# Time Discretization of the Nonlinear System with Variable Time-delayed Input using a Taylor Series Expansion

Hyung Jo Choi*, and Kil To Chong**

* Department of Mechatronics Engineering, Chonbuk National University, Jeonju, Korea
(E-mail: hyungjochoi@yahoo.co.kr)

**Department of Electronical & Information Engineering, Chonbuk National University, Jeonju, Korea
(E-mail: kitchong@chonbuk.ac.kr)

**Abstract**: This paper suggests a new method discretization of nonlinear system using Taylor series expansion and zero-order hold assumption. This method is applied into the sampled-data representation of a nonlinear system with input time delay. Additionally, the delayed input is time varying and its amplitude is bounded. The maximum time-delayed input is assumed to be two sampling periods. Them mathematical expressions of the discretization method are presented and the ability of the algorithm is tested for some of the examples. And 'hybrid' discretization scheme that result from a combination of the 'scaling and squaring' technique with the Taylor method are also proposed, especially under condition of very low sampling rates. The computer simulation proves the proposed algorithm discretized the nonlinear system with the variable time-delayed input accurately.

**Keywords:** Nonlinear system, time-varying delay, discretization, Taylor series

## 1. INTRODUCTION

Time-delay occurred during the information processing and data transmission in many engineering systems. In recent years, there were many systems that were controlled via network and transferred the data from a remote site owing to the development of networks, in the sense that time-delay, which possibly occurs during the data transmission through the network, is the most important factor for the system performance.

Many studies have been performed to solve the problems of time-delay during that time. Boutayeb designs an observer for the discrete system with state-delay and output-delay and presents a necessary and sufficient condition for the asymptotic stability using the Lyapunov method [1]. Luo and Chung propose a method of delay-dependent criterion that guarantees the asymptotic stability of the linear uncertain system with time-delay [2]. Nihtia, Damak, and Babary propose a design method of the real-time delay estimator for the input delay of SISO system with the finite dimension by transferring the time-delayed part of the system to the transport system, which has a type of linear partial differential equation [3].

As in the digital controller design issue, generally, the two methods are usually used. Firest, the controller design is to be performed in the continuou-time space based on the continuous-time model, and then it is transformed to a digital controller [4,5,6,7]. Although this method has been used in many of the existing studies, a time-delay system is left with the implementation of a digital controller that has many limitations due to the infinite dimension of a delayed item in the time-delayed system. Second, it directly designs a digital controller based on the discrete-time space model after transforming the continuous-time model to the discrete-time space model. By using this method, the implementation of a digital controller is more unrestricted because the problem of infinite dimension due to the time-delayed item in the discrete-time space can be solved by this method. Therefore, the second method is well suited to discretize the time-delayed system. This paper proposes a discretizing method for the nonlinear system with a time-delayed input using the well known existing discretizing algorithm [8],[9] and for the nonlinear system with variable time-delay using a Taylor series expansion [10],[11]. Generally, there are some difficulties for directly applying the results of the above studies to the actual system because the delayed input values are time-varying in a system. Therefore, this study proposes a discretizing algorithm for the nonlinear system with variable time-delayed input for an actual system, using a Taylor series expansion. Besides, the well-known 'scaling and squaring' technique, which is widely used for computing the matrix exponential, is applied to the nonlinear case, when the sampling period is too large.

This paper consists of the following chapters to explain the results of the study. Chapter 2 presents the existing method of discretizing for a system with variable time-delay using a Taylor series expansion. Chapter 3 derives a discretizing algorithm for the nonlinear system that has a variable time-delay in the input of what the algorithm proposed in this study. A discretizing algorithm using a Taylor series expansion is derive din the case that the values of delay are changed within, twice for the sampling period after observing the case that the values of delay are changed in a single sampling period. Chapter 4 proves that the proposed algorithm for time-varying delay has a good performance by using 'scaling and squaring' technique when a large sampling period is used. Chapter 5 performs a computer simulation for the algorithm proposed in this study. Finally, Chapter 6 provides the conclusion fo this study and direction of the future study.

