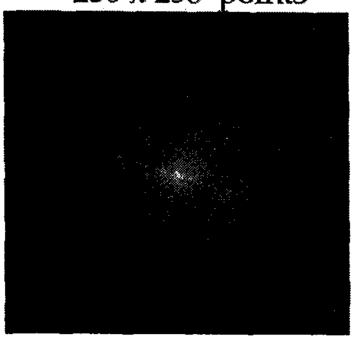
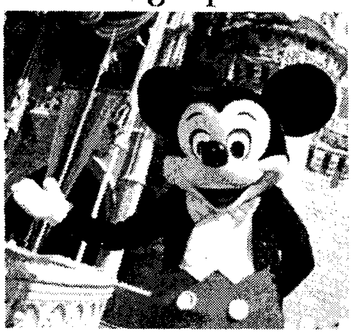


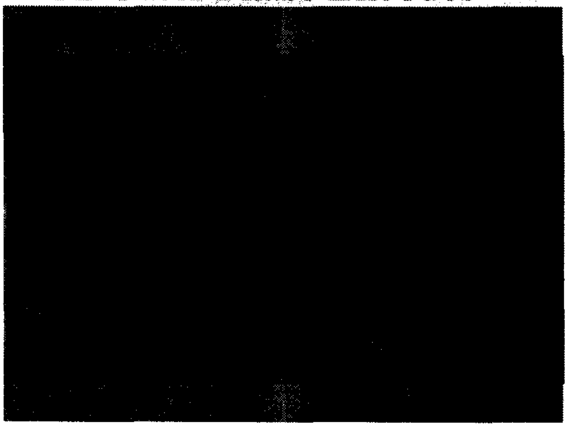
Image Space



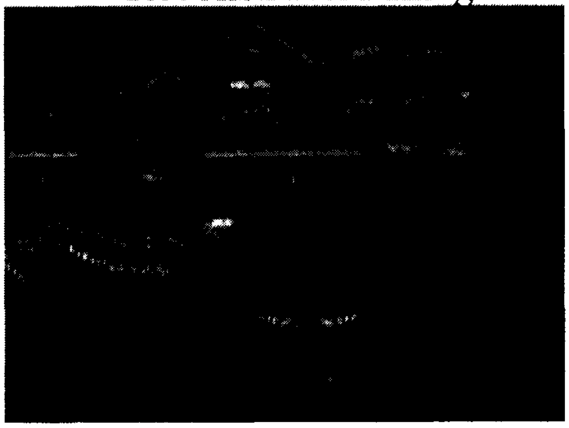
However, the question remains: While the central point in K-space corresponds to the average value in the image, what are the physical meanings for the center and peripheral regions of K-space. Shown to the left is the center 16 x 16 region of K-space and the image reconstructed from it. Obviously, there are regions of high and low signal; the contrast between various regions is quite high, and yet, the object remains difficult to discern. As the central region is expanded, the contrast remains about the same, yet resolution increases. A 30 x 30 region and then a 256 x 256 region (the whole data set) is shown below. Hence, the center of K- space is associated with contrast information, while the periphery is more associated with resolution, and edges.

This is furtherillustrated by beginning the collection with the outer regions of K-space and moving in to the center. The images below start with the collection of the high Ky phase encoding lines, and moving to the center.

32 Outer Phase Encodes

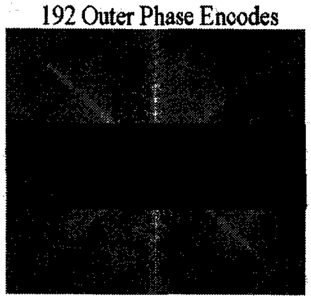


Reconstructed Image

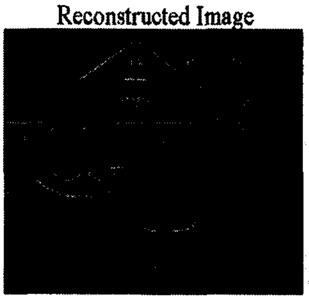


Note how the image has virtually no contrast between different regions when only the outer rows of data are reconstructed. Instead, the image consists almost entirely of edges, boundaries, and defining areas of sharp transitions in amplitude. However, as increased number of lines are added (left below) toward the center of K-space (corresponding to the low phase encoding amplitudes), more of the object becomes define, and yet, the contrast is not introduced until the center of K-space is covered (right below).

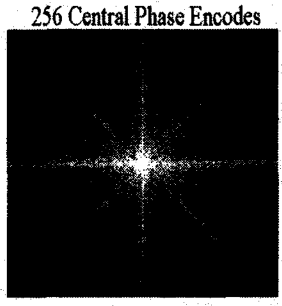
192 Outer Phase Encodes



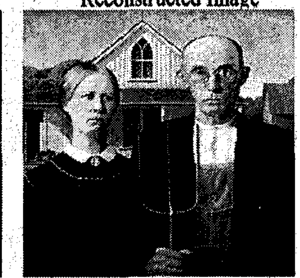
Reconstructed Image



256 Central Phase Encodes



Reconstructed Image



In the lecture, we will further develop the relationship between the various K-space trajectories, like spirals and radial scans, as well as the phase encoding order.

Summary

All MR pulse sequences have a variety of common features. These include application of a gradient along the sliceselection followed by application of an RF pulse at the Larmor frequency of the tissue plane of interest. Thereafter, the remaining gradient waveforms are applied in the remaining orthogonal axes to encode the spins to various spatial positions. Currently, data acquisition is presented in the context of collecting data in K-space. This "K-space"represents the Fourier transform of the object. All pulse sequences attempt to collect data throughout K-space, albeit via different paths (rectilinear (most common), spiral, or radial). However, within K-space, the central region corresponds to the region that encodes the contrast information. The peripheral regions define the edges and hence provide the resolution in the images.