### 다상 부스트 교호컨버터의 불연속전류모드 특성 해석

신휘범\*, 장은승\*, 이현우\*\*
\*경상대학교, \*\*경남대학교

# Analysis of Multi-Phase Interleaved Boost Converter in Discontinuous Inductor Current Mode

Hwi-Beom Shin\*, Eun-Sung Jang\*, and Hyun-Woo Lee\*\*
\*Gyeongsang National University, \*\*Kyungnam University

#### **ABSTRACT**

This paper presents the steady-state characteristics of the multi-phase interleaved converters in discontinuous inductor current mode(DICM). The full-order averaged model is derived, and the conversion ratio and efficiency are presented.

### 1. Introduction

Recently, the interleaved boost converter (IBC) has been studied for the application to power factor correction circuits and interface between the photovoltaic array, or battery source and the dc bus of AC inverter.[1]-[2] The IBC is composed of several identical boost converters connected in parallel. The converters are controlled by the interleaved switching signals, which have the same switching frequency and the same phase shifting. By virtue of paralleling the converters, the input current can be shared among the cells or phases, so that high reliability and efficiency can be obtained. As a consequence of the interleaving operation, the IBC exhibits both lower current ripple at the input side and lower voltage ripple at the output side.

When the IBC operates in DICM, the currents in the inductors are discontinuous but the input and/or output current may be continuous. The inductor size may be considerably reduced with a little affecting the input current harmonics. In this paper, the IBC in DICM is modeled with full order and the steady-state expressions are derived.

## Averaged modeling of multi-phase IBC in DICM

Fig. 1 shows the N-phase IBC, in which N boost converters are connected in parallel. Each converter consists of an inductor, active switch, and diode. The active switches are switched with sequence of  $S_1$ ,  $S_2$ , ...,  $S_N$ , during the PWM (pulse width modulation) period and the control signals are equally shifted each other. It is assumed that all converters operate in the discontinuous inductor current mode (DICM). Fig. 2 shows the current waveforms of the 4-phase IBC in DICM for example. The N phase inductor currents are shifted with the same delay of  $\tau(=T_N/N)$ .

The current dynamics of the kth phase inductor during a PWM period can be expressed from Fig. 2 as follows.

(i) 
$$mT_s+(k-1)\tau \le t < mT_s+(k-1)\tau+d_kT_s$$

$$\frac{d\hat{t}_k}{dt} = \frac{-r_k i_k + V_g}{L_k} \tag{1}$$

(ii)  $mT_s + (k-1)\tau + d_kT_s \le t < mT_s + (k-1)\tau + (d_k + q_k)T_s$ 

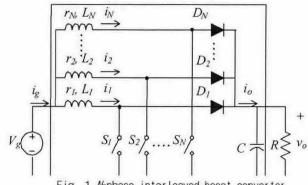


Fig. 1 N-phase interleaved boost converter

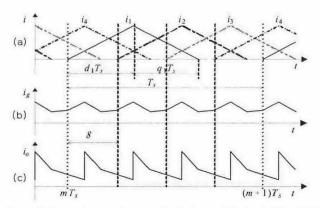


Fig. 2 Current waveforms of 4-phase IBC in DICM: (a) inductor currents, (b) input, (c) output current.

$$\frac{di_k}{dt} = \frac{-r_k i_k + V_g - v_o}{L_k} \tag{2}$$

(iii)  $mT_s+(k-1)\tau+(d_k+q_k)T_s \le t < (m+1)T_s+(k-1)\tau$ 

$$\frac{di_k}{dt} = i_k = 0 \tag{3}$$

where  $d_k$  and  $q_k$  denote the duty cycles of the active and diode switches, respectively.

The output current is a sum of phase current whose active switch is at OFF state and the diode is at ON-state. Hence, the output voltage can be written as

$$\frac{dv_o}{dt} = \frac{1}{C} \left\{ \sum_{k=1}^{N} q_k \dot{i}_k - \frac{v_o}{R} \right\} \tag{4}$$