## 2. DISCRETIZING OF THE NONLINEAR SYSTEM USING TAYLOR SERIES EXPANSION

### 2.1 Discretizing of the nonlinear system with time-delay

In this section, a discretizing algorithm for the nonlinear system with a constant time-delay is considered. Let us assume the nonlinear system is as shown in Eq. (1).

$$\frac{dx(t)}{dt} = f(x(t)) + g(x(t))u(t-D) \quad (1)$$

In case of applying the zero-order-hold (ZOH) assumption for the above system as in the linear system, the delayed values of input can be applied by the separated two different time intervals as shown in Eq. (2).

$$u(t-D) = \begin{cases} u(kT-qT-T) \equiv u(k-q-1) & kT \le t < kT+\gamma \\ u(kT-qT) \equiv u(k-q) & kT+\gamma \le t < kT+T \end{cases} \quad (2)$$

Therefore, the state values for $kT+\gamma$ can be obtained as presented in Eq. (3) due t the application of the input values of time interval $[kT, kT+\gamma)$ .

$$x(kT+\gamma) = \Phi_\gamma(x(kT), u(k-q-1)) \quad (3)$$

where $\Phi_\gamma$ can be derived directly by using Eq. (4).

$$x(k+1) = \Phi_T(x(k), u(k)) = x(k) + \sum_{l=1}^{\infty} A^{[l]}(x(k), u(k)) \frac{T^l}{l!} \quad (4)$$

In the same manner, the values of the state vector for $(k+1)T$ can be obtained as shown in Eq. (5) owing to the application of the input values of the time interval $[kT+\gamma, (k+1)T]$ .

$$x(kT+T) = \Phi_{T-\gamma}(x(kT+\gamma), u(k-q)) \quad (5)$$

The sampled-data representation of the nonlinear system (1) can be achieved by using Eq. (3) and (5) as follows.

$$x(k+1) = \Phi_T^D(x(k), u(k-q-1), u(k-q)) = \Phi_{T-\gamma}(\Phi_\gamma(x(k), u(k-q-1)), u(k-q)) \quad (6)$$

If the finite series truncation order $N$ is to apply for Eq. (6), the approximated sampled-data representation can be obtained as follows.

$$x(k+1) = \Phi_T^{N,D}(x(k), u(k-q-1), u(k-q)) \quad (7)$$

## 3. Discretizing of the nonlinear system with variable time-delayed input

### 3.1 For the time-delay is smaller than the sampling period

Let us consider the nonlinear system as shown in Eq. (8).

$$\frac{dx(t)}{dt} = f(x(t)) + g(x(t))u(t-D(t)) \quad (8)$$

Let us assume Eq. (8) will be discretized to get the sampling period of $T = t_{k+1} - t_k > 0$ . The values of time-delay for the $k$th sampling period can be expressed as follows.

$$D_k = q_k T + \gamma_k \quad (9)$$

where $q_k = 0$ and $0 \le \gamma_k < 1$ is a real number. Where the values of the delay are smaller than a single sampling period, the time interval for the $k$th sampling period can be divided into the two different sections of $[kT, kT+\gamma_k), [kT+\gamma_k, kT+T)$ based on the point of time when the time-delay occurs. This is because the maximum delayed input located within a single sampling period. In this case, the values of $\gamma_k$ become an important factor as the calculation of the state values because the delayed input values in the sampling periods can be decided by the values of $\gamma_k$ .

Therefore, the input values applied to the system can be expressed according to the interval as shown in Eq. (10) for the time-delay has occurred for the $k$th sampling period.

$$\begin{aligned} u(t-D_k) &= u(k-q_k-1) & kT \le t < (kT+\gamma_k) \\ &= u(k-q_k) & (kT+\gamma_k) \le t < (kT+T) \end{aligned} \quad (10)$$

The discretizing of the nonlinear system of Eq. (8) that has an input presented by Eq. (10) using a Taylor series expansion is shown as follows.

$$x(kT+\gamma_k) = x(kT) + \sum_{l=1}^{\infty} A^l(x(kT), u(k-q_k-1)) \frac{\gamma_k^l}{l!} \qquad kT \le t < (kT+\gamma_k) \quad (11)$$

$$x(kT+T) = x(kT+\gamma_k) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma_k), u(k-q_k)) \frac{(T-\gamma_k)^l}{l!} \qquad (kT+\gamma_k) \le t < (kT+T) \quad (12)$$

By using Eq. (11) into Eq. (12), the following equations can be acquired.

$$x(kT+T) = x(kT+\gamma_k) + \sum_{l=1}^{\infty} A^l((x(kT) + \sum_{l=1}^{\infty} A^l(x(kT), u(k-q_k-1)) \frac{\gamma_k^l}{l!}), u(k-q_k)) \frac{(T-\gamma_k)^l}{l!} \quad (13)$$

The approximation of Eq. (13) up to the order of $N$ is shown as follows.

$$x(kT+T) = x(kT+\gamma_k) + \sum_{l=1}^{N} A^l((x(kT) + \sum_{i=1}^{N} A^l(x(kT), u(k-q_k-1)) \frac{\gamma_k^l}{i!}), u(k-q_k)) \frac{(T-\gamma_k)^l}{l!} \quad (14)$$

Therefore, the nonlinear system that has a variable time-delayed input is smaller than one sampling gperiod can be discretized by the discretized time space model with the order of $N$ as shown in Eq. (14).

