

Minimum Variance FIR Smoother for Model-based Signals

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Abstract: In this paper, finite impulse response (FIR) smoothers are proposed for discrete-time systems. The proposed FIR smoother is designed under the constraints of linearity, unbiasedness, FIR structure, and independence of the initial state information. It is also obtained by directly minimizing the performance criterion with unbiased constraints. The approach to the MVF smoother proposed in this paper is logical and systematic, while existing results have heuristic assumption, such as infinite covariance of the initial state. Additionally, the proposed MVF smoother is based on the general system model that may have the singular system matrix and has both system and measurement noises. Thorough simulation studies, it is shown that the proposed MVF smoother is more robust against modeling uncertainties numerical errors than fixed-lag Kalman smoother which is infinite impulse response (IIR) type estimator.

Keywords: Minimum variance FIR smoother, Unbiasedness property, State estimation, Receding horizon strategy.

1. Introduction

In recent years, FIR(finite impulse response) type estimator designs that meet desired specifications in the frequency or spatial domain have been widely investigated and are now well established in signal processing areas [1], [2], [3], [4], [5], [6], [7], [8]. The FIR type estimators have several design and implementation advantages against the IIR type estimators. The FIR structure makes use of finite measurements and inputs on the most recent time interval called the receding horizon or horizon, while the IIR(infinite impulse response) structure utilizes all information up to estimation time. Even though FIR type estimators have many advantages over IIR type estimators in the signal processing area, IIR type estimators have been researched and used more widely for model-based signals that can be represented as a predictable signal from certain models and formalize a prior knowledge. Actually, the Kalman filter with an IIR structure has been the standard choice for the state estimation in state space model, and thus has been an important tool for signal processing. However, as applications became more numerous, some pitfalls of the Kalman filter were discovered such as the problem of divergence due to the lack of reliability of the numerical algorithm or to inaccurate modeling of the system under considerations. Therefore, to overcome these demerit, the FIR type estimators have been used as an alternative to the IIR type estimator for the state estimation in state space model.

For discrete-time systems, the FIR smoother without *a priori* initial state information can be represented as

$$\hat{x}_{k-h} = \sum_{i=k-N}^{k-1} H_{k-i} y_i + \sum_{i=k-N}^{k-1} L_{k-i} u_i. \quad (1)$$

The IIR structure has a similar form to (1) with $k - N$ replaced by the initial time k_0 . In IIR and FIR types, the

initial state means x_{k_0} and x_{k-N} , respectively. A strong unbiased condition for the FIR smoother (1) can be represented as

$$E[\hat{x}_{k-h}] = E[x_{k-h}] \quad \text{for any } x_{k-N}. \quad (2)$$

The constraint (1) will prevent to obtain a reasonable solution. However, among linear unbiased FIR estimators satisfying the conditions (1) and (2), some kind of optimal estimators will be obtained depending on some performance criterion.

The recursive forms of FIR smoothers for state-space models are derived in [9], [10]. The optimal FIR smoother [11] was developed from a Fredholm integral equation with some complex boundary indices. The horizon initial state was given by the state propagator and the discrete Lyapunov equation or was assumed to be unknown. However, general readers might find it hard to understand the derivation of complicated estimation algorithm. To reduce the computational burden of the optimal FIR smoother in [9], [10], [11], the fast algorithm for optimal FIR smoother for general state-space model was developed in [12]. The horizon initial state was given by the state propagator and the discrete Lyapunov equation. The problem associated with this approach is that its performance depends heavily on the accuracy of the state propagator, which is seriously influenced by uncertainty in the system initial state. Furthermore, although above stochastic FIR smoothers [9], [10], [11], [12] were developed for general state space models with system and measurement noises, it is not clear to understand the optimality and has some heuristic assumptions such as infinite covariance of the initial state.

To the authors knowledge, there seems to be no result on unbiased linear FIR smoothers with a priori built-in unbiasedness regardless of the initial state information as well as a clear optimality for general state space models with both

system and measurement noises. Therefore, a new unbiased FIR smoother will be proposed in this paper. The proposed unbiased FIR smoother is to be represented in a batch form and then in a recursive form. The proposed unbiased FIR smoother in this approach is logical and systematic, while the previous approaches have some heuristic assumptions such as infinite covariance of the initial state. Additionally, it does not require the system matrix inversion, i.e., H_{k-i} and L_{k-i} of (1) will be represented without using the system matrices inversion while the FIR smoothers in previous approaches have assumed that the system matrix is nonsingular since it requires the system matrix inversion.

