Two-Stage Estimator Design Using Stable Recursive FIR Filter and Smoother

Jong-Ju Kim* , Jae-Hun Kim**, and Joon Lyou***

*Agency for Defense Development, Yuseong P.O. Box 35-3, Taejeon 305-152, S. Korea
(Tel: +82-42-821-4412; Fax: +82-42-821-2224; Email: jjkim2261@hanmail.net)
**Agency for Defense Development, Yuseong P.O. Box 35-1, Taejeon 305-152, S. Korea
(Tel: +82-42-821-3172; Fax: +82-42-821-2221; Email: kjh1132@hanafos.com)
***Department of Electronics Engineering, Chungnam National University, Taejeon 305-764, S. Korea
(Tel: +82-42-821-5669; Email: jlyou@cnu.ac.kr)

Abstract: FIR(Finite Impulse Response) filter is well known to be ideal for the finite time state-space model, but it requires much computation due to its inherent non-recursive structure especially when the measurement interval grows to a large extent. And often a fixed-lag smoother based on the finite time interval is needed to monitor the soundness of the system model and the measurement model, but the computation burden of FIR-type smoother imposes much restriction of its usage for real-time application. Conventional recursive forms of FIR estimator[1]-[4] could not be used for real time applications, since they are numerically unstable in their recursive equations. To cope with this problem, we suggest a stable recursive form FIR estimator(SRFIR) and its usefulness is demonstrated for designing the real-time fixed-lag smoother on the finite time window through an example of detection of rate bias in the anti-aircraft gun fire control system.

Keywords: Finite Impulse Response(FIR), Recursive form of FIR filter(RFIR), Stable RFIR (SRFIR), Fixed-lag Smoother, Sub-optimal fixed-lag smoother, Anti-Aircraft Gun Fire Control System(AAGFCS)

1. Introduction

This paper deals with the problem of designing a recursive form of FIR filter and smoother for discrete-time state-space models with system noises. FIR filter has been known to be ideal in the finite time-basis model which shows the characteristics of BLUE(Best Linear Unbiased Estimator) with the fast response time and the BIBO(Bounded Input/Bounded Output) stability to the parameter changes[1]-[3]. However its critical drawback lies in its inefficient computation capability due to its non-recursive structure.
Bruckstein[4] derived first a recursive form of the finite time filter using the scattering theory and Kwon[1] later derived a recursive form of FIR filter using the orthogonal property of the estimated error to the measurement vector. But those recursive forms include the numerically unstable components in its smoother part and require re-initialization of the filter periodically to prevent divergence[1],[4],[5], which make those algorithms difficult to use in real time applications. So we suggested a stable recursive form of FIR filter which could approximate the FIR filter of minimum variance[6]-[8]. For detection of rate measurement bias in the Anti-Aircraft Gun Fire Control System(AAGFCS), we propose a new sub-optimal fixed-lag smoother which shows the least computation, since it results from combining the stable recursive form of FIR filter and FIR smoother using the information fusion approach[9]-[11].

2. Recursive-form FIR Filter

Kwon[1] derived a recursive form FIR estimator(RFIR) which uses the results of FIR filter ($\hat{x}(i|i;N)$) and FIR smoother(backward filter $\hat{x}(i-N|i;N)$) simultaneously, and it is expressed in a parametic form of the gains of FIR filter and FIR smoother designed on the finite measurement window $[i-N, i]$. And the equation of RFIR is composed of two coupled states of the forward FIR filter and backward FIR filter. Since the gains of RFIR can be expressed in a parametric form, Kwon's RFIR is introduced in this section. The system model is a linear discrete time-invariant state-space equation described as

$$x(i+1)=Ax(i)+Bw(i) \tag{1}$$
$$y(i)=Cx(i)+v(i) \tag{2}$$

where $x(i) \in R^n$ is the state vector for time index i, $w(i) \in R^p$, $v(i) \in R^m$ are zero mean white Gaussian noises and mutually uncorrelated with variances of Q and R, respectively. And $w(i)$, $v(i)$ are uncorrelated with initial state $x(0)$.
The recursive form of the augmented FIR filter and FIR smoother has been given as follows[1]

