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사각형 탱크 보강판의 유체구조 연성진동에 대한 이론적 연구

Analytical Study on Hydroelastic Vibration of Stiffened Plate for a Rectangular Tank

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ABSTRACT

In this paper, a theoretical study is carried out on the hydroelastic vibration of a rectangular tank wall. It is assumed that the tank wall is clamped along the plate edges. The fluid velocity potential is used for the simulation of fluid domain and to obtain the added mass due to wall vibration. In addition, the vibration characteristics of stiffened wall of the rectangular tank are investigated. Assumed mode method is utilized to the stiffened plate model and hydrodynamic force is obtained by the proposed approach. The coupled natural frequencies are obtained from the relationship between kinetic energies of a wall including fluid and the potential energy of the wall. The theoretical result is compared with the three-dimensional finite element method and then added mass effect is discussed due to tank length and potential mode.

1. Introduction

It is well known that the natural frequency of a submerged plated is significantly less than that of dry plate. This is mainly due to the added mass of fluid and this phenomenon is known as the fluid-structure interaction. In this regard, deep consideration should be taken into account for estimation of added mass and interactive modeling between structure and fluid.

Although many researches have been carried out for the field, but in general, natural frequency correction due to added mass is normally dependent upon empirical formula. However, empirical formula couldn't always give satisfactory results for every case.

Wall vibration within a rectangular tank with fluid has recently been studied. Kim[1] calculated on the added mass of thin rectangular plate vibrating elastically in an infinite ideal fluid. Lee[2] investigated experimentally on transverse vibration characteristics in water of rectangular plates having an inner free cutout. Jeong[3][4] derived a formulation on a plate using normalized functions which are admissible functions satisfying all the plate boundary condition. However, in this case only 100 percent filling case was dealt with. Lee[5] derived an expression of fluid domain using velocity potential imposing simple free surface boundary condition, keeping our interest in high frequency region. On the other hand, as for stiffened plate, Kim[6] investigated natural frequency of thin elastic rectangular plate with various boundary condition. Han[7] calculated natural frequency of stiffened plate having a concentrated mass using receptance method.

This paper describes an application of the hydroelasticity theory to the fluid-structure interaction problems in a rectangular tank. A tank wall with/without stiffeners are modeled using assumed mode methods and velocity potential method is adopted for the assessment of added mass.

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2. Theory

2.1 Formulation of a stiffened plate

Figure 1 shows a plate with stiffener. For the modeling of a rectangular plate, assumed mode method is adopted and mode shapes of beam are used.

$$w(x,y,t)=\sum_{m=1}^{p}\sum_{n=1}^{q}A_{mn}(t)X_m(x)Y_n(y) \qquad (1)$$

where,

$X_m(x), Y_n(y)$ are m[th] and n[th] mode shapes of a beam for x and y direction respectively, which is spatial coordinate of a plate, and $A_{mn}(t)$ is the time dependent generalized coordinate.

Using the assumed mode method, kinetic and potential energy can be calculated. This approach can be extended to the stiffened plate. In that case, kinetic and potential energy of the plate itself can be expressed as follows

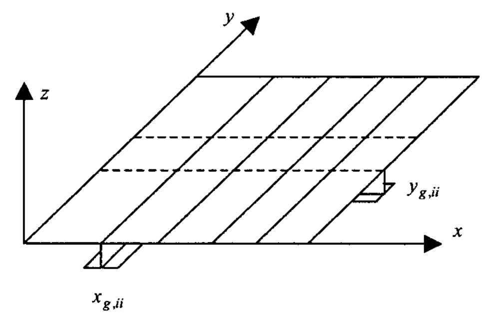

Fig.1 A plate with stiffeners and coordinate system

2.1.1 Kinetic energy of stiffened plate

$$T_p(t)=\frac{1}{2}\int\int \rho h\ \dot{w}(x,y,t)^2 dxdy$$

$$T_g(t)=\frac{1}{2}\rho\sum_{ii=1}^{n_g} A_{x,ii}\int \dot{w}(x,y_{g,ii},t)^2 dx$$

$$+I_{px,ii}\int\left(\frac{\partial\dot{w}(x,y_{g,ii},t)}{\partial y}\right)^2 dx\Bigg]dy$$

$$T(t)=T_p+T_g \qquad (2)$$

V_p, V_g are kinetic energy of plate and stiffener.

a , ρ , A_{xii}, I_{pxii} and $y_{g,ii}$ are stiffener length, density, sectional area, polar moment of inertia, y-axis location of the ii th stiffener.