### 3.2 For the time-delay is smaller than twice the sampling period

Let us consider the nonlinear system as shown in Eq. (8) again. In addition, assume that the time-delay for the $k$th sampling period is $D_k = q_k T + \gamma_k$, where $q_k = 0,1,2,...$ is an integer, and $0 \le \gamma_k < 1$ is a real number. Moreover, assume that the delayed values in the present sampling periods and previous sampling periods are already known. If the values of te variable time-delay are bigger than the sampling scale, the input values will be beyond the sampling periods and affect the next sampling period. As a result, a system, which has a single input, will be applied over two inputs according to the magnitude of a time-delayed. In this case, if the previous input before one step is inputted after the input of the present step, the input of the previous sep will be neglected and of the present step will only affect the system. Therefore, if the values of the variable time-delay are bigger than one sampling period, the verification of which input affects to the system for each sampling period shall be required. In this study, the factors for each time-delay are compared to solve these problems. The $k$th time-delay can be expressed as $D_k = q_k T + \gamma_k$, where $q_k$ presents multiple sampling period, and $\gamma_k$ shows where the location of the time-delay in the sampling interval is. It is possible to determine where the time-delayed inputs are applied to the system by checking the values of $q_k$. In case two inputs exist in a single sampling interval, it is possible to verify which numbers of the input are applied to the system by comparing the values of $\gamma_k$. These processes are shown as follows. First of all, in case of $q_{k-1}=0$ and $q_k=0$, the time-delayed input for the *k*th input can be applied to the *k*th period.

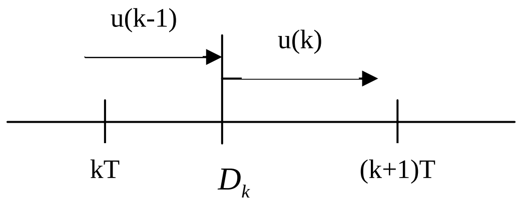


Fig. 1 The input values when q(k-1)=0 and q(k)=0

Therefore, the input values in the *k*th period, are as shown in Fig. 1, can be determined as follows;

$$u(t)=u(k-q_k-1) \qquad kT \le t < kT+\gamma_k$$
$$u(t)=u(k-q_k) \qquad kT+\gamma_k \le t < (k+1)T \qquad (15)$$

The (*k-1*)th input will affect the system, because the *k*th input cannot affect the *k*th period for $q_{k-1}=0$ and $q_k=1$. Therefore, the input values in this period are as shown in Fig. 2 can be expressed as follows:

$$u(t)=u(k-q_k) \qquad kT \le t < (k+1)T \qquad (16)$$

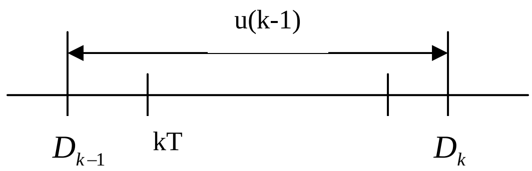


Fig 2. The input values when q(k-1)=0 and q(k)=1

In the case of $q_{k-1}=1$ and $q_k=0$, there are two input values in a single sampling interval. Therefore, it is necessary to check which input can be affected firstly to the system. First of all, for the condition of $\gamma_{k-1} \ge \gamma_k$, the (*k-1*)th input will be neglected because the *k*th input is applied to the system before the (*k-1*)th input affect the system. Therefore, the input values are as shown in Fig. 3 defined as follows.

$$u(t)=u(k-q_k-2) \qquad kT \le t < kT+\gamma_k$$
$$u(t)=u(k-q_k) \qquad kT+\gamma_k \le t < (k+1)T \qquad (17)$$

