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An Optimal Pricing and Inventory control for a Commodity with Price and Sales-period Dependent Demand Pattern

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Abstract

This paper deals with an integrated problem of inventory control and dynamic pricing strategies for a commodity with price and sales-period dependent demand pattern, where a seller and customers have complete information of each other. The problem consists of two parts; one is each buyer's benefit problem which makes the best decision on price and time for buyer to purchase items, and the other one is a seller's profit problem which decides an optimal sales strategy concerned with inventory control and discount schedule. The seller's profit function consists of sales revenue and inventory holding cost functions. The two parts are closely related into each other with some related variables, so that any existing general solution methods can not be applied. Therefore, a simplified model with single seller and two customers in considered first, where demand for multiple units is allowed to each customer within a time limit. Therewith, the model is generalized for a n-customer-classes problem. To solve the proposed n-customer-set problem, a dynamic programming algorithm is derived. In the proposed dynamic programming algorithm, an intermediate profit function is used, which is computed in case of a fixed initial inventory level and then adjusted in searching for an optimal inventory level. This leads to an optimal sales strategy for a seller, which can derive an optimal decision on both an initial inventory level and a discount schedule, in $O(n^2)$ time. This result can be used for some extended problems with a small customer set and a short selling period, including sales strategy for department stores, Dutch auction for items with heavy holding cost, open tender of materials, quantity-limited sales, and cooperative buying in the on/off markets.

I. Introduction

1.1 Background and motivation

In the past, companies would fix the price of a product or service over a relatively long time period; that is, posted prices were usually static. This was mainly because of the absence of accurate demand information, high transaction costs associated with changing prices, and huge investment required for necessary software and hardware to implement any associated dynamic pricing strategy.

In the last couple of decades, the variety of goods sold in the market has significantly increased, while the product life cycles have become shorter. Although improved supply chain practices and production technologies have helped to increase responsiveness, long lead times and shorter selling seasons have resulted in larger forecasting errors and an inability to change inventory levels in response to demand. As a result, it has been realized that production and inventory decisions have to be made in advance with little demand information before the actual selling begins. Given that the inventory levels and the length of the selling season (period) are predetermined, pricing decision becomes increasingly important in balancing demand and supply. Determining the right price to charge a customer for a product is a complex task, requiring that a company know not only its own operating cost and availability of supply, but also how much the current customer values the product and what future demand will be. Therefore, to charge a customer the right price, a company must have a wealth of information about its customer vase and be able to set and adjust its prices at minimum cost. Recently, the internet provides tremendous opportunities for implementing dynamic pricing mechanisms, since it is easier to collect information about markets and customers, and to change prices electronically rather than physically.

One dynamic posted pricing mechanism that is increasingly used by companies on the internet is markdown pricing. In this mechanism, a seller posts a group of items for opening price and buyers bid the quantities they want at that price. After certain duration, the price drops to a lower one. After some time, the price drops again. The markdowns continue until all items are sold or until the price drops to the minimum level set by the seller. The rationale behind and the advantages of a markdown pricing strategy are well-known in the fashion industry as well as many other industries that operate in highly seasonal and/ or short-life products. A markdown mechanisms provides the seller with the flexibility to reach buyers with different valuations, i.e., to price discriminate, and a means of progressively reducing her inventory. However, it is not clear under which circumstances such a mechanism would maximize the seller's profits. Furthermore, it is less clear how a potential buyer should behave under such a pricing strategy. A buyer who participates in a price markdown mechanism faces the following dilemma: If the buyer's valuation of the item is higher than the current price, he will have a positive surplus if he bids at the current price level. However, the buyer could wait for the next round of markdown and potentially have a higher surplus, provided that there are a sufficient number of remaining units.

While markdown pricing mechanisms have been in use long before the advent of the internet, they have received only a brief glance from both the economics and operations management literature. In particular, many fundamental design and associated strategic behavior dimensions have not yet been addressed: At which price levels or when will a buyer bid and how much will he bid given a particular markdown strategy? What is the optimal number and discount time points of price level?

The objective of this paper is to integrate inventory control and dynamic pricing together, and to analyze a markdown pricing mechanism under complete information.

1.2 Literature review

The practice of price markdowns over time was addressed earlier in Stokey[11] and then in Praag[5]. These papers have considered the optimal markdowns to employ, given that customers had single unit demand and the seller had unlimited capacity. Stokey[11] have found the surprising result that, if productions costs are constant, the seller is better off utilizing a single monopoly price. Elmagharaby[2] has extended the problem of Stokey[11] to a multi-unit demand framework with capacity constraints.

