평직에 대한 투과율 계수의 균질화

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Asymptotic Expansion Homogenization of Permeability Tensor for Plain Woven Fabrics

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ABSTRACT

Homogenization method is adopted to predict the permeability tenor for glass fiber plain woven fabrics. Calculating the permeability tensor numerically is an encouraging task because the permeability tensor is a key parameter in resin transfer molding (RTM). Based on multi-scale approach of the homogenization method, the permeability for the micro-unit cell within fiber tow is computed and compared with that obtained from flow analysis for the same micro-unit cell. It is found that they are in good agreement. In order to calculate the permeability tensor of macro-unit cell for the plain woven fabrics, the Stokes and Brinkman equations which describe inter-tow and intra-tow flow respectively are employed as governing equations. The effective permeabilities homogenized by considering intra-tow flow are compared with those obtained experimentally. Control volume finite element method (CVFEM) is used as a numerical method. It is shown that the asymptotic expansion homogenization method is an attractive method to predict the effective permeability for heterogeneous media.

1. Introduction

Liquid composites molding (LCM) including Resin transfer molding (RTM), Structural Reaction Injection Molding (SRIM), and Vacuum Assisted RTM (VARTM) is a promising process for fabricating large, integrated, and high performance product. Since the liquid composite molding is a closed molding process using coupled mold or vacuum, Volatile Organic Compound (VOC) and scrap rate can be minimized. Moreover, it is convenient to optimize and standardize the manufacturing process. In order to facilitate rapid and efficient resin impregnation into the fiber preform, porous Resin Distribution Medium (RDM) like in VARTM is increasingly employed. In that case, the resin flow becomes three dimensional flow, which is, however, generally regarded as two dimensional flow in the traditional LCM. Therefore, the three dimensional permeability tensor which is a measure of the resistance of flow through the preform is needed to analyze the process precisely. The permeability tensor for a variety of fiber preforms have been numerically predicted and experimentally measured for the past few decades [1-3]. In this study, the permeability tensor for plain woven fabrics is computed by using the asymptotic expansion homogenization method, which is formulated based on control volume finite element method (CVFEM). It is well-known that the resin flows through inter-tow and intra-tow regions are governed by the stokes and the Brinkman equations respectively when pressure gradient is applied to the preform. The Stokes and the Brinkman equations are simultaneously used in the implementation of the homogenization method for the corresponding regions. In order to obtain the permeability experimentally, a radial flow experiment was performed and the results were compared with value predicted numerically.

2. Homogenization theory

The main idea of homogenization method is to replace a real heterogeneous medium with an equivalent homogeneous one which has the same average macroscopic behavior. The homogenization method assumes that all physical quantities vary in multi-scales including local and global scales and the quantities are periodic with respect to the local scale due to the periodicity of the geometrical microstructure. As the periodic dimension approach zero, the homogenized effective material properties are obtained and the asymptotic behavior of medium can be calculated. The asymptotic homogenization is carried out by substituting the differential equations with rapidly oscillating coefficients into the differential equations with constant or slowly varying coefficients. It has been reported that the homogenization method has a rigorous mathematical background and it is in particular useful for microstructures with complex and irregular configuration. Because of these attractive features, the homogenization method has been widely used in the past few years in prediction of such physical properties as elastic constants and thermal conductivity and in topology optimization of structures. Figure 1 shows the hierarchic structure of periodic unit cells in the plain woven fabrics: the first level indicates the periodic macro-unit cells of the plain woven fabrics and the second level does the periodic micro-unit cells within the fiber tow.

3. Experiment

The radial flow experiment was carried out to measure the in-plane permeability of plain woven fabrics. It is known that since the radial flow method can prevent the edge effect which results in race-tracking phenomenon, it is more accurate than unidirectional flow method. The mold was designed so that the thickness of mold cavity could be altered readily and the inlet pressure could be maintained at a constant level. The advancement of flow front was recorded by a digital camcorder during filling and the permeability was obtained by means of the advancement. A silicone oil (dimethyl siloxane polymer, from Dow Corning) was employed for the experiment and its viscosity was known as 9.7×10^{-2} Pa·s.

4. Results and discussion

In order to verify the homogenization theory adopted in this study, the permeabilities calculated by homogenization theory are compared with those obtained by flow simulation in the case of the micro-unit cell with square and hexagonal arrays of the filaments. Figure 2 shows that the homogenized effective permeability has good agreement with that using flow simulation. Generally speaking, more than three different flow simulations are needed in order to obtain a three dimensional permeability tensor for the unit cell, which is a task requiring a lot of time and cost. However, the homogenization method can obtain the permeability tensor from just one numerical analysis. Therefore, the homogenization method is more efficient than that obtained by flow simulation. In this study, the effective permeability for the micro-unit cell in the case of hexagonal packing is used as the input data to calculate the permeability for the macro-unit cell. The cross section of tow is modeled as elliptic and the yarn paths are described by the sinusoidal functions based on the real geometry of plane woven fabrics. As the plain woven fabric is compacted, it is assumed that the minor axis of the elliptical cross-section is reduced, while the major axis of the ellipse keep constant. The effects of nesting and shifting between neighboring layers caused by stacking of fabrics are not considered. Each glass fiber tow consists of about 4000 filaments whose radius is 10 μm. When total fiber volume fraction is 0.18, the size of a representative macro-unit cell is 3.5×20.0×20.0 mm^3. The finite element mesh used in the prediction of permeability contains 8872 nodes and 21234 tetrahedral elements. The effective permeability for the macro-unit cell is plotted as a function of fiber volume fraction as shown in Fig. 3. It is shown that in-plane permeability is about one order of magnitude as high as out-of-plane permeability and both of them are decreased with the increase in the fiber volume fraction.

