# 네트워크 기반 추종 제어기 설계

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## Networked Tracking Controller Design

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#### **ABSTRACT**

An  $H_2$  tracking controller is proposed for networked control systems. The network induced delay is assumed to be time varying and vary in the known range. The proposed controller guarantees stability and  $H_2$  performance for all time varying delay in the known range. The proposed controller is verified using a simple networked motor control system.

### 1. Introduction

Control systems in which control loops are closed through a serial network is called networked control systems (NCSs). Recently, NCSs have received a lot of attention due to their flexibility and easy maintenance. The main disadvantage of NCSs is network induced time delay in the control loop. Since the time delay problem is unavoidable in NCSs, the problem has been studied extensively.

Depending on the network type and scheduling methods, the time delay characteristics in NCSs can be modelled as constant, time-varying, and stochastic. In the case of constant time delay [1], it is relatively easy to design controllers. In [2], dynamic scheduling methods are proposed and network-induced delay is assumed to be time-varying. And maximum allowable delay bound (MADB) for a given controller is derived: if the network-induced time delay is smaller than MADB, the closed-loop system is stable. The derived bound is rather conservative and less conservative bound is derived in [3]. In both cases, controller synthesis problems are not

considered. In [4], an LQG controller is proposed for a NCS where time delay is a stochastic process. It is assumed that the network-induced time delay is measurable, for example, by using time-stamped packet. We note that control of time varying delay systems is also considered in a general framework of time delay systems [5]. We also note that time varying delay of packets can be modelled as intermittent transmission and this approach is pursued in [6].

In this paper, we propose a controller for a NCS with time-varying delay, where the delay is known to vary in the known range. The time-varying delay is treated as parameter variation in the system and robust control technique is used to design a controller. An H2 servo control problem is formulated in the framework of NCSs.

## 2. Problem Formulation

Time-varying delay  $\tau_k$  includes communication delay (sensors to controller plus controller to actuators) and controller computation time. Controller computation time can be considered constant; however, communication delay is time-varying depending on the network traffic.

In this paper, the following network assumptions are made.

- (N1) Plant outputs are sampled with the fixed period T and the sampling is synchronized.
- (N2) Delay  $\tau_k$  is time-varying and its bounds are known:

$$\tau_{\min} \le \tau_k \le \tau_{\max} \tag{1}$$

(N3) Actuator updates are synchronized.

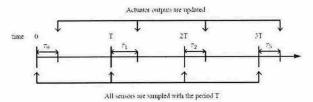


Fig. 1 Timing diagram of the networked control system

Periodic synchronized sampling of plant outputs can be achieved in many ways: for example, the controller can send a message periodically for sampling synchronization. Assumption 3 can be achieved by broadcasting actuator commands to all actuators.

The networked servo control problem can be formulated in a nonstandard sampled-data control framework (see Fig. 2).

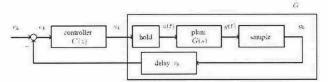


Fig. 2 Networked servo control problem as a nonstandard sampled-data control problem

The tracking control objective in this paper is to find a stabilizing controller C(z), which minimizes

$$J(C) = \sum_{k=0}^{\infty} (\|e_k\|_2^2 + \beta^2 \|u_{k-1}\|_2^2)$$
 (2)

We assume that the continuous time plant G(s) is a linear, time-invariant system given by

$$G(t) = Ax(t) + Bu(t)$$
  
 $y(t) = Cx(t)$ 

where x in the state, y is the output, and u is the control input.

As the controller C(z), we use a linear time-invariant discrete controller

$$\begin{array}{l} \zeta_{k+1} = A_c \zeta_k + B_c e_k \\ u_k = C_c \zeta_k + D_c e_k \end{array} \tag{3}$$