Some other researches have also analyzed the sale of multiple units via dynamically decreasing prices, with a focus mainly on retail price markdowns, as in Lazear[10], Pashigan[12], Pashigan[13], Wang[14], and Warner[15]. These researches have studied alternative pricing strategies and their impacts via theoretical models and experiments. Most of them were done under the assumption that a customer will make a purchase immediately if the price is below the customer's valuation. This assumption, which might be reasonable in a retail environment, overlooks the strategic behavior of customers. In case of business-to business sales, the potential number of customers is smaller. Customers may have more information about each other's demands and valuations, and they are likely to act strategically. For example, if the current price of a product is below a customer's valuation, but the customer does not expect too much competition in purchasing the product, he might wait for the next price markdown instead of purchasing the product at the current price.

Some authors have relaxed the assumption of single unit demand and considered the impact of strategic bidding behavior in multi-unit auctions, as in Tenorio[3], Tenorio[4], and Katzman[9]. However, their models are mostly limited to selling most 3 units, while their analyses are not easily extendable to multiple units. Png[16] considers the benefit of a most-favored-customer protection policy when using a two-step pricing discrimination in the face of uncertain demand. Under most-favored-customer protection, the seller may guarantee to refund customers who make a purchase at a higher price if the price drops in the future.

This paper is to extend the work of Elmaghraby[2], noticing that a markdown pricing mechanism is equivalent to a multi-unit Dutch auction with discriminatory pricing rule. Elmaghraby[2] has analyzed biding behavior and auction design in case of multiple units and multiple buyers, considering strategic behaviors of the buyers and the seller. His problem is extended in this paper to consider a constraint of time schedule, customers' time limits, discount schedule, and discount time points and prices. It can be used not only for some specific auctions but also for general business strategies like a department sales strategy.

1.3 Organization of this paper

The organization of the paper is briefed as follows. Chapter 2 is devoted to the problem description and formulation. Chapter 3 analyzes the solution properties and proposes a dynamic programming. In Chapter 4, a parameter analysis is performed. Chapter 5 gives some concluding remarks.

II. Problem Description

2.1 Notation and Assumptions

The proposed problem considers a decision system for a seller's optimal discount schedule. There are one seller and two or more buyers who want to purchase more than one item. Each buyer has a valuation (strategy) which represents a price of an item to be willing to buy it. If the price is higher than his valuation, he may give up purchasing the item. Otherwise, the buyers will indicate their intention to purchase, as they tell the seller their demands and want to buy the item as cheap as possible. It is assumed that the seller faces n buyers with fixed valuations per item unit $v_1 > v_2 > ... > v_n$ and demands, at most $D_1, D_2, ..., D_n$, and if the price is equal to the valuation, then the buyer will prefer to purchase the units. Each buyer is available only by its time limit to give up the chance to buy, where $t_1 < t_2 < ... < t_n$. A seller has K identical units of each item at the beginning and does not consider additional ordering. The initial inventory K is less than the total demand (sum of all the buyers' demands). The seller posts his decision on discount prices and markdown times at the beginning of the sales. An m-step markdown pricing, where $p_1 > p_2 > ... > p_m$, and discount points, where $T_1 = 0 < T_2 < ... < T_m$, are considered. Both the seller and the buyers have complete information about all the discount prices, buyers' valuations, time limits and demands, and all the buyers are present at the beginning.

The inventory holding cost depends on the number of remaining units and the time lapsed in storage from the beginning. The unit holding cost is fixed, and the additional ordering is not considered.

The following notation will be used throughout the rest of this paper.

Parameters :

n : number of the classifications of the buyers

v_j : highest valuation of the buyer j

t_j : time limit of the buyer j

D_j : maximum unit of the buyer j's demand

q_{ji} : number of the items bid by the buyer j at the i th step

Q_{ji} : number of the units awarded to the buyer j at the i th step

h : inventory cost per unit and time

$I(K,t)$: inventory level at time t with the initial stock K

R_j : expected benefit of the buyer j, $j=1...n$, $R_j=\sum_{i=1}^{m}(v_j-p_i)Q_{ji}$

R_s : expected profit of the seller, $R_s=\sum_{i=1}^{m}\sum_{j=1}^{N}p_iQ_{ji}-h\int_{t=0}^{\infty}I(t)dt$