$$\hat{X}(i+1;N)=L\,\hat{X}(i;N)+K\,Y(i) \tag{3}$$

where

$$\hat{X}(i;N)=\begin{bmatrix}\hat{x}(i|i;N)\\ \hat{x}(i-N|i;N)\end{bmatrix} \tag{4}$$

$$Y(i)=\begin{bmatrix}y(i)\\ y(i-N)\end{bmatrix} \tag{5}$$

$$L=\begin{bmatrix}L_{11}L_{12}\\ L_{21}L_{22}\end{bmatrix} \tag{6}$$

$$K=\begin{bmatrix}K_{11}K_{12}\\ K_{21}K_{22}\end{bmatrix} \tag{7}$$

Each component of the matrix L and K is defined as follows

$$L_{11}=M_1, \quad L_{12}=M_1H(N)CM_2$$

$$L_{21}{=}{-}G(0)CA, \quad L_{22} = M_3$$
$$K_{11}{=}H(0), \quad K_{12} = -M_1 H(N)[I + CM_2 G(N)]$$
$$K_{21}{=}G(0), \quad K_{22} = G(0)CAH(N) - M_3 G(N)$$

where

$$M_1{=} \ [I - H(0)C]A$$
$$M_2{=} \ [I - G(N)C]^{-1}$$
$$M_3{=} \ [\bar{P}^{-1}(0)A - G(0)CAH(N)] \ M_2$$
$$\bar{P}(0){=}I - BQB^T P^{-1}(0)$$

And the gain matrices $G(0), H(0), G(N)$, and $H(N)$ are determined by the recursive algorithm suggested in [1].

3. A Stable RFIR

The RFIR described in the previous section implies an unstable part, and the eigenvalues of L_{22} are located outside the unit circle. We proposed a stabilizing solution to this RFIR in [8] , and summarize the results in this section.

The computation of RFIR through the equation of (3) can not be perfect considering possible computation error existing in the real world. If there exists some error during computation of the estimate, then the computed state diverges from the true value due to instability of L_{22}. Considering this effect the introduction of the random error into the smoother part can be used to construct a stable estimator while taking RFIR itself as a unstable plant. Therefore the system model for SRFIR(Stable RFIR) is taken as

$$\hat{X}(i+1;N){=}L \ \hat{X}(i;N) + K \ Y(i) + \bar{U}(i) \tag{8}$$

where $\bar{U}(i) = \begin{bmatrix} 0 \\ u(i) \end{bmatrix}$ is added to reflect the modelling error of the equation. Here we take $u(i)$ as a zero-mean white gaussian vector ,which has no correlation with $w(i), v(j)$ and $x(0)$ for all i, j , with covariance of $E[u(i)u(i)^T] = Q_c$. Further Q_c can be taken arbitrarily so that $(L_{22}, \sqrt{Q_c})$ may meet the controllable condition.

To construct the estimator, the measurement equation at time $i - N$ can be used as

$$y(i-N){=}Cx(i-N) + v(i-N) \tag{9}$$

In (8) and (9) if (C, L_{22}) meets the observable condition we can construct the stable state estimator for the RFIR such as

$$\bar{X}(i{+}1){=}L\bar{X}(i){+}KY(i){+}\bar{\lambda}[y(i{-}N){-}\bar{y}(i{-}N)] \tag{10}$$
$$\bar{y}(i-N){=}\bar{C}\bar{X}(i) \tag{11}$$

where

$$\bar{\lambda}{=} \begin{bmatrix} 0 \\ \lambda \end{bmatrix}, \ \ \bar{C} = \begin{bmatrix} 0 \ C \end{bmatrix} \tag{12}$$

