

On-line System Identification using State Observer

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Abstract: This paper deals one of the methods of system identification, especially on-line system identification in time-domain. The algorithm in this study needs all states of the system as well input to it for system identification. In this reason, Kalman filter is used for state estimation. But in order to implement a state estimator, the fact that a system model must be known is logical contradiction. To overcome this, state estimation and system parameter estimation are performed simultaneously in one sample. And the result of the system parameter estimation is used as basis to state estimation in next sample. On-line system identification comes, in every sample by performing both processes of state estimation and parameter estimation that are related mutually and recursively. This paper demonstrates the validity of proposed algorithm through an example of an unstable inverted pendulum system. This algorithm can be useful for on-line system identification of a system that has fewer number of measurable output than system order or number of states.

Keywords: system identification, on-line system identification, parameter estimation, state observer, Kalman filter

1. INTRODUCTION

In general, system identification is finding out mathematical system model through experiment data. There are too many methods of system identification. They can be classified in main two categories. The first one is frequency-domain system identification and another is time-domain system identification. In this paper time-domain system identification is dealt. In other words, this paper is focused on estimating parameters of system model in time-domain.

In system identification process the most important thing is what model structure should be chosen to represent a real system. The system identification method is determined after system model structure would be fixed. But a few system identification methods in time-domain cannot choose model structure. It is possible to say that they would rather be able not to be determined as a structure, which stands for a real system more properly. For example, we have a system but we don't have any information about it except input data and output data. In this situation we can find out a mathematical model about this system using some system identification methods. But it is very hard to get system model as a meaningful structure, that is, controllable canonical form or observable canonical form. The output data have physical properties, neither do the states in identified system because we cannot assign model structure to have states with physical properties. This problem arises in the methods that system parameter is estimated through singular value decomposition (SVD) of Markov parameters such as eigen system realization (Juang, 1994) and subspace identification (Overschee and Moor, 1996), etc. Due to this reason, if it is possible to identify a system to mathematical model we want to, it is really useful to understand system's behaviour and design a controller to satisfy specified performance.

The reason why this study deals with on-line system identification is that because the data used in parameter estimation are not only output data from real system also estimated states. In order to identify the real system as a system model having the structure we want, all states must be

measured through output. But it is possible in not all system. A state observer is used in order to estimate states in the system that not all states can be measured as output. These estimated states are used instead of the immeasurable states through output. So mathematical system model is needed to implement state observer but nothing is known about system except input data and output data. To work out this problem, on-line system identification method is used. On-line system identification means that acquiring data to be used in parameter estimation and identification process are performed simultaneously in the same sample.

2. SYSTEM IDENTIFICATION

2.1 System parameter estimation state model

This study is focused on linear time-invariant (LTI) system in discrete time only. LTI system model is subjected as follows

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k), \quad (1)$$

$$\underline{y}(k) = C\underline{x}(k). \quad (2)$$

In (1) and (2), the parameters that should be estimated are system matrixes A, B and C . Using (1) and (2), if transform those equations into more convenient form,

$$\begin{aligned} \underline{y}(k) &= C A \underline{x}(k-1) + C B \underline{u}(k-1) \\ &= [C \cdot A \quad C \cdot B] \begin{bmatrix} \underline{x}(k-1) \\ \underline{u}(k-1) \end{bmatrix} \end{aligned} \quad (3)$$

Taking transpose in (3),

$$\underline{y}'(k) = [\underline{x}'(k-1) \quad \underline{u}'(k-1)] [C \cdot A \quad C \cdot B]'. \quad (4)$$

Now suppose that available sample data is from $k=1, 2, \dots, N$. Next equation is accomplished.

$$_N = \begin{bmatrix} \underline{y}'(1) \\ \underline{y}'(2) \\ \vdots \\ \underline{y}'(N-1) \\ \underline{y}'(N) \end{bmatrix} = \begin{bmatrix} \underline{x}'(0) & \underline{u}'(0) \\ \underline{x}'(1) & \underline{u}'(1) \\ \vdots & \vdots \\ \underline{x}'(N-2) & \underline{u}'(N-2) \\ \underline{x}'(N-1) & \underline{u}'(N-1) \end{bmatrix} \begin{bmatrix} CA & CB \end{bmatrix} \quad (5)$$