Variables :

K : initial stock level

m : number of markdown steps

p_i : i th markdown price, $i=1...m$

T_i : beginning of time to sell at the price p_i, $i=1...m$

2.2 Problem Formulation

The objective of the proposed problem is to maximize the seller's profit, which is represented by a combination of sales revenue and inventory cost over a finite number of periods. The integrated discount pricing and inventory decision problem can be mathematically expressed as follows:

Buyer j's benefit Problem

Maximize $$R_j=\sum_{i=1}^{m}(v_j-p_i)Q_{ji} \quad (1)$$

subject to

$$Q_{ji}\le q_{ji} \quad \forall i \quad (2)$$

$$\sum_{i=1}^{m}q_{ji}\le D_j \quad (3)$$

$$q_{ji}=0, if\ p_i>v_j \quad \forall i \quad (4)$$

$$Q_{ji}, q_{ji}\ge 0 \quad \forall i \quad (5)$$

Seller's profit Problem

Maximize $$R_s=\sum_{i=1}^{m}\sum_{j=1}^{N}p_iQ_{ji}-h\int_{t=0}^{\infty}I(K,t)dt \quad (6)$$

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{N}Q_{ji}\le K\le\sum_{j=1}^{N}D_j \quad (7)$$

$$p_i>p_{i+1} \quad \forall i \quad (8)$$

$$p_i, K \ge 0 \quad \forall i \quad (9)$$

In the buyer's benefit problem, a function (1) represents a pattern of the common buyer who wants to buy an item at the cheapest price. Constraint (2) makes the number of items for buyer j to receive less than the number that he wants to buy. And the maximum demand of the each buyer can be derived as in constraint (3). Constraint (4) represents the situation where if the price at the current time point is higher than his valuation, then no purchasing is made.

In the seller's profit problem, objective function (6) can represents total sales revenue except inventory cost. A capacity of the initial inventory is represented as in constraint (7). And constraint (8) enforces a markdown discount strategy.

III. Solution Approach

The proposed problem has many decision variables including initial inventory, number of discount steps, discount prices and time points which are interrelated to one another so as to get the problem hard to solve. Specifically, the decisions of the buyers and the seller on when and how many items they will buy and also on discount strategies may affect the decisions of the rest of the buyers. Moreover each buyer's pattern may influence the seller's decision. In order to make it easy to find the optimal solution, this paper starts with a two-buyer-set problem, which is then extended to a general case, called n-buyer-set problem.

3.1. Two-buyer-set problem

This section considers a seller who faces two customers with the valuations $v_1>v_2$, where customer j wants to purchase up to D_j, until reaching their time limit t_j, j =1,2. This problem without the buyer's time limit constraint has been considered by Elmaghraby[2].

For the proposed problem the following property can be derived for the situation where $D_1<K$, and $D_2<K$, $D_1+D_2>K$.

Property 1. Under the given conditions, $D_1<K$, and $D_2<K$, $D_1+D_2>K$, the following decisions are derived;

(a) The optimal markdown points are determined at the time points $T_1=0, T_2=0+\varepsilon\ or\ t_1+\varepsilon$.

(b) The optimal number of the markdown steps is at m*=2.
(c) The optimal prices are determined at $p_1^* \le v_1$ and $p_2^* = v_2$.
Proof)
(a) By the assumption on buyers' behaviors, the buyers will not wait but immediately make a purchasing decision with valuations when the price does not change and no discount schedule is expected later because there is no benefit to wait. If decisions are made, then purchasing will be made just after changing prices. According to the seller's profit function, a discount time point will be set just after one of the buyers' time limits at which each buyer gives up purchasing items. Otherwise, because an additional holding cost occurs, it would not be an optimal solution. Therefore, searching for discount time points can be done at the set of time 0 and buyers' time limit points. Accordingly, if there are two buyers, available time points in the optimal solution set will be at the time 0 and t_1(buyer 1's time limit).
(b) Suppose that two discounts are made :
i) In case of $p_2 < v_2$: after the time t_1 is elapsed, there are no competition to buy but only the second buyer remains, so that discount is not necessary. Therefore, the relation $p_2 = p_3$ holds.
ii) Otherwise, in case of $p_2 > v_2$: the second buyer may be willing to buy items after the time t_1 is elapsed, and so leads to the relation $p_1 = p_2$.
Therefore, in the two-buyer-set situation, an optimal step number is the value 2.
(c) In case of p1>v1; no one may be willing to buy, and so the relations $p_1^* \le v_1$ and naturally $p_2^* \le v_2$ holds. At the last step, that is, discount step 2, the optimal price is hold at $p_2^* = v_2$, because any items remaining after buyer 1 purchases the units for his demand at the higher price are sold to buyer 2 at the price p_2 which is the lower price.