In fact the observable condition of (C, L_{22}) was proved using the equation of L_{22}[8]. The error equation of SRFIR is obtained by subtracting (10) from (8) such that

$$\widetilde{e}(i+1) \equiv \hat{X} - \bar{X} = (L - \bar{\lambda} \ \bar{C}) \ \widetilde{e}(i) - \bar{\lambda} \ \bar{v}(i) + \bar{U}(i) \tag{13}$$

where $\quad \bar{v}(i) = y(i-N) - \bar{y}(i-N)$

Taking expectation of above equation we can get the mean equation of the estimator such that

$$M_e(i+1) = (L - \bar{\lambda}\bar{C})M_e(i) \tag{14}$$

And the covariance equation can be derived as follows

$$\begin{aligned} \widetilde{P}_e(i+1) = \ & \bar{L}\widetilde{P}_e(i)\bar{L}^T + \bar{\lambda}\bar{R}\bar{\lambda}^T + \bar{Q} \\ & -\bar{L}E[\widetilde{e}(k)\bar{v}(i)^T]\bar{\lambda}^T - \bar{\lambda}E[[\bar{v}(i)\widetilde{e}(i)^T]\bar{L}^T \\ & +\bar{L}E[\widetilde{e}(i)\bar{U}(i)^T] + E[\bar{U}(i)\widetilde{e}(i)^T]\bar{L}^T \\ & -E[\bar{U}(i)\bar{v}(i)^T]\bar{\lambda}^T - \bar{\lambda}E[\bar{v}(i)\bar{U}(i)^T] \end{aligned} \tag{15}$$

where

$$\bar{L}{=}L - \bar{\lambda}\bar{C}$$
$$\bar{R}{=}E[\bar{v}(i-N)\bar{v}(i-N)^T]$$
$$\bar{Q}{=}\begin{bmatrix} 0 & 0 \\ 0 & Q_c \end{bmatrix}$$

Similar to [8] all the correlation terms can vanish using the asymptotically zero mean property of the error and non-correlation condition of $u(i)$ with other variables. Hence the covariance equation can be simply represented by

$$\widetilde{P}_e(i+1){=}\bar{L} \ \widetilde{P}_e(i)\bar{L}^T + \bar{\lambda}\bar{R}\bar{\lambda}^T + \bar{Q} \tag{16}$$

The above equation is the well-known Lyapunov equation, and from the Kalman filter theory the limiting positive definite optimal covariance exists if and only if the unstable component of L belongs to the observable and controllable subspace by the measurement and the system model error(process noise), respectively[9]. From the fact that $(L_{22}, \sqrt{Q_c})$ can be arbitrarily taken to meet the controllable condition and (C, L_{22}) meets observable condition only if (C, A) meets the observable property, the limiting positive definite covariance exists.

To find the optimal weight matrix of λ we choose the performance index as the minimum covariance, that is the trace of the covariance of the estimate error. It can be derived as following.

$$\begin{aligned} \widetilde{P}_e(i+1) = \ & \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22}{-}\lambda C \end{bmatrix} \widetilde{P}_e(i) \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22}{-}\lambda C \end{bmatrix}^T \\ & + \begin{bmatrix} 0 & 0 \\ 0 & \lambda\bar{R}\lambda^T \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & Q_c^T \end{bmatrix} \end{aligned} \tag{17}$$

The trace of the covariance is $J(\lambda) = trace(\widetilde{P}_e(i+1))$. And the variation of J with respect to λ can be written by

$$\begin{aligned} \delta J(\lambda) = \ & 2tr \begin{bmatrix} 0 & 0 \\ 0 & -\delta\lambda C \end{bmatrix} \widetilde{P}_e(i) \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22}{-}\lambda C \end{bmatrix}^T \\ & + \begin{bmatrix} 0 & 0 \\ 0 & \delta\lambda\hat{R}\lambda^T \end{bmatrix} \\ & = -2tr\delta\lambda[C\{\widetilde{P}_{21}(i)L_{21}^T + \widetilde{P}_{22}(i)(L_{22}^T - C^T\lambda^T \ )\} - \bar{R}\lambda^T] \end{aligned} \tag{18}$$

For $\delta J(\lambda) = 0$, it follows that

$$\lambda = (L_{21}\widetilde{P}_{21}(i)^T + L_{22}\widetilde{P}_{22}(i)^T)C^T(CP_{22}C^T + \bar{R})^{-1} \tag{19}$$

To reduce the computation burden, we can use a constant weight matrix which can be computed beforehand. It is well-known that the constant weight matrix also ensures stability but results in some degraded performance during the transition period. To complete the SRFIR design, we have to determine the amount of variance of the system model error (Q_c),normally it is called the procedure of *"filter tuning"*. Through this procedure we can achieve the optimal performance of SRFIR.