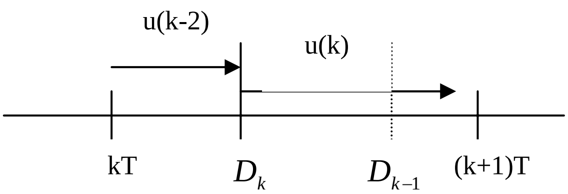


Fig. 3 The input values when q(k-1)=1 & q(k)=0,
r(k-1) > r(k)

On the other hand, for the condition of $\gamma_{k-1} < \gamma_k$, the input of both the (*k-1*)th and the *k*th affect the system. Hence, the input values are as shown in Fig. 4 defined as follows:

$$u(t)=u(k-q_k-2) \qquad kT \le t < kT+\gamma_{k-1}$$
$$u(t)=u(k-q_k-1) \qquad kT+\gamma_{k-1} \le t < kT+\gamma_k$$
$$u(t)=u(k-q_k) \qquad kT+\gamma_k \le t < (k+1)T \qquad (18)$$

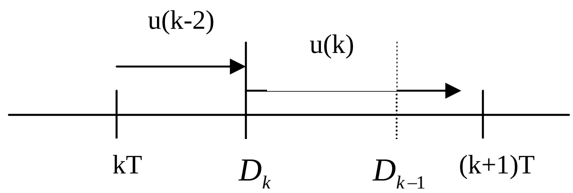


Fig. 4 The input values when q(k-1)=1 & q(k)=0,
r(k-1) < r(k)

Finally, in the case of $q_{k-1}=1$ and $q_k=1$, the (*k-1*)th input can only affect the *k*th sampling period. Therefore, the input values are as shown in Fig. 5 defined as follows:

$$u(t)=u(k-q_k-1) \qquad kT \le t < kT+\gamma_{k-1}$$
$$u(t)=u(k-q_k) \qquad kT+\gamma_{k-1} \le t < (k+1)T \qquad (19)$$

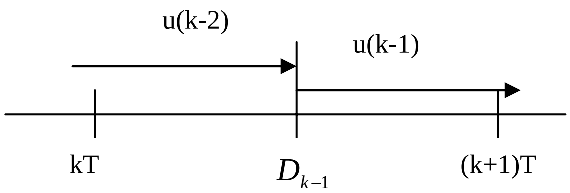


Fig. 5 The input values when q(k-10=1 and q(k)=1

By substituting the results into the approximated equation wit the order of *N* using a Taylor series expansion, the discretizing equation for the nonlinear system with variable time-delay in which the values of a time-delay are less than twice of the sampling period can be obtained as follows. First of all, in the case of $q_{k-1}=0$ and $q_k=0$, the values of the state vector can be expressed as follows:

$$x(kT+\gamma_k)=x(kT)+\sum_{l=1}^{N} A^l(x(kT),u(k-q_k-1))\frac{\gamma_k^l}{l!}$$
$$kT \le t < kT+\gamma_k$$
$$x(kT+T)=x(kT+\gamma_k)+\sum_{l=1}^{N} A^l(x(kT+\gamma_k),u(k-q_k))\frac{(T-\gamma_k)^l}{l!}$$
$$kT+\gamma_k \le t < (k+1)T \qquad (20)$$

For $q_{k-1}=0$ and $q_k=1$, the values of the state vector can obtained as follows:

$$x(kT+T)=x(kT)+\sum_{l=1}^{N} A^l(x(kT),u(k-q_k))\frac{(T-\gamma_k)^l}{l!}$$
$$kT \le t < (k+1)T \qquad (21)$$

For $q_{k-1}=1$ and $q_k=0$, the values of the state vector can be expressed as follows in the case of $\gamma_{k-1} \ge \gamma_k$.

$$x(kT+\gamma_k) = x(kT) + \sum_{l=1}^{N} A^l(x(kT), u(k-q_k-2))\frac{\gamma_k^l}{l!}$$
$$kT \le t < kT+\gamma_k$$
$$x(kT+T) = x(kT+\gamma_k) + \sum_{l=1}^{N} A^l(x(kT+\gamma_k), u(k-q_k))\frac{(T-\gamma_k)^l}{l!}$$
$$kT+\gamma_k \le t < (k+1)T \quad (22)$$