This completes the proof.

Based on property 1, a fast discount strategy and a later discount strategy can be derived.

3.1.1 Fast discount strategy

In this strategy, the discount point is at the time point 0+ ε. Therefore, it is similar to one of the Elmaghraby[2]'s.

Property 2. The proposed problem is the same as the Dutch Auction problem without time limit considered.

Proof) It is straightforward, refering to Elmaghraby[2].

According to Elmaghraby[2], Nash's equilibrium theory is used to find the optimal solution as follows,

(1) q11*=D1, if $(v_1 - p_1)D_1 \ge (v_1 - p_2)\frac{K + D_1 - D_2}{2}$

(2) An optimal price : $p_1^* = v_1 - \frac{K + D_1 - D_2}{2D_1}(v_1 - v_2)$ and $p_2^* = v_2$

(3) The seller's profit : $Rs = (v_1 - v_2)\frac{D_1 + D_2}{2} + K\frac{3v_2 - v_1}{2}$

(4) The optimal inventory level :
$$K^* = \begin{cases} D_1 + D_2 - \delta & ,if\ v_2 > v_1/3 \\ D_1 + \delta & ,otherwise \end{cases}$$

3.1.2 Later discount strategy

In this strategy, the discount point is at the time point $t_1 + \varepsilon$, that is, the discount point is set after the buyer 1's time limit when buyer 1 cannot buy items at the discount price. Therefore, this problem is different from that of the fast discount strategy, as shown in the following property;

Property 3. The associated decisions are derived as follows;

(a) $p_1^* = v_1,\ p_2^* = v_2$

(b) $Q_{11} = D_1, Q_{22} = K - D_1$

(c) The optimal inventory level :
$$K^* = \begin{cases} D_1 + D_2 - \delta & ,if\ v_2 > ht_1 \\ D_1 + \delta & ,otherwise \end{cases}$$
and the seller's optimal profit is $Rs^* = v_1 D_1 + v_2(K^* - D_1) - ht_1(K^* - D_1)$.

Proof)

(a) In case of $p_1 < v_1$; buyer 1 makes a decision to buy the item at the number of D_1 units, buyer 2 can get at least $K - D_1$ units. That is, buyer 1 can buy the items, so that there is no competition against buyer 2. Therefore, the optimal prices to maximize the seller's profit in a discount schedule are at $p_1^* = v_1,\ p_2^* = v_2$.

(b) From the result of (a), that is, $p_1^* = v_1, p_2^* = v_2$; buyer 1 can get D_1 units, and buyer 2 can get $K - D_1$ units.

(c) The seller's profit function is linear in variable K, and $v_2 - hf_1$ is a gradient of the function. Therefore, if $v_2 - hf_1 > 0$, then the optimal K is at its maximum value; otherwise, if $v_2 - hf_1 < 0$, the minimum value of K is an optimal solution.

This completes the proof.

3.1.3. Optimal Solution

An optimal solution for the two-buyer-set problem can be found by comparing the former two strategies in the Sections 3.1.1 and 3.1.2.

An optimal initial inventory level is derived as;

(1) In Section 3.1.1, the fast discount strategy leads to

$$K^* = \begin{cases} D_1 + D_2 - \delta & ,if\ v_2 > v_1/3 \\ D_1 + \delta & ,otherwise \end{cases}$$

(2) In Section 3.1.2, the later discount strategy leads to

$$K^* = \begin{cases} D_1 + D_2 - \delta & ,if\ v_2 > ht_1 \\ D_1 + \delta & ,otherwise \end{cases}$$

The seller can compare the optimal profits of two solutions at each strategy to choose the better strategy and initial inventory level.

3.2. Multi-buyer-set problem

This section considers a general model with multiple buyer sets. There are n classes of buyers with valuations per unit, $v_1 > v_2 > ... > v_n$, demands $D_1, D_2, ..., D_n$, and time limits $t_1 < t_2 < ... < t_n$. A selling strategy in this case is given as an m-step markdown mechanism with price p_i at step i. It is assumed that initial inventory level K is less than the sum of all buyers' demands but more than each buyer's demand,

Property 4. At the condition of $D_j < K < \sum_{j=1}^{n} D_j, \forall j$, the following decisions are derived.