Substituting system matrixes with θ ,

$$_N = \Phi_N^T \theta \quad (6)$$

With regression matrix Φ_N^T , (data matrix)

$$\Phi_N^T = \begin{bmatrix} \underline{x}'(0) & \underline{u}'(0) \\ \underline{x}'(1) & \underline{u}'(1) \\ \vdots & \vdots \\ \underline{x}'(N-2) & \underline{u}'(N-2) \\ \underline{x}'(N-1) & \underline{u}'(N-1) \end{bmatrix} \quad (7)$$

With (5)-(7), we can estimate the system parameter as θ by least-square estimation (LSE) method

$$\theta = [C \cdot A \quad C \cdot B]^T = (\Phi_N^T \Phi_N)^{-1} \Phi_N^T \quad (8)$$

As previously states all of the state is possible to measure through output, that is, C is identity matrix, estimated parameters in (8) changes to

$$\theta = \begin{bmatrix} A & B \end{bmatrix}^T = (\Phi_N^T \Phi_N)^{-1} \Phi_N^T \quad (9)$$

But if not all states can be measured, that is, C is not identity matrix or number of output is less than system order, estimated parameters θ cannot cover whole parameters in real system. To prove this problem, estimated states can be used in regression matrix (7) instead of state in regression matrix

2.2 State estimation for parameter estimation

Now suppose estimated state as $\underline{x}(k)$, then we get next new regression matrix,

$$\Phi_N^T = \begin{bmatrix} \underline{x}'(0) & \underline{u}'(0) \\ \underline{x}'(1) & \underline{u}'(1) \\ \vdots & \vdots \\ \underline{x}'(N-2) & \underline{u}'(N-2) \\ \underline{x}'(N-1) & \underline{u}'(N-1) \end{bmatrix} \quad (10)$$

Using this regression matrix, system parameter is estimated in similar method with (8) and (9). To estimate state, so many state observers are useful, e.g. Kalman filter, Luenberger observer etc. In this paper Kalman filter is used for state estimation.

2.3 On-line system identification

In this time you can notice that there are no ways to

implement state observer whatever the observer is. That is, the system model is known to implement all of the state observers including Kalman filter. But anything is not known about the system except input and output data of the system. In other words, any state observer that is necessary to system identification (system parameter estimation) cannot be implemented. On-line system identification is possible to apply to solve this contradiction. On-line system identification is a recursive method that estimates system parameters using sampled data in every sample. As applying this on-line system identification method, we can predict all states in the system with the results of estimation of parameters in every sample time. Under this situation, the LSE method should be changed into recursive form in order to be able to estimate parameters in every sampling time. That is, we use recursive LSE (Ljung, 1999) to estimate parameters of the system. But parameter estimation is possible with Kalman filter. So Kalman filter is used also in system parameter estimation. The equations of Kalman filter for parameter estimation are as follows.

$$\theta(k+1) = \theta(k) + K_p(k)[\underline{y}(k+1) - \underline{y}_p(k+1)] \quad (11)$$

$$K_p(k+1) = P_p(k)C^T[C^T P_p(k)C' + R_p(k)]^{-1} \quad (12)$$

$$P_p(k+1) = [I - K_p(k+1)C']P_p(k) \quad (13)$$

$$\theta = \begin{bmatrix} A & B \end{bmatrix}^T \quad (14)$$

where $\underline{y}_p(k+1) = [\underline{x}(k) \quad \underline{u}(k)] \cdot \theta(k)$ and $C = I_n$, n is system order. Eventually Kalman filter for parameter estimation (11)-(14) brings to same results as those of recursive LSE. After system parameter estimation by Kalman filter, estimated parameters are used again in state estimation by another Kalman filter that is a state observer in same sampling time. Another Kalman filter for state estimation is as follows.