Time-delay elements in Gis as follows:

$$\int_{0}^{r_{k}} \exp(A(T-r)) B dr \tag{4}$$

Defining a nominal value  $\tau_{nom}$  of  $\tau_k$  and

$$\Delta(\tau_{k}, \tau_{nom}) = \int_{\tau}^{\tau_{k}} \exp(A(T - r)B) dr$$
 (5)

we can express (4) as follows:

$$\int_{0}^{\tau_{k}} \exp(A(T-r))Bdr = \int_{0}^{\tau_{norm}} \exp(A(T-r))Bdr$$

$$\Delta(\tau_{k}, \tau_{norm})$$

Note that all time-varying elements are in  $\Delta(\tau_k, \tau_{nom})$  and  $\Delta(\tau_k, \tau_{nom})$  will be treated as norm-bounded uncertainty, where  $B_{\Delta}$  is a matrix bound satisfying

$$\Delta \Delta' \leq B_{\Delta}$$
 for all  $\tau_{\min} \leq \tau_k \leq \tau_{\max}$ 

 $B_{\Delta}$  can be found from a simple optimization problem. We skip all the details and only present the outline of main results due to space limitation. We can transform the objective function optimization problem into an  $H_2$  optimization problem of the following system:

$$\overline{x_{k+1}} = \underline{A_k x_k} + B_1 w_k + B_{2,k} u_k$$

$$z_k = \underline{C_1 x_k}$$

$$y_k = \overline{C_2 x_k}$$

where  $\overline{A}_{k}$   $\overline{B}_{1}$ ,  $\overline{B}_{2,k}$   $\overline{C}_{1}$ , and  $\overline{C}_{2}$  are defined using system parameters A B and C

To find a stabilizing controller minimizing (2) is equivalent to find a stabilizing controller minimizing  $\|T_{zu}\|_2^2$ , where is the closed-loop system from  $w_k$  to  $z_k$ . We derived linear matrix inequalities conditions to derive a controller.

## 3. Experiment

To verify the proposed controller, a simple networked control system of a DC motor is constructed. The system consists of five DSP boards (TMS320F241 DSP), which are connected through the CAN network. Only two boards (sensor board and controller board) are actually involved in the DC motor control and the others are used for dummy traffic generation and monitoring.

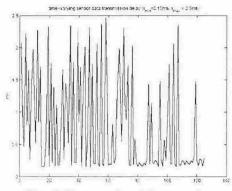


Fig. 3 Time-varying delay example

The sensor board sends the motor speed to the controller board through CAN network with the period T = 4ms. The controller board generates control commands and the DC otor is controlled by the motor driver board. Note that the controller board and the motor driver board is hard-wired.

The transmission speed of the CAN network is 1 Mbps and physical transmission of one message packet is about 150 us. Dummy board 1 and 2 are used to generate dummy message packets. The priority of dummy message packets is higher than that of sensor message packets. Thus sensor message packet transmission is delayed when there are dummy message packets. Sensor data transmission delay is time-varying depending on dummy message generation rate. Note that network traffic load is arbitrarily specified by adjusting dummy message generation rate.

 $H_2$  servo controller is obtained for different  $au_{
m mex}$  values with  $\beta \!=\! 100$ . The computed  $H_2$  norm and  $au_{\it nom}$  are given in Table I.

Table 1  $H_2$  norms and  $au_{nom}$  for different  $au_{max}$ 

| $	au_{max}$ | $H_2$ norm | $	au_{nom}$ |
|-------------|------------|-------------|
| 0.6ms       | 4.1614     | 0.372 ms    |
| 0.9ms       | 4.2029     | 0.521 ms    |
| 1.5ms       | 4.3769     | 0.818 ms    |
| 2.5ms       | 4.9331     | 1.3 ms      |

Step response of different  $\tau_{\text{mex}}$  values are given in Fig. 4. It can be seen that the step response are relatively insensitive to time-varying delays for all cases.

#### 4. Conclusion

In this paper, we proposed a servo controller for networked control systems with time-varying delays. In networked control systems, there is inevitable time delay in data transmission and the delay in many cases is time-varying depending on the network delay. The proposed servo controller guarantees the closed-loop stability for all time-varying delays belonging to a certain interval. As the performance index,  $H_2$  norm is used. The controller can be computed easily by solving linear matrix inequalities.

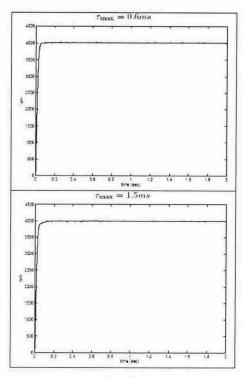


Fig. 4 Step response for different  $\tau_{mex}$  values

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