(a) Everyone can buy at least one unit of the commodity before the last discount period.

(b) The markdown points are at the time points 0+ε or some of $t_i + \varepsilon$, i=2⋯n

(c) All buyers who does not violate the time limit until the last discount being made can get as many as they want to buy, that is, D_j.

(d) The optimal markdown prices are at some of the values v_j, j=1⋯n

Proof)

(a) At the step i, suppose there is one who cannot buy any units but gives up purchasing because of $p_i > v_j$. At the step i, among the buyers giving up purchasing because of the time limits, all buyers in the segments from buyer $j+1$ cannot buy items, so that after the discount time point t_j, only holding cost can be changed without sales profit. By moving the discount time point to just after the discount time point $t_i - 1$, a better solution can be guaranteed. Therefore, as an optimal solution, it is not necessary to consider buyers who cannot buy anything during the sales period.

(b) It is the same as the step of the proof of Property 1.

(c) Only at the last step, competition is incurred. Buyers passed over their time limits before the last discount time point can buy items as many as at their full demand, D_j.

(d) The seller's profit function is linear in p_i, so that the relation $p_i^* \le \min\{v_j \mid$buyer j passed over their time limit during the step $i\}$ holds, showing one of the extreme points, v_j.

This completes the proof.

Following Property 4, a yes-or-no decision on discount can be made at each buyer's time limit point. In general, it is necessary to compare 2^n alternatives for such yes-or-no decisions. It would also be too many equations to be considered to find an optimal solution by using differential equation evaluating. Therefore, a dynamic programming approach may be recommended for any small-sized cases.

3.2.1 Dynamic programming

Suppose that the seller can sell the item to all buyers and the current time point is just after the ith buyer's time limit. Then, at that time, seller's intermediate profit can be calculated from the beginning. Let function $f(\cdot)$ represent the seller's intermediate profit which can be calculated as follows.

Definition. The seller's intermediate profit function $f(\cdot)$ is defined as follows;

$$f(0, j) = v_j \sum_{k=1}^{j} D_k, \quad j = 1 \ldots n$$

$$f(1, j) = v_j \sum_{k=1}^{j} D_k - ht_1 \sum_{k=j+1}^{n} D_k, \quad j = 1 \ldots n$$

$$f(i, j) = \max\{f_1(i, j), f_2(i, j)\}, \quad i = 2 \ldots n, j = 1 \ldots n,$$

where the function $f_1(i, j) = f(i-1, j) - h(t_i - t_{i-1}) \sum_{k=j+1}^{n} D_k$ represents the situation where the item is sold to j buyers before the ($i-1$) th buyer's time limit, so that purchase is made without discount just after i th time limit, and

$$f_2(i, j) = f(i-1, i-1) + v_j \sum_{k=i}^{j} D_k - h(t_i - t_{i-1}) \sum_{k=j}^{n} D_k$$

represents the situation when the item is sold to i buyers before the ($i-1$)th buyer's time limit, so that purchase is made by j buyers at discount at i th time limit.

Accordingly, n×n $f(i, j)$ matrix is derived. From the matrix, an optimal discount schedule can be derived with K representing sum of all the buyers' demand. Before the current time point(after the i th buyer's time limit), the maximum of the function value can be found, while the later schedule can be independently from the preceding on. Therefore, an optimal discount price and time schedule can be determined.

Therewith, an optimal initial inventory level can be determined as follows;

The initial inventory level can be derived by adjusting the intermediate function matrix. If the seller has only K items less than all demands at the beginning, the function $f(\cdot)$ with turn out to have more inventory cost. Therefore, if the inventory cost is adjusted as $f(i, j) + ht_i \sum_{k=j+1}^{n} D_k, \forall i, j$, an optimal initial inventory level can be determined. Excepting the last buyer, the following functions can be computed for all the buyers one by one;

$$f^*(i, j) = \max\left\{ f(i, j) + ht_i \sum_{k=j+1}^{n} D_k, \forall i, j \right\}$$

This function is modified to adjust any surplus inventory cost, $ht_i \sum_{k=j+1}^{n} D_k$, with each of the buyers indexed from n to 1 so as to find the index $j*$ which indicates the maximum value of the function, resulting in an optimal initial inventory level represented by sum of the available buyers' demand, $K^* = \sum_{j=1}^{j*} D_j$.