5. Conclusions

The permeability tensor for plain woven fabrics is calculated by using asymptotic expansion homogenization method, which is well-known method of reducing the computational size of problem by decoupling the scales into a single micro-scale and a single macro-scale. The asymptotic expansion homogenization method is the reasonable method based on rigorous mathematics and it is applicable to localization as well as homogenization. Unlike most of studies, the homogenization method is implemented by using control volume finite element method (CVFEM)

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and is verified by comparing the permeabilities obtained by the method with the permeabilities computed by flow simulation in the micro-unit cell. It is found that the former have good accordance with the latter. In order to consider the contribution of both inter-tow and intra-tow flow in the macro-unit cell, the Stokes and the Brinkman equations are adopted in the formulation procedure of the homogenization method and are calculated simultaneously. There is a little difference between the numerical and experimental results. It is explained by the approximated geometry of unit cell or the nesting effect of preform layers. However, it is found that calculating time and effort can be reduced by applying the homogenization method to calculation of the effective permeability compared with the typical method using flow field.

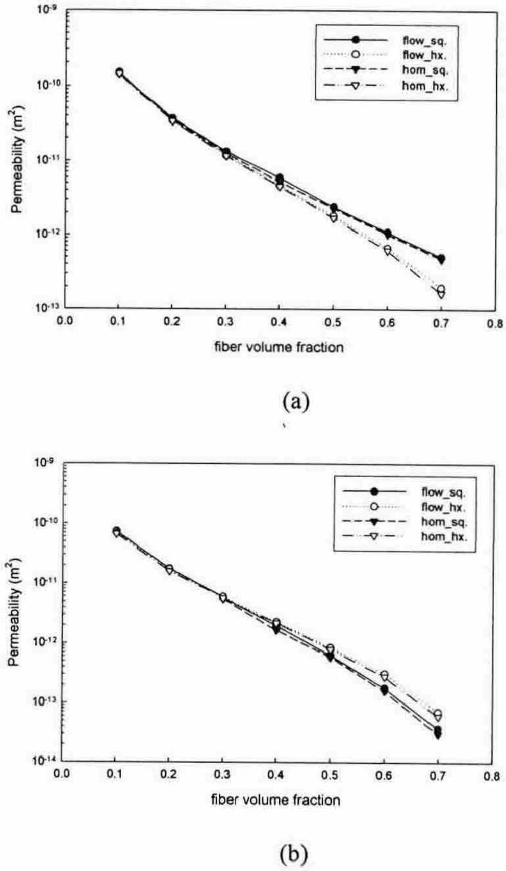

Fig. 2 Effectively permeability for the micro-unit cell vs. fiber volume fraction calculated (a) in the axial and (b) in the transverse flow directions.

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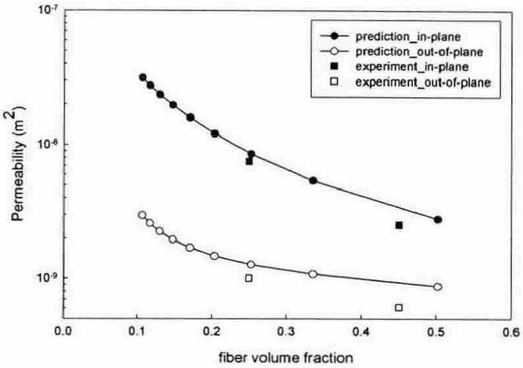

Fig. 3 Comparison between predicted and measured permeabilities with respect to fiber volume fraction.

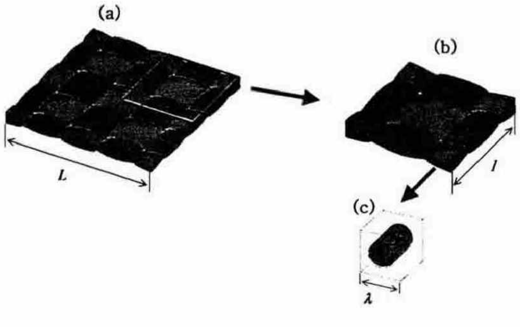

Fig. 1 Schematic diagram of homogenization method: (a) macrostructure of plain woven fabrics, (b) periodic macro-unit cell, (c) periodic micro-unit cell.