By the way the above covariance means just the covariance of the relative error between the FIR filter and the SRFIR, it is necessary to compute the amount of the real covariance of the error between the true state($X(i)$) and the estimate of SRFIR ($\bar{X}(i)$). We can describe the relation here as following Let's denote

$$\bar{e}(i) = X(i) - \bar{X}(i) \tag{20}$$

$$\hat{e}(i) = X(i) - \hat{X}(i) \tag{21}$$

$$\widetilde{e}(i) = \hat{X}(i) - \bar{X}(i) \tag{22}$$

Then

$$P_{SRFIR}(i) = E[\bar{e}(i)\bar{e}(i)^T] \tag{23}$$

$$P_{FIR}(i) = E[\hat{e}(i)\hat{e}(i)^T] \tag{24}$$

$$P_e(i) = E[\widetilde{e}(i)\widetilde{e}(i)^T] \tag{25}$$

And it follows that

$$\begin{aligned}\bar{e}(i)=&X(i) - \bar{X}(i) \\ =&[X(i) - \hat{X}(i)] + [\hat{X}(i) - \bar{X}(i)] \\ =&\hat{e}(i) + \widetilde{e}(i)\end{aligned} \tag{26}$$

Hence

$$\begin{aligned}P_{SRFIR}(i)=&E[\bar{e}(i)\bar{e}(i)^T] \\ =&E[\hat{e}(i) + \widetilde{e}(i)][\hat{e}(i) + \widetilde{e}(i)]^T \\ =&E[\hat{e}(i)\hat{e}(i)^T + \widetilde{e}(i)\widetilde{e}(i)^T \\ &+\hat{e}(i)\widetilde{e}(i)^T + \widetilde{e}(i)\hat{e}(i)^T]\end{aligned} \tag{27}$$

Using the fact that the error of FIR filter is orthogonal to the measurement vector and $\bar{X}(0)$[1], it can be shown that in the steady state condition,

$$\begin{aligned}\lim_{i\to\infty} E[\hat{e}(i)\widetilde{e}(i)^T]=&\lim_{i\to\infty} E[\hat{e}(i)\hat{X}(i)^T] \\ &- \lim_{i\to\infty} E[\hat{e}(i)\bar{X}(i)^T] \\ =&0\end{aligned} \tag{28}$$

Therefore the following relation holds true for the steady state

$$P_{SRFIR}(\infty) = P_{FIR}(\infty) + P_e(\infty) \tag{29}$$

4. A suboptimal fixed-lag smoother using SRFIR

The state of RFIR is made up of two state vectors which are jointly coupled each other by the equation of RFIR written in (3). That is

$$\hat{X}(i)=\begin{bmatrix}\hat{x}(i|i;N) \\ \hat{x}(i-N|i;N)\end{bmatrix} \tag{30}$$

The measurement window of RFIR corresponds to $[i-N, i]$. We can simply construct the N-frame fixed-lag smoother exploiting the structure of RFIR. Let the estimate and covariance of the filter and the smoother for the time $i - N$ as $x_f(i-N), P_f(i-N)$ and $x_s(i-N), P_s(i-N)$, respectively. The optimal smoother can be derived through combining the information of the filter and smoother to meet the statistically optimal criterion such as the least square method or the information fusion[9],[10]. Hence the equation of N-frame fixed-lag smoother defined in the time interval $[i-2N, i]$ can be given as following[6]

$$\begin{aligned}x^*(i-N)=&P(i-N)[P_f^{-1}(i-N)x_f(i-N) \\ &+P_s^{-1}(i-N)x_s(i-N)]\end{aligned} \tag{31}$$

where

$$P^{-1}(i-N)=P_f^{-1}(i-N) + P_s^{-1}(i-N) \tag{32}$$

Replacing the equation of RFIR as that of SRFIR we can get the suboptimal fixed-lag smoother which is numerically efficient combining the recursive equation of SRFIR. The computational advantage of the proposed fixed-lag smoother can be utilized maximally for real time computation in a suboptimal sense.