This paper is organized as follows. In Section 2, the MVF smoothers for general discrete-time system is proposed in a standard FIR form. The performance of the proposed MVF smoother will be compared with fixed-lag Kalman smoother in section 3. Finally, conclusions are presented in Section 4.

2. MVF Smoother for Discrete-Time System

Consider a linear discrete-time state space model:

$$x_{k+1} = Ax_k + Bu_k + Gw_k, \quad (3)$$

$$y_k = Cx_k + v_k, \quad (4)$$

where $x_k \in \mathfrak{R}^n$ is the state, $u_k \in \mathfrak{R}^l$ and $y_k \in \mathfrak{R}^q$ are the input and measurement, respectively. At the initial time k_0 of the system, the state x_{k_0} is a random variable with a mean \bar{x}_{k_0} and a covariance P_{k_0} . The system noise $w_k \in \mathfrak{R}^p$ and the measurement noise $v_k \in \mathfrak{R}^q$ are zero-mean white Gaussian and mutually uncorrelated. Q and R denote the covariances of w_k and v_k which are assumed to be positive definite matrices. These noises are uncorrelated with the initial state x_{k_0} . (A, C) of the system (3)-(4) is assumed to be observable so that all modes are observed at the output and the stabilized observer can be constructed. In order to obtain FIR smoothers, it is necessary to relate the most recent information such as measurements and inputs to the estimate state. The system (3)-(4) will be represented in a batch form on the most recent time interval $[k-N, k]$ called the horizon. The finite number of measurements and inputs will be expressed in terms of the state x_{k-N} at the initial time $k-N$ on the horizon $[k-N, k]$ as follows:

$$Y_{k-1} = \tilde{C}_N x_{k-N} + \tilde{B}_N U_{k-1} + \tilde{G}_N W_{k-1} + V_{k-1}, \quad (5)$$

where

$$Y_{k-1} \triangleq [y_{k-N}^T \ y_{k-N+1}^T \ \cdots \ y_{k-1}^T]^T, \quad (6)$$

$$U_{k-1} \triangleq [u_{k-N}^T \ u_{k-N+1}^T \ \cdots \ u_{k-1}^T]^T, \quad (7)$$

$$W_{k-1} \triangleq [w_{k-N}^T \ w_{k-N+1}^T \ \cdots \ w_{k-1}^T]^T,$$

$$V_{k-1} \triangleq [v_{k-N}^T \ v_{k-N+1}^T \ \cdots \ v_{k-1}^T]^T,$$

and \tilde{C}_N , \tilde{B}_N , and \tilde{G}_N are obtained from

$$\tilde{C}_i \triangleq \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{i-1} \end{bmatrix}, \quad (8)$$

$$\tilde{B}_i \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ CB & 0 & \cdots & 0 & 0 \\ CAB & CB & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & CB & 0 \end{bmatrix}, \quad (9)$$

$$\tilde{G}_i \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ CG & 0 & \cdots & 0 & 0 \\ CAG & CG & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{i-2}G & CA^{i-3}G & \cdots & CG & 0 \end{bmatrix}, \quad (10)$$

for $2 \leq i \leq N$. In case of $i = 1$, \tilde{C}_1 is set to C and \tilde{B}_1 and \tilde{G}_1 are defined as zero matrices. The noise term can be shown to be Π_N given by

$$\Pi_N = \tilde{G}_N Q_N \tilde{G}_N^T + R_N, \quad (11)$$

where

$$Q_N = [\text{diag}(\overbrace{Q \ Q \ \cdots \ Q}^N)]$$

and

$$R_N = [\text{diag}(\overbrace{R \ R \ \cdots \ R}^N)]. \quad (12)$$

An FIR smoother with a batch form for the state x_{k-h} can be expressed as a linear function of the finite measurements Y_{k-1} (6) and inputs U_{k-1} (7) on the horizon $[k-N, k]$ as follows:

$$\hat{x}_{k-h|k-1} = HY_{k-1} + LU_{k-1} \quad (13)$$

where

$$H \triangleq [H_N \ H_{N-1} \ \cdots \ H_1],$$

$$L \triangleq [L_N \ L_{N-1} \ \cdots \ L_1],$$

and matrices H and L will be chosen to minimize a given performance criterion later.