Utilizing the modeling approach<sup>[3]</sup>, the averaged inductor current in the kth phase can be written as

$$\frac{d\overline{i}_{k}}{dt} = \frac{1}{L_{k}} \left\{ -r_{k}\overline{i}_{k} - q_{k}\overline{v}_{o} + (d_{k} + q_{k})V_{g} \right\}$$

$$\frac{d\overline{v}_{o}}{dt} = \frac{1}{C} \left\{ \sum_{j=1}^{N} \frac{q_{k}}{d_{k} + q_{k}} \overline{i}_{j} - \overline{v}_{o} \right\} \tag{5}$$

where  $k=1, 2, \cdots, N$ . If the converter operates in CICM,  $q_k$  becomes the complement of  $d_k$ , namely  $q_k$  =1- $d_k$ . When the converter operates in DICM,  $q_k$  depends on the operating condition. Fig. 2 shows the inductor current in DICM where it is assumed that the inductor resistance is negligible. During  $d_k T_s$ , the active switch is closed, so that the inductor current increases with the slope of  $V_g/L_k$  During  $q_k T_s$ , the passive switch conducts and the inductor current decreases with the slope of  $(V_g-v_o)/L_k$ . For the remainder of the switching period neither the active nor the passive switch conducts and the inductor current remains at zero if the parasitic resonant elements of the switches are negligible. The averaged inductor current can be written from Fig. 2 as

$$\overline{i}_{k} = \frac{1}{T_{s}} \int_{t}^{t+T_{s}} i_{k}(\tau) d\tau = \frac{V_{g}}{2L_{k}} d_{k} T_{s} (d_{k} + q_{k})$$
(6)

This equation is valid under  $T_s << L_k/r_k$ . The duty cycle of the passive switch  $q_k$  can be expressed as

$$q_{k} = \begin{cases} \frac{2L_{k}}{d_{k}T_{s}V_{g}} \overline{i_{k}} - d_{k} & \text{for DICM} \\ 1 - d_{k} & \text{for CICM} \end{cases}$$
(7)

Substituting into yields the full-order averaged model for the multi-phase IBC in DICM such as

$$\frac{d\overline{i}_{k}}{dt} = -\left[\frac{r_{k}}{L_{k}} + \frac{2}{d_{k}T_{s}}\left(\overline{v_{o}} - 1\right)\right]\overline{i_{k}} + \frac{d_{k}}{L_{k}}\overline{v_{o}}$$

$$\frac{d\overline{v_{o}}}{dt} = \frac{1}{C}\sum_{i=1}^{N}\left(\overline{i_{j}} - \frac{T_{s}V_{g}d_{j}^{2}}{2L_{i}}\right) - \frac{1}{RC}\overline{v_{o}} \tag{8}$$

where  $k = 1, 2, \dots, N$ .

### 3. Steady-State Analysis

The steady-state solutions can be derived by using either (5) or (8) with (7). Herein, the characteristics will be derived from (5) and (7), which results in more concise expressions. The output voltage and inductor currents in steady state can be calculated from (5) as

$$V_o = \frac{V_g}{\Delta_N} \sum_{j=1}^N \frac{Q_j}{r_j} \tag{9}$$

$$I_{k} = \frac{V_{g}}{r_{k}} \frac{1}{\Delta_{N}} \left\{ \frac{D_{k} + Q_{k}}{R} - \sum_{j=1}^{N} \frac{Q_{j}}{r_{j}} \frac{(Q_{j}D_{k} - Q_{k}D_{j})}{D_{j} + Q_{j}} \right\}$$
(10)

where  $\Delta_N = \frac{1}{R} + \sum_{j=1}^N \frac{Q_j^2}{r_j(D_j + Q_j)}$ ,  $k = 1, 2, \dots, N$  and  $D_k$  and  $Q_k$  denotes the steady-state value of the duty cycles  $d_k$  and  $q_k$ , respectively. The equation (7) in steady state can be written as

$$Q_k = \frac{RK_k}{V_g D_k} I_k - D_k \tag{11}$$

where  $K_k=2L_k/RT_s$  and it will be used for deriving the DICM condition. The steady-state solution can be derived from (9), (10), and (11).