3.2.2 Computational time

As discussed above, a full enumeration search will require the computational complexity order of $O(2^n)$, but the proposed dynamic programming requires the complexity order of $O(n^2)$. It means that larger n-buyer-set problems can be solved in the dynamic programming approach.

3.3. Considering seller's time limit

Until now, only the buyers' time limits have been considered. In this section, an additional constraint, called seller's time limit, is considered.

Many items like food and drugs have fixed and short lifetime. They can be classified into seasonal, perishable, and deteriorating items. For such items, each seller may have to consider the lifetimes at the associated sales schedule, because they may affect profit. By selling such items at discount price after certain time being elapsed, the seller's profit may be reduced. Clearance sale may also be considered prior to launching any new products. Moreover, during lifetime, item values are often considered as being constant, regardless of their remaining lifetimes.

Now, it is discussed on deriving the seller's time limit. If the seller's time limit is longer then the last m-step discount time, it will be not necessary to consider. Therefore, this section is to consider the case that seller's time limit is shorter than the last discount time (determined in Section 3.2). There are two alternative ways to consider; one is a way to reduce initial inventory level from $K*$, and the other one is to discount more at the last available discount time. The first way will lead to reduce seller's revenue and inventory cost, while the second way will lead to sell more items at lower price.

From the beginning to the seller's time limit L, an optimal schedule is the same as in Section 3.2. At the last discount step before L, it must be determined how many items the seller would sell.

Let $J = \arg\max\{j \mid t_j = T_i < L\}$. Then, an optimal seller's profit by considering seller's time limit is determined as follows;

$f^*(i,j) = \max\{f(J,j)\}, \quad j = J+1 \ldots n$

Then, an optimal inventory level can be found by searching the function $f^*(i,j)$ from J to the last buyer n. The optimal initial inventory level is equal or less than the optimal solution of the original n-buyer-set problem.

IV. Computational Results

The proposed dynamic programming gives an optimal solution in a sufficiently short time. Parametric test is made for computational evaluation. By changing the value of h(holding cost per unit and time), an optimal decision can be traced.

For the computational evaluation, the number of the buyers is set at the value 30. The associated valuations are randomly selected from 1 to 100, then, multiplied by 10, and arranged decreasingly. Their time limits are randomly selected from 1 to 100, and arranged increasingly. Their demands are also randomly selected from 1 to 10. Their numerical values are listed as in Table 1.

Buyer index	time limit	valuation	demand	Buyer index	time limit	valuation	demand
1	1	980	2	16	45	420	1
2	4	970	4	17	54	410	5
3	7	950	3	18	60	400	9
4	8	900	3	19	70	390	1
5	9	890	1	20	71	380	1
6	18	870	6	21	72	370	7
7	26	780	7	22	74	330	1
8	27	770	3	23	81	250	4
9	29	760	1	24	86	160	2
10	31	750	1	25	88	130	6
11	32	740	1	26	89	110	2
12	36	700	9	27	90	60	1
13	37	690	3	28	94	50	7
14	41	680	10	29	97	40	3
15	42	460	7	30	99	30	4

Table 1. Test case of 30 buyers

As the computational results for each value of the unit holding cost h by use of the proposed dynamic programming, Table 2 shows the optimal initial inventory level K, the optimal number of steps m, and the maximum of seller's profit.

h	K	m	R_s
0	115	30	58640
1	100	22	54055
2	90	19	50608
3	90	12	47779
4	86	9	45466
5	85	7	43715
6	85	6	42046
7	85	6	40382
8	76	6	38862
9	76	4	37919
10	61	3	37150
11	61	3	36811
12	54	1	36720
13	54	1	36720

Table 2. Computational result for each value of the unit holding cost

The computational result is also depicted as in Figures 1 and 2, showing that the optimal decisions appear in a step function. This is caused because the objective function is a piecewise linear function in h, depending on data.

Figure 3 shows how maximum of the seller's profit changes. Unlike the decisions of initial inventory level and number of the discount steps, the seller's profit changes more smooth by. This is caused because at each linear piece of the function, profit value decreases linearly as holding cost reduces.

If h is a value more than 11, no discount strategy will be the best regardless of the inventory unit cost. That is, if the inventory cost is high, then it will be better to sell without price discount at the beginning, while if the inventory cost is low, it will be more profitable to apply any discount strategy.