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) \quad (15)$$

$$P(k+1) = AP(k)A' + \Gamma Q(k)\Gamma' \quad (16)$$

$$\underline{x}(k+1) = \underline{x}(k+1) + K(k+1)[\underline{y}(k+1) - \underline{y}(k+1)] \quad (17)$$

$$K(k+1) = P(k+1)C^T[C^T P(k+1)C' + R(k+1)]^{-1} \quad (18)$$

$$P(k+1) = [I - K(k+1)C']P(k+1) \quad (19)$$

where $\underline{y}(k+1) = C \cdot \underline{x}(k+1)$.

In brief, we can estimate parameters in system by on-line through (11)-(14) using input and output data from real system at present sample time k . After that, the result from parameter estimation is based to estimation of states that will be caused in next sample time $k+1$ through (15)-(19). These states estimation results from (15)-(19) will be referred again by parameter estimation process at the next sample time $k+1$. As these recursive processes are repeated, system parameter estimation results will be convergent to the real value of parameters of system.

3. SIMULATION AND RESULTS

In this section, we validate the algorithm described in previous section by simulation. The plant used in simulation is inverted pendulum system SIP02 manufactured by Quanser

Industrial. Inverted pendulum is nonlinear system inherently, but we identify the nonlinear inverted pendulum system as linear system. That is, the model structure to be identified is linear model. This system has one input and four outputs and the system order is 4. First of all, we need to have nonlinear dynamic equation of inverted pendulum. The nonlinear dynamic equation is referred in a manual of SIP02.

Inverted pendulum is an unstable system, so we need stabilizing controller in order to acquire data to be used in parameter estimation safely. A stabilizing controller is constructed by LQR controller, and LQR gain is determined base on the parameters in Quanser manual. Next figure is a block diagram made with Simulink model.

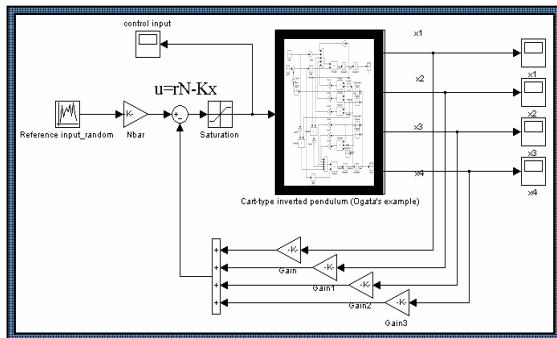


Fig. 1 Simulink model of nonlinear inverted pendulum system with LQR stabilizing controller.

In figure 1 the data used in parameter estimation are input to the nonlinear system and output from it. The input data is control input from the stabilizing controller to nonlinear system model, and the output data are the system outputs that are composed 4 elements - position of cart, velocity of cart, angle of pole and angular velocity of pole.

The results of the parameter estimation of the linearized system with this nonlinear system are listed in next table.

Table 1 Comparisons parameters in matrix and eigen value of real linearized system with identified system.

	Linearized System	Identified System
A	$\begin{bmatrix} 1.0013 & 0.01 & 0 & -0.0013 \\ 0.2591 & 1.0005 & 0 & -0.2534 \\ 0.0001 & 0.0000 & 1.0000 & 0.0094 \\ 0.0144 & 0.0000 & 0 & 0.8900 \end{bmatrix}$	$\begin{bmatrix} 1.0010 & 0.0099 & 0.0000 & -0.0012 \\ 0.1545 & 0.9721 & 0.0005 & -0.2159 \\ -0.0001 & -0.0000 & 1.0000 & 0.0095 \\ -0.0326 & -0.0123 & -0.0001 & 0.9062 \end{bmatrix}$
B	$\begin{bmatrix} 0.0002 \\ 0.0333 \\ 0.0001 \\ 0.0144 \end{bmatrix}$	$\begin{bmatrix} 0.0002 \\ 0.0294 \\ 0.0001 \\ 0.0128 \end{bmatrix}$
Eigenvalues	<p>1.0000 1.0494 0.9556 0.8868</p>	<p>1.0000 1.0421 0.9572 0.8799</p>

The sample time for this simulation of system identification is 0.01 second, and systems listed in Table 1 are discrete-time systems. The initial conditions are listed next table.

Table 2 Initial values of system parameters and states.