If N phases of the IBC are symmetrical such as  $r_j = r$ ,  $L_j = L$ , and  $D_j = D$  for  $j = 1, \dots, N$ , the equations (9), (10), and (11) can be rewritten as

$$M = \frac{V_o}{V_g} = \frac{1}{\Delta} \frac{NQ}{r} \tag{12}$$

$$I_{k} = \frac{V_{g}}{rA} \frac{D+Q}{R} \tag{13}$$

$$Q = \frac{RK}{V_s D} I_k - D \tag{14}$$

where 
$$\Delta = \frac{1}{R} + \frac{NQ^2}{r(D+Q)}$$
,  $k = 1, 2, \dots, N$ . From

(13) and (14), the following quadratic equation for the unknowns Q can be obtained as

$$NQ^{2} - \left(\frac{K}{D} - \frac{r}{R}\right)Q - \left(\frac{K}{D} - \frac{r}{R}\right)D = 0$$
 (15)

From (15), we can derive the relation as

$$\Delta = \frac{K}{rD} \tag{16}$$

Hence, the equations in (12) and (13) can be simplified as

$$M = \frac{NDQ}{K} \tag{17}$$

$$I_{k} = \frac{V_{g}D(D+Q)}{RK} \tag{18}$$

A solution of Q in (15) can be found as

$$Q = \frac{K}{2ND} \left\{ \delta + \sqrt{\delta \left( \delta + \frac{4ND^2}{K} \right)} \right\} \tag{19}$$

where  $\delta = 1 - rD/RK$ . Substituting (19) into (17) yields the voltage conversion ratio M(D) as

$$M(D) = \frac{1}{2} \left\{ \delta + \sqrt{\delta \left( \delta + \frac{4ND^2}{K} \right)} \right\} \tag{20}$$

The kth inductor and input currents in steady state can also be derived from (18) as

$$I_{k} = \frac{V_{g}}{NR\delta} M^{2}, \quad I_{g} = N \cdot I_{k}$$
 (21)

The converter efficiency can be calculated as

$$\eta = \frac{V_o^2/R}{V_g I_g} = 1 - \frac{RD}{rK} = \delta \tag{22}$$

It can be seen that efficiency becomes higher at smaller duty cycle. If  $\delta$  = 1, the system becomes lossless. If the inductors are loss-less and the converter is symmetrical, the steady-state characteristics can be obtained only by inserting the relation of  $\delta$  = 1 into the above equations. It can be verified that the steady-state solutions in (20) and (21) have the same expressions in [4] in case of the single-phase IBC operating in DICM.

Fig. 3 shows that the voltage conversion ratio increases as the number of phase increases. Fig. 4(a) implies that the small resistance of inductor reduces

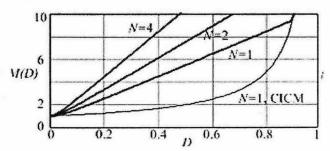


Fig. 3 Voltage conversion ratios in DICM (K=0.1, r=0).

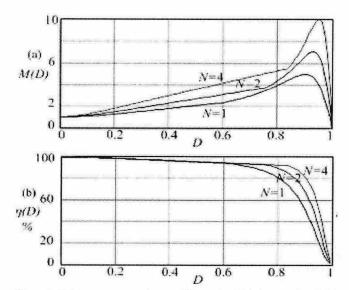


Fig. 4 Voltage conversion ratio and efficiency in DICM when K=0.1, r=0.01.

the conversion ratio considerably. The efficiency may not be affected by the phase number over the useful duty cycle range as shown in Fig. 4(b).

### 4. Conclusions

The multi-phase interleaved boost converter operated in DICM has been modeled with full order and analyzed in steady state. This model can be used to derive the transfer functions of interest. The general and analytical expressions for efficiency, voltage conversion ratio and inductor currents will be useful to design the converter circuits.

This work was financially supported by MOCIE through IERC program.

### 참 고 문 헌

- [1] P. Lee, Y. Lee, D. K. W. Cheng, and X. Liu, "Steady-State Analysis of an Interleaved Boost Converter with Coupled Inductors," IEEE Trans. on Ind. Electronics, Vol. 47, No. 4, pp. 787-795, 2000, Aug.
- [2] H. B. Shin, J. G. Park, S. D. Chang, and H. C. Choi, "Generalized analysis of multi-phase interleaved boost converter", Int. J. of Electronics, Vol. 92, No. 1, pp.1-20, 2005, Jan.
- [3] Sun, D. M. Mitchell, M. F. Greuel, P. T. Krein, and R. M. Bass, "Averaged Modeling of PWM Converters Operating in Discontinuous Conduction Mode," IEEE Trans. on Power Electronics, vol. 16, no. 4, pp. 482–492, 2001, July.
- [4] R. W. Erickson and D. Maksimovic, Fundamentals of Power Electronics, 2nd Edition, Kluwer Academic Publishers, Massachusetts, 2001.