For $\gamma_{k-1} < \gamma_k$, the value of the state vector can be obtained as follows:

$$x(kT+\gamma_{k-1}) = x(kT) + \sum_{l=1}^{N} A^l(x(kT), u(k-q_k-2))\frac{\gamma_{k-1}^l}{l!}$$
$$kT \le t < kT+\gamma_{k-1}$$
$$x(kT+\gamma_k) = x(kT+\gamma_{k-1}) + \sum_{l=1}^{N} A^l(x(kT+\gamma_{k-1}), u(k-q_k-1))\frac{(\gamma_k-\gamma_{k-1})^l}{l!}$$
$$kT+\gamma_{k-1} \le t < kT+\gamma_k$$
$$x(kT+T) = x(kT+\gamma_k) + \sum_{l=1}^{N} A^l(x(kT+\gamma_k), u(k-q_k))\frac{(T-\gamma_k)^l}{l!}$$
$$kT+\gamma_k \le t < (k+1)T \quad (23)$$

## 4. 'SCALING AND SQUARING' TECHNIQUE

When the Taylor series method is applied it provides an accurate result. However, the order *N* must be very large in order to achieve the desired accuracy, if the sampling interval *T* is also large. this is due to the probability that when *T* is considerably large $A^{[l]}T^l/l!$ might become extremely large (due to the finite-precision arithmetic0 before it becomes small at higher powers, when convergence takes over. In the case of linear system this phenomenon occurs when calculating $e^{AT}$ and $\int_o^T e^{AT}dt$, which causes overflow of the computer number representation.

A 'scaling and squaring' technique, which is also called 'extrapolation to the limit' technique in the numerical analysis literature, can be applied to solve this kind of problem. this technique Is popularly used to calculate the exponential matrix $\exp(AT)$ for large sampling period by applying 'scaling and squaring' technique one can subdivide the sampling interval *T* into two or more subintervals of equal length. An appropriate positive integer *m* can be chosen s that $T/2^m$ is small enough to calculate the exponential matrix. In this case the sampling period *T* is subdivided into $2^m$ equally spaced subintervals of length $T/2^m$ and the exponential matrix is calculated for a short interval $T/2^m$. Finally, 'squaring' the matrix $\exp(AT/2^m)$ *m* times performs the computation of $\exp(AT)$:

$$\exp(AT) = (((\exp(A\frac{T}{2^m}))^2)^{\cdots})^2$$

By applying the Taylor series method the popular 'scaling and squaring' technique is easily extended to the nonlinear case. When doing a particular analogue, one can use nonlinear operators and powers of operators to substitute matrices and matrix products. Subsequently, the key idea utilized in the nonlinear analogue of the 'scaling and squaring' technique remains the same as the linear case.

In the nonlinear case, when *T* is large enough, one can divide the interval $[t_k, t_{k+1})$ to $2^m$ equally spaced subintervals, and use a small Taylor expansion order *N* with a time step $T/2^m$, for the $2^m$ intermediate subintervals to substitute the large order *N'* used in the single-step Taylor method case.

Assume now that $\Omega(N',T): R^n \to R^n$ is the operator that corresponds to the Taylor expansion of order *N'* with a time step *T*, and when it acts on $x(kT)$ the outcome is:

$$x(kT+T) = \Omega(\tilde{N},T)x(kT)$$

where $\Omega(\tilde{N},T)(\bullet) = I + \sum_{l=1}^{\tilde{N}} A^{[l]}(x(k),u(k))\frac{T^l}{l!}$

Using the operator notation the resulting discrete-time system is written as follows;

$$x(kT+T) = \left[\Omega(N,\frac{T}{2^m})\right]^{2^m} x(kT)$$

The above ASDR can be viewed as the direct result of combining the Taylor's method and the 'scaling and squaring' technique.

Knowing how to choose the parameters of *N* and *m* is important. Different values of *N* and *m* can reflect different requirements of the discretization performance. In fact the criterion for electing an appropriate *m* involves comparing the magnitude of the sampling period *T* with the fastest time constant $1/\rho$ of the original continuous-time system. If *T* is small compared to $2/\rho$, then we can set $m=0$, and apply the single-step Taylor series method. Since *T* is small, a low order *N* single-step Taylor discretization method is usually sufficient to meet the expected accuracy requirement. When *T* is larger than the fastest time constant $1/\rho$, we apply the 'scaling and squaring' discretization technique. the sampling interval is therefore subdivided into $2^m$ subintervals, and a low-order *N* single step Taylor discretization method is applied to each subinterval. These subdivisions require that the following inequality hold:

$$\frac{T}{2^m} < \frac{2}{\rho}$$

Since the requirements for numerical convergence and stability are also met. The positive integer *m* is assigned as:

$$m = \max\left(\left[\log_2\left(\frac{T}{\theta}\right)\right]+1, 0\right)$$

where $\theta < 2/\rho$ is arbitrarily chosen and $[x]$ denotes the integer part of the number $x$. It is evident, that smaller values of the arbitrarily selected number $\theta$ would result in more stringent bounds on $T/2^m$.