Augmenting (3) and (5) yields the following linear model:

$$\begin{bmatrix} Y_{k-1} \\ 0 \end{bmatrix} = \begin{bmatrix} \tilde{C}_N & 0 \\ A^{N-h} & -I \end{bmatrix} \begin{bmatrix} x_{k-N} \\ x_{k-h} \end{bmatrix} + \begin{bmatrix} \tilde{B}_N & 0 & \cdots & 0 \\ A^{N-h-1}B & \cdots & B & 0 & \cdots & 0 \end{bmatrix} U_{k-1} + \begin{bmatrix} \tilde{G}_N & 0 & \cdots & 0 \\ A^{N-h-1}G & \cdots & G & 0 & \cdots & 0 \end{bmatrix} W_{k-1} + \begin{bmatrix} V_{k-1} \\ 0 \end{bmatrix}. \quad (14)$$

By using the equation (14), the MVF smoother (13) can be rewritten as

$$\begin{aligned}
\hat{x}_{k-h} &= \begin{bmatrix} H & -I \end{bmatrix} \begin{bmatrix} Y_{k-1} \\ 0 \end{bmatrix} + LU_{k-1}, \\
&= \begin{bmatrix} H & -I \end{bmatrix} \begin{bmatrix} \tilde{C}_N & 0 \\ A^N & -I \end{bmatrix} \begin{bmatrix} x_{k-N} \\ x_{k-h} \end{bmatrix} \\
&+ \begin{bmatrix} H & -I \end{bmatrix} \begin{bmatrix} \tilde{B}_N & 0 & \dots & 0 \\ A^{N-h-1}B & \dots & B & 0 & \dots & 0 \end{bmatrix} U_{k-1} \\
&+ \begin{bmatrix} H & -I \end{bmatrix} \begin{bmatrix} \tilde{G}_N & 0 & \dots & 0 \\ A^{N-h-1}G & \dots & G & 0 & \dots & 0 \end{bmatrix} W_{k-1} \\
&+ \begin{bmatrix} H & -I \end{bmatrix} \begin{bmatrix} V_{k-1} \\ 0 \end{bmatrix} + LU_{k-1}, \tag{15}
\end{aligned}$$

and taking the expectation on both sides of (15) yields the following equation:

$$\begin{aligned}
E[\hat{x}_{k-h}] &= (H\tilde{C}_N - A^{N-h})E[x_{k-N}] + E[x_{k-h}] \\
&+ \begin{bmatrix} H & -I \end{bmatrix} \begin{bmatrix} \tilde{B}_N & 0 & \dots & 0 \\ A^{N-h-1}B & \dots & B & 0 & \dots & 0 \end{bmatrix} U_{k-1} \\
&+ LU_{k-1}.
\end{aligned}$$

To satisfy the unbiased condition, i.e., $E[\hat{x}_{k-h}] = E[x_{k-h}]$, without respect to the initial state and the input, the following constraints are required:

$$\begin{aligned}
H\tilde{C}_N &= A^{N-h}, \tag{16} \\
L &= - \begin{bmatrix} H & -I \end{bmatrix} \\
&\times \begin{bmatrix} \tilde{B}_N & 0 & \dots & 0 \\ A^{N-h-1}B & \dots & B & 0 & \dots & 0 \end{bmatrix} \tag{17}
\end{aligned}$$

Substituting (16) and (17) into (15) yields the simplified equation as

$$\begin{aligned}
\hat{x}_{k-h} &= x_{k-h} + H\tilde{G}_N W_{k-1} \\
&- \begin{bmatrix} A^{N-h-1}G & \dots & G & 0 & \dots & 0 \end{bmatrix} W_{k-1} \\
&+ HV_{k-1}, \tag{18}
\end{aligned}$$

$$\begin{aligned}
e_{k-h} &\triangleq \hat{x}_{k-h} - x_{k-h} \\
&= H\tilde{G}_N W_{k-1} \\
&- \begin{bmatrix} A^{N-h-1}G & \dots & G & 0 & \dots & 0 \end{bmatrix} W_{k-1} \\
&+ HV_{k-1}. \tag{19}
\end{aligned}$$