\hat{A}_0	\hat{B}_0	\hat{x}_0	$\hat{\dot{x}}_0$
$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$

As we can see Table 1 and 2, the parameters in identified system are almost identical with the ones in real system. And the convergence of estimated parameters can be confirmed by depicting the variation of the elements in the matrixes A and B as time goes.

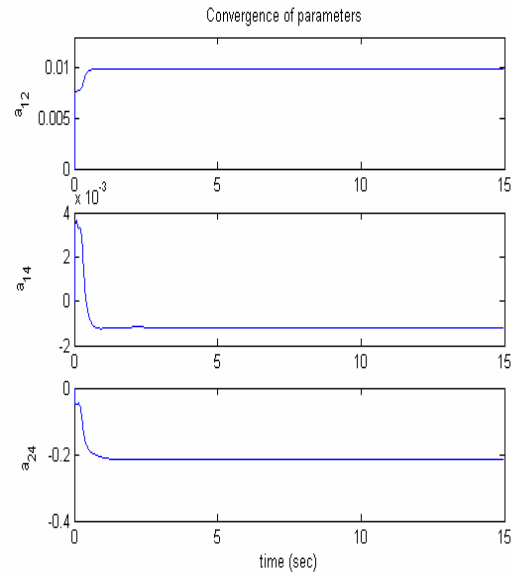


Fig. 2. Convergence of parameters a_{12} , a_{14} and a_{24} .

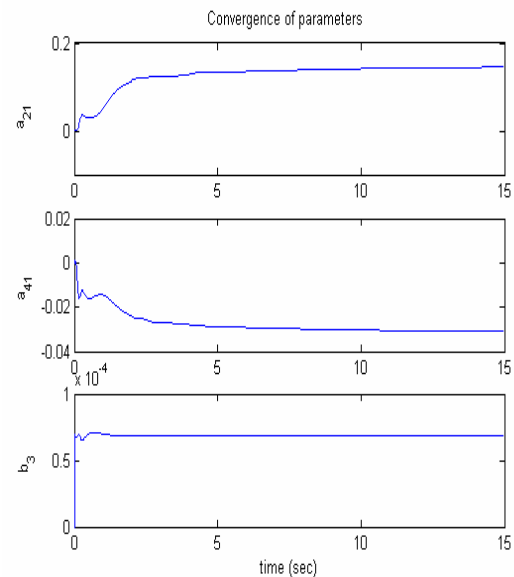


Fig. 3. Convergence of parameters a_{21} , a_{41} and b_3 .

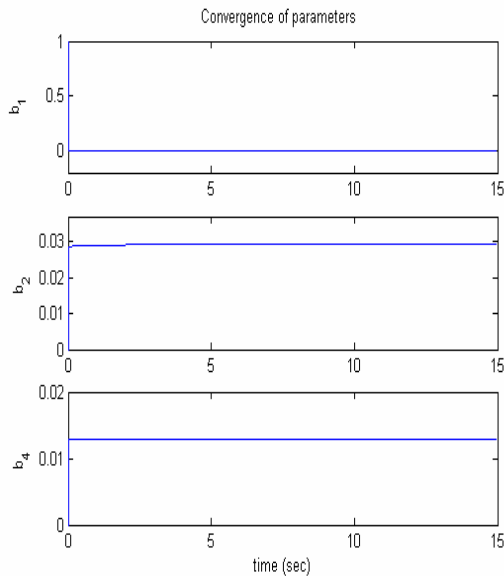


Fig. 4. Convergence of parameters b_1 , b_2 and b_4 .

Table 3 Definition of each parameter in matrix.

\hat{A}_0	\hat{B}_0
$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$	$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$

The graphs show that all elements in each matrix well converge. This means that method to estimate parameters of system using states estimated through state observer is indeed reasonable.

4. CONCLUSION

Till now we studied on-line system identification with state observer. This study confirmed proposed algorithm through system identification of the nonlinear inverted pendulum system into linearized system. Even though the example it estimation of parameters of the system whose state is measured by output, if we alternate the proposed algorithm, parameters of the system that not all states can be measured by output in, is possible to be estimated.

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