## 5. COMPUTER SIMULATION

### 5.1 Simple chemical processing system

In order to verify the algorithm proposed in this study, a simulation was performed for the typical system of CSTR. The system equation can be expressed s shown in Eq. (40).

$$x'(t) = -x^2(t) - 3x(t) + u(t-D(t))(1-x(t)) \quad (40)$$

The initial condition is $x(0)=0$, and the input and time-delay is applied by a sinusoidal wave. This simulation consists of the two cases; the conditions are changed by the period of sinusoidal delay and of sampling period. In addition, this study assumes and compares that the results of the method of the Taylor series and Matlab ODE Solver proposed by this study are the exact values of the system. It is possible to verify that the basis of what the results of Matlab ODE Solver can be used as a reference value from the study of Park [12, 13].

Fig. 6 shows the state values and relative errors for the system in which the input is given by $u(t) = 0.9 * \sin((t - D(t))/4)$, time-delay is $D(t) = 0.04 * \sin(t/4) + 0.05$ for the sampling period of $T = 0.05\sec$. As shown in Figure 6, the maximum errors for the state values are not exceeded over 1%. Figure 7 presents the state values and relative errors for the system in which the input is given by $u(t) = 0.9 * \sin((t - D(t))/4)$, the time-delay is $D(t) = 0.04 * \sin(t/4) + 0.05$ for the sampling period of $T = 0.01\sec$. Figure 8 shows the results of the system that has the same input and time-delay except for the sampling period, which is given by $T = 0.005\sec$. The RMS values for each case present 0.0018 for the sampling period of $T = 0.05\sec$. In case of the sampling period of $T = 0.01\sec$, the RMS value is 0.0014 and is 0.0009 for $T = 0.01\sec$. As a result, it is shown that the RMS value is decreased by the shorter sampling period.

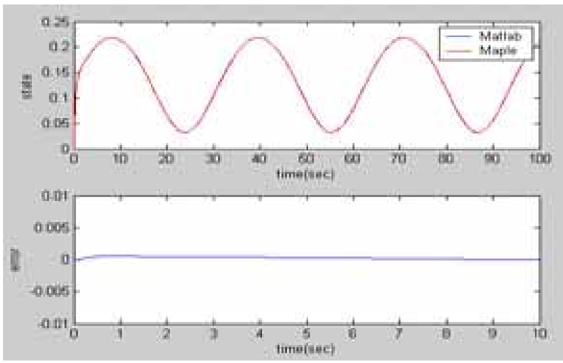


Fig. 6 State errors and values of CSTR, T=0.05sec

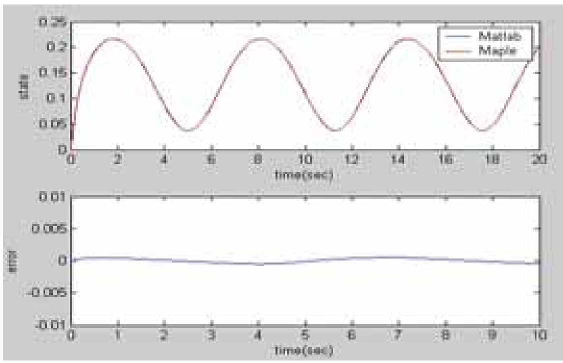


Fig 7 Sampling period of delay is changed, T=0.01sec

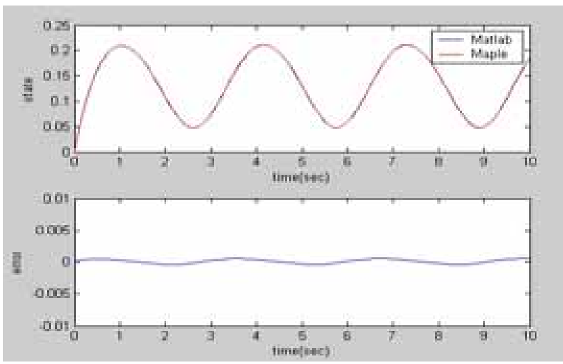


Fig 8 Sampling period of delay is changed, T=0.005sec

### 5.2 Second-order nonlinear system

Another simulation for the second-order nonlinear system with a variable time-delay that is a little more complex than that of the first-order nonlinear system presented in Section 4.1. The system equation is shown in Eq. (41).