5. Two Stage Estimator

We consider a problem of estimating target states in the anti-aircraft gun fire control system. If rate measurement sensor becomes biased under some operational situation, the bias can be eliminated by the estimator which uses the position measurement only combining it with the detection mechanism of the rate estimate bias[6]. However when the target model changes from the presumed model, it is not easy for the conventional-type filter which uses position measurement only to track the variable model properly and to eliminate the rate bias at the same time. The position filter has more elimination capability of the measurement noise proportional to the size of the time window, but it needs to have smaller window to be able to adapt to the model change. When the window size is reduced to deal with the increased dynamic bandwidth, it shows ill side-effects of differentiation rather than smoothing of measurement noise, for it is less aided by the information of the model dynamic.

But if we use the noncausal estimator such as the fixed-lag smoother based on the finite time model, it can solve the above two problems at a time even though the estimate is performed not at the current time but at the past time. The smoother can reject the dynamic model bias at the delayed time point, for it can estimate the model uncertainty from the time point of estimation to the current time point helped by the measurement information[9],[12]. Also it can achieve the good measurement noise rejection due to the increased information from past and future measurements simultaneously.

Nevertheless the fixed-lag smoother suffer time delay problem and it can not provide the current time estimate properly. So we take two different estimators. One is the FIR filter using position and velocity(PVFIR), which is used as a main target state estimator(MTSE). The other one is the SRFIR-type fixed-lag smoother using only position, which is used as an auxiliary target state estimator(ATSE) to compute the rate estimate error of MTSE. In case that the rate bias is sufficiently slowly time varying compared with the fixed-lag time interval of the smoother, we can effectively compensate for the PVFIR filter by scaling the computed rate bias using the weight between 0 and 1.0 depending on the changing speed of the rate bias. See Figure 1 for the structure of this two stage estimator.

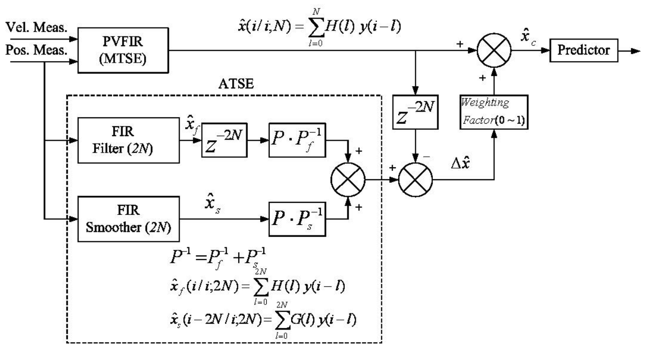

Fig. 1. Structure of Two Stage Estimator

5.1. Main Target State Estimator(MTSE)

The main target state estimator is designed to take the form of FIR filter which uses both position and velocity measurements in the interval of $[i-N, i]$. The filter model is defined in (1) and (2), and the size of measurement window(N) and the variance of the system model uncertainty(Q) is chosen simultaneously to minimize the covariance of the filtered estimate referenced to the given real system once the variance of the measurement sensor(R) is known a priori(filter tuning). To use unit variance matrix(I) instead of R, the measurement equation can be easily re-scaled for symmetric positive definite R matrix by using the decomposition of $R = \Lambda^T \Lambda$ [11].

The information of the initial filter state is considered as unknown zero while computing the FIR filter gain [1]-[3],[9],[11]. The estimate of the FIR filter using position and velocity(PVFIR) is given as

$$\hat{x}(i; N) = \sum_{l=0}^{N} H(l) y(i-l) \tag{33}$$

The filter gain can be computed by the algorithm suggested by [1]-[3],[6]-[8].

5.2. Auxiliary Target State Estimator(ATSE)

The ATSE is utilized to monitor and eliminate the bias of the rate estimate in the MTSE. This estimator, which takes the form of SRFIR fixed-lag smoother, can be easily designed by incorporating the SRFIR filter and the SRFIR smoother which are given in Section 4 using the information fusion approach[9],[11].