2.1.2 Potential energy

$$V_p(t)=\frac{1}{2}\int\int D_E\left(\begin{aligned}&\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)^2\\&+2\upsilon\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+2(1-\upsilon)\left(\frac{\partial^2 w}{\partial xy}\right)^2\end{aligned}\right)dxdy,$$

$$V_g(t)=\frac{1}{2}\sum_{ii=1}^{n_g}\left[\begin{aligned}&EI_{x,ii}\int\left(\frac{\partial^2 w(x,y_{g,ii},t)}{\partial x^2}\right)^2 dx\\&+GI_{px,ii}\int\left(\frac{\partial^2 w(x,y_{g,ii},t)}{\partial x\partial y}\right)^2 dx\end{aligned}\right]$$

$$V(t)=V_p+V_g \qquad (3)$$

V_p, V_g are potential energy of plate and stiffener.

D_E , υ are bending rigidity of plate, passion ratio and EI_x , GJ_x , n_g are equivalent bending rigidity, torsional rigidity of x-axis stiffener and number of stiffeners respectively.

Natural frequency of the stiffened panel can be obtained from Eq. (4) applying Lagrange equation to kinetic and potential energies.

$$[K]q+[M]\ddot{q}=0, \qquad (4)$$

It should be noted that non-zero off-diagonal terms exists in Eq. (4) because the admissible functions are not the eigen-mode of the considered problem.

2.2 Hydrodynamic modeling

Figure 2 shows a rectangular tank filled with liquid and coordinate system. a , b , H and L are breadth, height of the plate, liquid filling level and length of the tank, respectively. The right most wall shown in brick pattern is taken into account for the formulation. However, application to other plates will be very similar and straightforward.

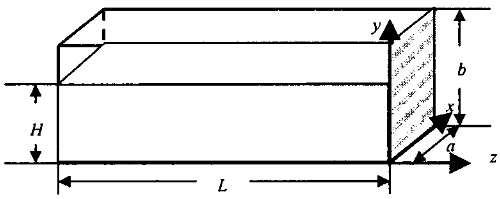

Fig.2 A rectangular tank filled with liquid

Velocity potential is used for the estimation of hydrodynamic force assuming incompressible and irrotational flow. Consequently, Laplace equation is the governing equation as shown in Eq. (5).

$$\nabla^2\Phi=\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0 \qquad (5)$$

Velocity potential can be obtained from Laplace equation with adequate boundary conditions, which are rigid wall boundary condition (normal velocity of fluid at tank walls = 0), elastic wall boundary condition (normal velocity of fluid at a tank wall is the same as velocity of the plate) and free surface boundary condition on disturbed free surface.

In general, free surface boundary condition is expressed as in Eq. (6).

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial y}=0 \quad \text{at } y=H \qquad (6)$$

However, it should be noted that because natural frequency of elastic wall is relatively high compared with water wave frequency, high frequency approximation can be applied to the general free surface boundary condition, which is simplified as Eq. (7).

$$\phi=0 \qquad \text{at } y=H \qquad (7)$$

Force applied to the wall can also be calculated with the Bernoulli's equation (8) and velocity potential.

$$p=-\rho\frac{\partial\Phi}{\partial t}\bigg|_{z=0} \qquad (8)$$

Consequently, force can be obtained as Eq. (9).

$$F_s=\int_0^H\int_0^a p\,dxdy \qquad (9)$$

Because it is known that added mass is proportional to the acceleration of the wall, the added mass can be obtained as Eq. (10).

$$F_s=\sum_{r=1}^{\infty}A_{sr}\ddot{a}_r(t) \qquad (10)$$

$$M_{add,sr}=A_{sr}$$

2.3 Natural frequency of stiffened plate in contact with liquid

Natural frequency of stiffened plate can be calculated with the structure modal mass, modal stiffness and added mass.

$$[K_{\text{modal}}][a_{add}]-[\Lambda_{add}][M_{\text{modal}}+M_{add}][a_{add}]=0,$$

$$[\Lambda_{add}]=[\varpi^2_{add}] \qquad (11)$$

3. Example and validation

Based upon the proposed methodology, a couple of numerical calculations are carried out for the validation and compared with NASTRAN, which is three dimensional finite element program.