$$x''(t) = x'(t)(1 - x^2(t)) - x(t) + u(t - D(t)) \quad (41)$$

The initial conditions are $x(0) = 0.1, x'(0) = 0$ and the input and time-delay is applied by a sinusoidal wave. In order to apply the discretizing algorithm using a Taylor series, the system will be changed by the state-space equation. If we assume the state f the system as follows,

$$X_1 = x, \quad X_2 = x' \quad (42)$$

The state variables can be expressed as follows.

$$\begin{aligned} X_1' &= f_1(X) + g_1(X)u = X_2 \\ X_2' &= f_2(X) + g_2(X)u = X_2(1 - X_1^2) - X_1 + u \end{aligned} \quad (43)$$

The same simulation formed as the CSTR is applied to the second-order nonlinear system, and various experiments are also achieved. Figure 7 and 8 show the results of computer simulation for the system in which the input is given by $u(t) = \sin(10 * (t - D(t)))$, time-delay is $D(t) = 0.0009 * \sin(t) + 0.001$ for the sampling period of $T = 0.001\sec$. As shown in Figure 7 and 8, the state values in the continuous-time are close to the discrete-time. In addition, it is shown that the errors of state values between the two different time domain are quite small. The RMS value for the state $X_1$ is $9.2093 \times 10^{-5}$ and $X_2$ is $4.0962 \times 10^{-4}$.

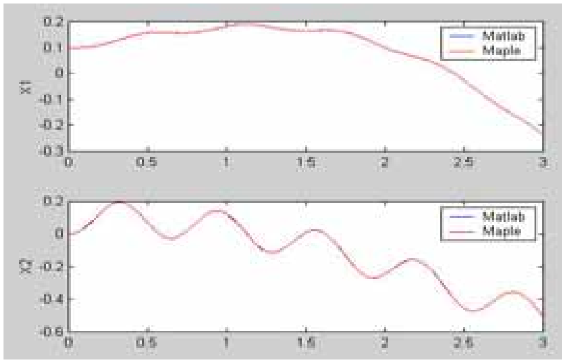


Fig 12 State values of using Matlab and Maple (T=0.001s)

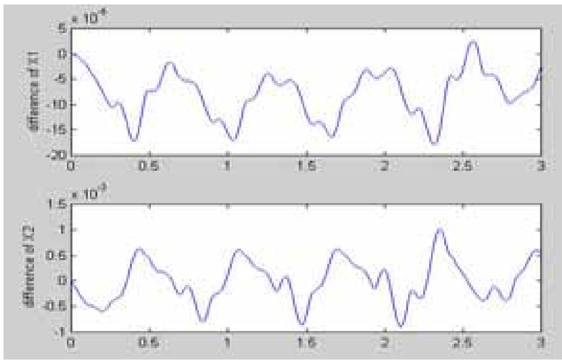


Fig 13 State differences between Matlab and Maple (T=0.001s)

From the foregoing results of the experiments we see that the values of state error will be decreased by the decrement of the sampling period of the system. In the case of the same sampling period, the state errors are decreased by the smaller changes of the time-delay. It is considered that this is because the zero-order-hold assumption is applied to he time-delayed values in the simulation. that is, there are some errors that the values of time-delay are not exactly followed by the sampling period in the continuous-time space, because the values of the variable time-delay in the discrete-time space is constant in the sampling period while the values are maintained at the point of time of the sampling In addition, it is shown that the errors of the delayed valued affect the calculation of the state values so that the state errors will occur. As seen from the preceding investigations, we see that the performance of discretizing algorithms for the values of the variable time-delay in the study on the dicretizing of the nonlinear system with variable time-delay is very important.

### 5.3 Scaling and squaring technique

In this section, we will show that 'scaling and squaring' technique the system more precise results in discretization when sampling period of system is greater than proper one.

Consider the nonlinear CSTR systems as following:

$$x'(t) = -x^2(t) - 3x(t) + u(t - D(t))(1 - x(t))$$

Sampling period is assumed to be *T=2sec*. According to [13], the proper sampling period of CSTR should be less than 0.6 sec, so our assumption is reasonable. Fig. 14 shows the result of discretization with scaling and squaring technique. As shown at Fig 14, the maximum error is less than $10^{-4}$. Therefore, 'scaling and squaring' technique is useful when the system has a large sampling period.