Since the matrix gain L can be obtained from H by (17), the object now is to find the optimal gain matrix H_B , subject to the unbiasedness constraints (16) and (17), in such a way that the estimation error of the estimate x_{k-h} as follows :

$$H_B = \arg \min_H E[e_{k-h}^T e_{k-h}] \tag{20}$$

For convenience, the matrix gain H can be partitioned as

$$H^T = \begin{bmatrix} h_1 & h_2 & \dots & h_n \end{bmatrix}, \tag{21}$$

and i -th rows of $\begin{bmatrix} A^{N-h-1}G & \dots & G & 0 & \dots & 0 \end{bmatrix}$ and A^{N-h} are denoted by β_i^T and α_i^T , respectively. Since the

unbiasedness constraint is $H\tilde{C}_N = A^{N-h}$, the i th unbiasedness constraint can be rewritten as

$$\tilde{C}_N^T h_i = \alpha_i. \tag{22}$$

for $1 \leq i \leq n$. Calculating $e_{k-h,i}^2$ form $e_{k-h,i} \triangleq \hat{x}_{k-h,i} - x_{k-h,i}$ and taking the expectation of it, we then obtain

$$\begin{aligned}
E[e_{k-h,i}^2] &= (h_i^T \tilde{G}_N - \beta_i^T) Q_N (h_i^T \tilde{G}_N - \beta_i^T)^T \\
&+ h_i^T R_N h_i. \tag{23}
\end{aligned}$$

Observe that the error variance for the i th state depends only on the i th row of the smoother gain matrix H . We, therefore, establish the following performance criterion:

$$\begin{aligned}
E[e_{k-h,i}^2] &= (h_i^T \tilde{G}_N - \beta_i^T) Q_N (h_i^T \tilde{G}_N - \beta_i^T)^T \\
&+ h_i^T R_N h_i + \lambda_i^T (\tilde{C}_N^T h_i - \alpha_i), \tag{24} \\
&= h_i^T (\tilde{G}_N Q_N \tilde{G}_N^T + R_N) h_i - \beta_i^T Q_N \tilde{G}_N^T h_i \\
&- h_i^T \tilde{G}_N Q_N \beta_i + \beta_i^T Q_N \beta_i \\
&+ \lambda_i^T (\tilde{C}_N^T h_i - \alpha_i), \tag{25}
\end{aligned}$$

where λ_i is the i th vector of a Lagrange multiplier, which is associated with the i th unbiased constraint. Under the constraint (22), the smoother gain h_i will be chosen to minimize (25) with respect to h_i and λ_i ($i = 1, 2, \dots, n$). In order to minimize $E[e_{k-h,i}^2]$, two necessary conditions

$$\frac{\partial E[e_{k-h,i}^2]}{\partial h_i} = 0 \quad \text{and} \quad \frac{\partial E[e_{k-h,i}^2]}{\partial \lambda_i} = 0$$

are necessary, which give

$$\begin{aligned}
h_i &= (\tilde{G}_N Q_N \tilde{G}_N^T + R_N)^{-1} (\tilde{G}_N Q_N \beta_i - \frac{1}{2} \tilde{C}_N \lambda_i) \\
&= \Pi_N^{-1} (\tilde{G}_N Q_N \beta_i - \frac{1}{2} \tilde{C}_N \lambda_i), \tag{26}
\end{aligned}$$

where the inverse of Π_N is guaranteed since $\Pi_N = \tilde{G}_N Q_N \tilde{G}_N^T + R_N$ is positive definite.

Pre-multiplying (26) by \tilde{C}_N^T , we have

$$\tilde{C}_N^T h_i = \tilde{C}_N^T \Pi_N^{-1} (\tilde{G}_N Q_N \beta_i - \frac{1}{2} \tilde{C}_N \lambda_i) = \alpha_i, \tag{27}$$

where a second equality comes from (22).