5.3. Comparison with the conventional fixed-lag smoother

Most of the conventional fixed-lag smoothers are designed using the all past measurement data of time interval $[0, i]$, deriving from the Kalman filter equations[9]-[11]. Such fixed-lag smoother can also be represented by the equation of (25) combining the two kinds of information[9],[10], replacing the forward filter by the Kalman filter which compute the filtered estimate at $i - 2N$ using $[0, i-2N]$ measurement data, and keeping the backward filter the same as the aforementioned FIR smoother.

However when the system model changes frequently, the performance of the conventional smoother may be poorer than that of FIR-type fixed-lag smoother which is based on the finite-time model. This is demonstrated by an example during review of simulation in the next section.

6. Simulation Results

The real measured target path is used to assess the effectiveness of the target state estimator. The target is a small remotely-controlled aircraft for test purpose, and it is often under the effect of wind or short period of maneuver. During simulation the sampling time T is 0.02 second, and N is chosen to be 16 points for MTSE and 64 points for ATSE (32 points each for FIR filter and smoother, respectively) from compromise between the performance and computational ease. Q has been tuned for the Kalman filter and the same value is used for FIR estimator, and R is selected unity taking the appropriate dimension.

We used the error of the aim point as a performance criteria to evaluate the effectiveness of the target state estimator in this simulation. Figure 2 shows the flight path in $3D$ space, and Figure 3 shows the time of flight that is used for prediction of the aim point.

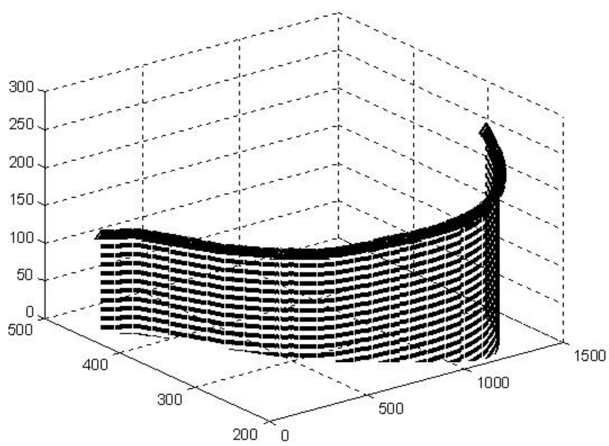

Fig. 2. 3D Target moving path

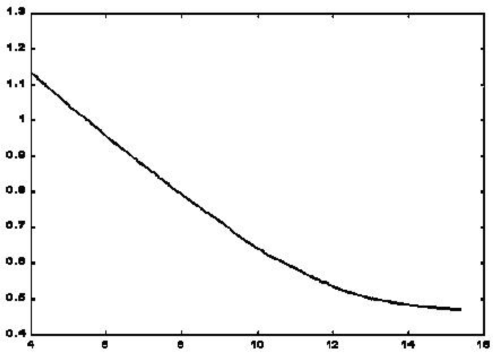

Fig. 3. Time of flight

The errors of the fixed-lag smoothers between the conventional type and SRFIR type are shown in Figure 4(azimuth) and Figure 5(elevation).

Figure 4, Figure 5:
1: SRFIR fixed-lag smoother
2: Conventional fixed-lag smoother

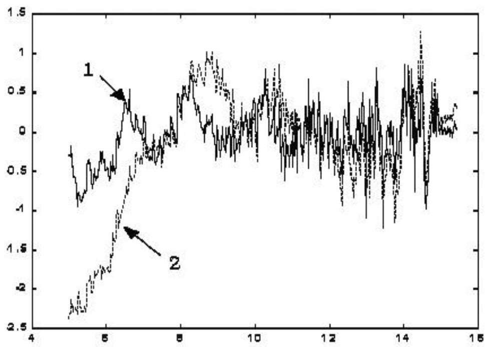

Fig. 4. Error of fixed-lag smoothers(azimuth)