V. Conclusion

This paper deals with an integrated problem of inventory control and markdown dynamic pricing for a commodity with price and sales-period dependent demand pattern under complete information, where a seller and customers have complete information of each other to make their decisions.

The problem consists of two parts; one is each buyer's benefit function which makes the best decision on price and time for buyer to purchase items, and the other one is a seller's profit problem which decides an optimal sales strategy concerned

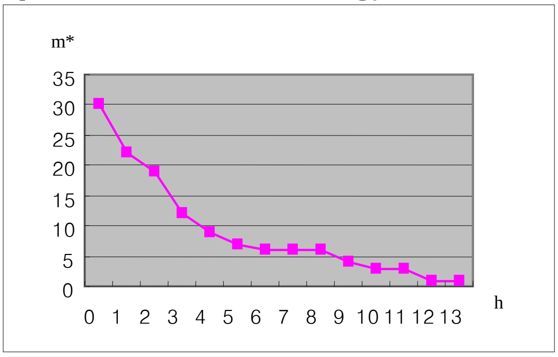

Figure 1. Optimal number of discount steps varying in holding unit cost

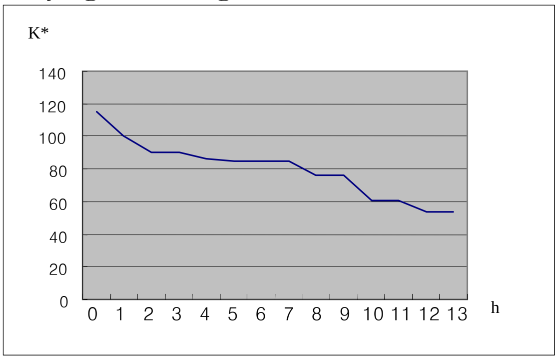

Figure 2. Optimal initial inventory number varying in holding unit cost

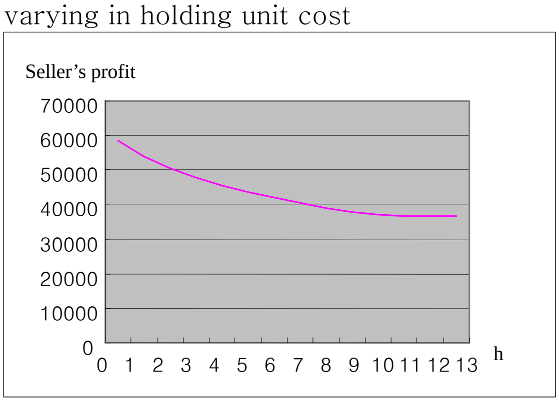

Figure 3. Maximum of the seller's profit varying in holding unit cost

with inventory control and discount schedule. The seller's profit problem consists of sales revenue and inventory holding cost functions. The two parts are closely related into each other with some related variables, so that any existing general solution methods can not be easily applied. Therefore, a simplified model with single seller and two customers is considered first, where demand for multiple units is allowed to each customer within a time limit. Therewith, the model is generalized for a n-customer-classes problem.

In general, to solve the proposed n-customer-set problem, a yes-or-no decision on discount can be made at each buyer's time limit point. Moreover, it would be too many equations to be considered to find an optimal solution by using differential equation evaluating. Therefore, a dynamic programming approach may be recommended for any small-sized cases. This leads to an optimal sales strategy for a seller, which can derive an optimal decision on both an initial inventory level and a discount schedule, in $O(n^2)$ time.

The result of this paper may be directly used for sales strategy of department stores, which is similar to a two-buyer-set problem; one is a set of buyers who want to have a brand-new item at the beginning of the sales period with higher valuation, and the other one is a set of buyers who want to buy the item as cheap as possible with lower valuations regardless of late purchasing. If their demands are deterministic and all the other information is known, the result of this paper can be well applied for such sales strategies. Like department store sales problem, some other problems with a small customer set and a short selling period can be treated immediately, including problems in Dutch auction for items with heavy holding cost, open tender of materials, quantity-limited sales, and cooperative buying in the on/off markets. However, those problems may be hard to treat if demand is uncertain, so that it may be needed to investigate market and customers carefully in treating them.

As a further study, an extended problem with decreasing valuations, and additional ordering allowed may immediately be considered. Moreover, various customers' behaviors and demand functions may also be considered.

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