3.1 Plate without stiffener in water

Figure 3 shows a rectangular tank filled with water. There's no stiffeners attached to the side wall. The thickness of the wall is 1.4 mm.

Calculation was carried out for 5 cases, varying the water filling level, i.e. 10%, 25%,50%,75% and 90% of tank depth.

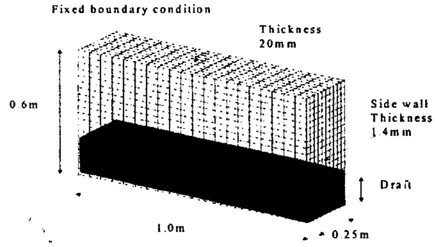

Fig.3 A rectangular tank filled with water

Clamped boundary condition was imposed on all edges of walls. Table 2 shows the natural frequencies obtained by the proposed theory and NASTRAN. It is found that the proposed theory agrees very well with NASTRAN result. The least natural frequency is the most important for tank design, and it has 1.2% error at 90% of draft.

Table 1 Natural frequency of the sidewall (without stiffener)

Draft (m)	Air(0%)			0.05(10%)		
Mode	NAS.	Theo.	ERR. (%)	NAS.	Theo.	ERR. (%)
1st	128.0	129.2	-0.9	127.9	129.1	-0.9
2nd	151.0	154.4	-2.2	150.2	154.0	-2.5
3rd	194.0	200.1	-3.1	190.9	198.3	-3.8
4th	259.0	267.5	-3.3	250.1	262.4	-4.9
5th	345.0	343.3	0.5	327.4	343.2	-4.8
Draft (m)	0.15(25%)			0.3(50%)		
Mode	NAS.	Theo.	ERR. (%)	NAS.	Theo.	ERR. (%)
1st	87.0	93.4	-7.4	44.6	45.6	-2.3
2nd	132.0	133.1	-0.8	91.7	93.2	-1.6
3rd	167.0	170.0	-1.8	143.0	146.4	-2.4
4th	221.0	227.0	-2.7	160.0	159.3	0.5
5th	214.0	232.3	-8.5	182.0	197.2	-8.4
Draft (m)	0.45(75%)			0.54(90%)		
Mode	NAS	Theo.	ERR. (%)	NAS	Theo.	ERR. (%)
1st	32.8	33.2	-1.3	29.1	29.4	-1.2
2nd	60.5	60.8	-0.5	52.6	53.0	-0.7
3rd	97.2	98.9	-1.8	81.3	81.9	-0.7
4th	149.0	145.6	2.3	122.6	123.7	-0.9
5th	149.0	156.1	-4.7	145.7	142.6	2.1

3.2 Plate with one stiffener

One stiffener (flat bar) is attached at the middle of a side tank wall. Thickness of the wall and stiffener are 1.4 mm and 10 mm x 1.4 mm, respectively. Dry condition and liquid contact condition are dealt with.

Table 2 shows the calculation results. In case of water contact condition, 0.45m is the 75% filling level case.

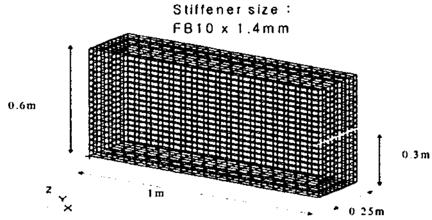

Fig.4 Tank model with one stiffener

Table 2. Natural frequency of side wall in air and water (1 stiffener)

Mode	Stiffened plate					
	in Air			in Water(draft :0.45m)		
	Theory	NAS.	Discrepancy.(%)	Theory	NAS.	Discrepancy.(%)
1st	154.6	151.8	1.8	39.2	38.3	2.3

It is found that only 2.3% of discrepancy of the fundamental natural frequency between the proposed theory and NASTRAN. Figure 5 shows the mode shape of the dry and water contact condition. Calculation result of theory coincides with that of NASTRAN.