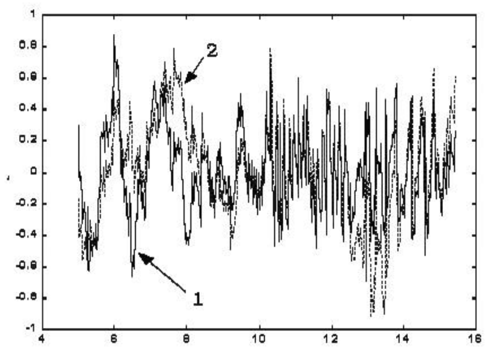

Fig. 5. Error of fixed-lag smoothers(elevation)

The proposed SRFIR fixed-lag smoother shows the superior performance to the conventional type which uses the all past measurements and can be regarded as the suboptimal fixed-lag smoother(SFS), since the latter one assumes the fixed system model(constant acceleration model) during the entire past period while the real motion conforms to the fixed model assumption only for a short duration. The difference of performance is contrasted in azimuth direction, for the target conducts the geometric turn mostly in such direction.

In Figure 6(azimuth) and Figure 7(elevation), the aim point error of the proposed two stage estimator is compared with that of the other kind Kalman filters.

Figure 6, Figure 7:
1: FIR fixed-lag smoother; ATSE
2: Main FIR filter(PVFIR, CA); MTSE
3: Two stage estimator(PVFIR, CA)
4: Kalman filter(PVKF, CV)
5: Kalman filter(PVKF, CA)

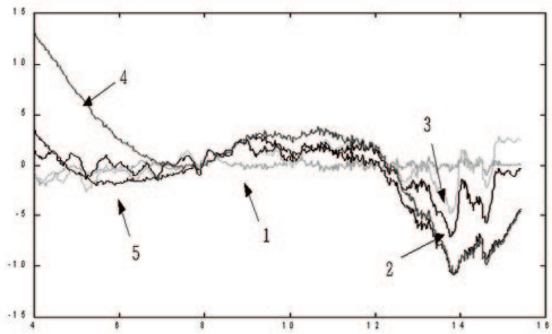

Fig. 6. Error of aim point(azimuth)

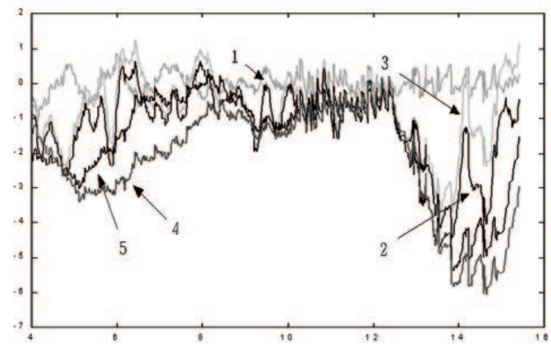

Fig. 7. Error of aim point(elevation)

The FIR filter using position and velocity based on the acceleration model which uses position and rate measurement shows the best performance compared with all Kalman filters such as PKF(position measurement) + CV(constant velocity model), PKF + CA(constant acceleration model), PVKF(position and velocity measurement) + CV, PVKF + CA in most of tracking time. However all filters including PVFIR show the aiming bias in the nearest crossing path due to mostly rate sensor bias. In this case the PVFIR + SFS(suboptimal fixed-lag smoother) can compensate the rate sensor bias well, and we can find the improved performance in Figures 6 and 7. The computed rate bias is made by comparing the rate estimate between PVFIR and SFS, and it is scaled down to half (0.5) and added to the current rate estimate of the MTSE considering the time varying effect of the rate estimate bias.

7. Conclusion

A two-stage estimator has been proposed which can achieve the fast tracking capability and eliminate the rate measurement bias at the same time. For the fast tracking capability we use the FIR-type main tracking filter which uses the position and rate measurement as well. To cancel the estimated rate bias error of the main filter when the rate measurements go biased, the second fixed-lag smoother using the position measurement only but using the wider measurement interval than the main filter has been used parallel to the main filter. To reduce the amounts of computation of the fixed-lag smoother, we proposed a suboptimal fixed-lag smoother using a stable recursive form FIR filter devised by ourselves. Through an application to the anti-aircraft gun fire control system, usefulness of our two-stage estimator has been demonstrated.

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