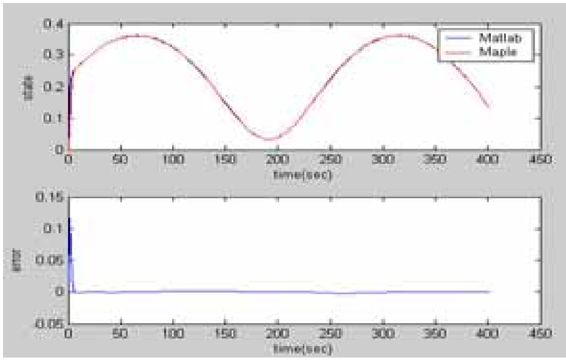


Fig. 14 Results of scaling and squaring technique

## 6. CONCLUSIONS

This paper proposes a discretizing method for the nonlinear system that has a variable time-delay input using a Taylor series expansion. The computer simulations with various examples are applied to verify the proposed algorithms. The time-delay is to be restricted to less than twice of the sampling period for each simulation commonly, and the ZOH assumption is applied for the discretizing of the variable time-delay for each sampling period. From the results of the experiments, the values of state error will be decreased by the smaller sampling period of the system. In case of the same sampling period, the state errors are decreased more by the smaller changes of time-delay. By following the results of the simulation, the maximum state errors do not exceed over 1% for the state values in the continuous-time space, and the errors show around $10^{-2}$% compared with the system that has a proper sampling period and time-delay. In addition, when the sampling period of discretization is greater than a proper one, we showed that 'scaling and squaring' technique can be used to get a discretization algorithm which shows better results. Moreover, it is also shown that the discetizing algorithms proposed in this paper make a satisfactory result, which can be verified by the RMS values for each error. In order to reduce the maximum state errors, we would like to apply the first-order-hold assumptions for the values of variable time-delay as a study in the future.

### ACKNOWLEDGMENTS

This work was supported by Korea Science and Engineering Foundation (KOSEF).

## REFERENCES

[1] M. Boutayeb, Observer design for linear time-delay systems, *System & Control Letters*, 2001

[2] Ren C. Luo, Long-Yen Chung, Stabilization for Linear Uncertain Systems with Time Latency, *IEEE Transaction on Industrial Electronics*, 2002

[3] M. Nihtila, T. Damak, J. P. Babary, On-line estimation of the time delay via orthogonal collocation, *Simulation Practice and Theory*, 1997

[4] Heung-Jae Im, Wan-Kyun Chung, and Il-Hong Suh, Predictive control of bilateral teleoperation with short-time delay, *Journal of Control, Automation and Systems*

[5] Chen, C. T., *Linear system Theory and Design.* Holt, Rinhart and Winston, Orlando, 1984

[6] Grobner, W., *Die Lie-Reihen und ihre Anwendungen.* VEB Deutscher Verlag der Wissenschaften, Berlin, 1967

[7] Hohmann, S., Konarad, A. and Krebs, V., Exact sampled-data representation of continuous-time nonlinear systems by finite polynomials with exactly determined coefficients, *Proceedings of the 2001 American Control Conference*, Vol. 2, pp. 1628-1633, 2001

[8] Franklin, G. F., Powell, J. d. and Workman, M. L., *Digital Control of Dynamic System*, Addison-Wesley, New York, 1988

[9] Vaccaro, R. J., *Digital Control*, McGraw-Hill, New York, 1995

[10] Isidori, A., *Nonlinear Control Systems: an introduction.* Springer-Verlag, Berlin, 1989

[11] Vidyasagar, M., *Nonlinear System Analysis*, Prentice Hall, Englewood Cliffs, 1978

[12] Ji Hyang Park, K. T. Chong, Kazantzis, Parlos, "Time Discretization of Nonlinear Systems with Delayed Multi-input Using Taylor Series", *International Journal of KSME*, Vol. 18, No. 7, pp. 1107-1120, 2004

[13] Ji Hyang Park, K. T. Chong, Kazantzis, Parlos, "Time discretization of non-affine nonlinear systems with delayed input using Taylor series", *International Journal of KSME*, Vol. 18, No. 8, pp. 1297-1305, 2004