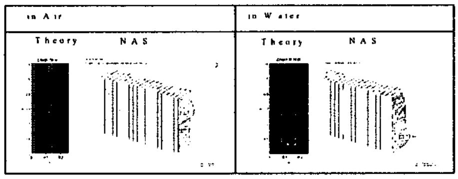

Fig.5 Mode shape of stiffened plate in air and water

3.3 plate with three stiffeners

As shown in Fig.6, three stiffeners(flat bar) are attached to the tank wall. Thickness of the wall and stiffener are 1.4 mm and 10 mm x 1.4 mm, respectively. Dry condition and liquid contact condition are dealt with. Table 3 shows the calculation results. In case of water contact condition, 0.45m is the 75% filling level case.

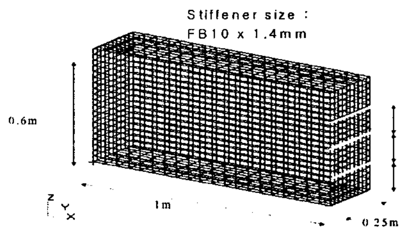

Fig.6 Tank model with three stiffeners

Table 3 Natural frequency of side wall in air and water (3 stiffeners)

Mode	Stiffened plate					
	in Air			in Water (draft : 0.45m)		
	Theory	NAS.	Discrepancy.(%)	Theory	NAS.	Discrepancy.(%)
1ST	286.8	272.6	4.9	101.3	94.4	6.8

Table 3 shows that the discrepancy is 6.8%. This difference seems to be large but it is mainly from the stiffened plate model, which has already 4.9% discrepancy in air, rather than the fluid-structure interaction model. Figure 7 shows the mode shape of the dry and water contact condition. It is found that the mode shapes of two calculations are very similar.

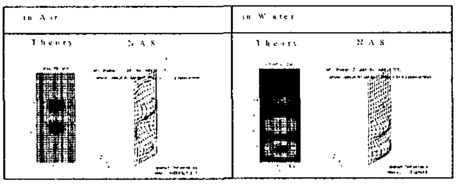

Fig.7 Mode shape of stiffened plate in air and water

4. Added mass effect due to tank length

As for vibration of tank, little attention has been shown to the question of added mass effect due to tank length (L). According to Eq.(12), added mass effect for tank lengths is R_{ij} shown in Fig.8 to first potential mode.

$$R_{ij} = \frac{1}{\lambda_{ij} \cdot \tanh \lambda_{ij} L} \qquad (12)$$

L, H, a, λ_{ij} are tank length, liquid filling draft, length of plate on x direction, characteristic value of velocity potential respectively. Added mass effect is exponentially decreased as tank length is increased and it is converged when tank length is reached to about same length of draft (H). That is, natural frequency increases to the saturated frequency as tank length (L) increases. Figure 8 shows natural frequency corresponding

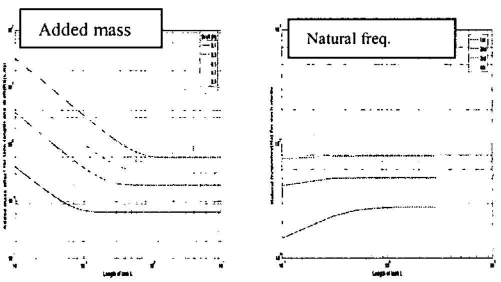

Fig.8 Added mass effect due to tank length

tank length (L) for each mode at draft 0.6m. It is noted that natural frequency tends to converge when tank length (L) reaches draft (H).

5. Conclusion

An analytical method to estimate the natural frequency of the stiffened wetted plate is developed. Through out the tests, it is found that this approach can properly deal with the fluid-structure interaction irrespective of the liquid filling level. The proposed scheme is embedded to stiffened plate and the calculation results show very good agreement with three-dimensional finite element method. In addition, the change of natural frequency is investigated for various tank lengths and it is found that natural frequency of tank wall increases as tank length increases. However, it is noted that when tank length reaches the similar dimension of draft (H), natural frequency tends to converge.

Some improvement can be expected when hydrodynamic and stiffened plate model is slightly revised.

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