From (27), h_i can be obtained as

$$\begin{aligned}
h_i^T &= [\alpha_i^T - \beta_i^T Q_N \tilde{G}_N^T \Pi_N^{-1} \tilde{C}_N] \\
&(\tilde{C}_N^T \Pi_N^{-1} \tilde{C}_N)^{-1} \tilde{C}_N^T \Pi_N^{-1} + \beta_i^T Q_N \tilde{G}_N^T \Pi_N^{-1}.
\end{aligned}$$

By reconstructing h_i , we can obtain the gain matrix H as

$$\begin{aligned}
H &= \begin{bmatrix} A^{N-h} & A^{N-h-1}G & \dots & G & 0 & \dots & 0 \end{bmatrix} \\
&\times \begin{bmatrix} W_{1,1} & W_{1,2} \\ W_{1,2}^T & W_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{C}_N^T \\ \tilde{G}_N^T \end{bmatrix} R_N^{-1}. \tag{28}
\end{aligned}$$

where

$$W_{1,1} \triangleq \tilde{C}_N^T R_N^{-1} \tilde{C}_N, \tag{29}$$

$$W_{1,2} \triangleq \tilde{C}_N^T R_N^{-1} \tilde{G}_N, \tag{30}$$

$$W_{2,2} \triangleq \tilde{G}_N^T R_N^{-1} \tilde{G}_N + Q_N^{-1} \tag{31}$$

By the constraint (17), we can obtain the smoother gain L from (28). The MVF smoother can be written as

$$\begin{aligned} \hat{x}_{k-h} &= \begin{bmatrix} A^{N-h} & A^{N-h-1}G & \dots & G & 0 & \dots & 0 \end{bmatrix} \\ &\times \begin{bmatrix} W_{1,1} & W_{1,2} \\ W_{1,2}^T & W_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{C}_N^T \\ \tilde{G}_N^T \end{bmatrix} R_N^{-1} \left(Y_{k-1} - \tilde{B}_N U_{k-1} \right) \\ &+ \begin{bmatrix} A^{N-h-1}B & \dots & B & 0 & \dots & 0 \end{bmatrix} U_{k-1}. \end{aligned} \quad (32)$$

As can be seen in (32), the inverse of the system matrix A does not appear in the smoother coefficients.

3. Numerical Example

To demonstrate the proposed MVF smoother, numerical examples on the discretized model of an F-404 engine [13] are simulated. The corresponding dynamic model is written as

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} 0.9305 + \delta_k & 0 & 0.1107 \\ 0.0077 & 0.9802 + \delta_k & -0.0173 \\ 0.0142 & 0 & 0.8953 + 0.1\delta_k \end{bmatrix} x_k \\ &+ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w_k, \end{aligned} \quad (33)$$

$$y(t) = \begin{bmatrix} 1 + 0.1\delta_k & 0 & 0 \\ 0 & 1 + 0.1\delta_k & 0 \end{bmatrix} x_k + v_k, \quad (34)$$

where δ_k is an uncertain model parameter. The system noise covariance Q_k is 0.02 and the measurement noise covariance R_k is 0.02. The horizon length is taken as $N = 10$ and the delay factor is set as $h = 3$. We perform simulation studies for the system (33) with temporary modeling uncertainty. As mentioned previously, the MVF smoother is believed to be robust against temporary modeling uncertainties since it utilizes only finite measurements on the most recent horizon. To illustrate this fact and the fast convergence, the MVF smoother and the fixed-lag Kalman smoother are designed with $\delta_k = 0$ and compared when a system has actually temporary modeling uncertainty. The uncertain model parameter δ_k is considered as

$$\delta_k = \begin{cases} 0.1, & 50 \leq k \leq 100, \\ 0, & \text{otherwise.} \end{cases} \quad (35)$$

Figure (1) compares the robustness between the proposed MVF smoother and fixed-lag Kalman smoother which given temporary modeling uncertainty (35) for the second state which is related to turbine temperature. It is seen that the estimation error of the MVF smoother is remarkably smaller than that of the fixed-lag Kalman smoother on the interval where modeling uncertainty exists. In addition, it is shown that the estimation error is converged much faster than that of the fixed-lag Kalman smoother after temporary modeling uncertainty disappears. Therefore, the proposed MVF smoother is very useful than IIR type estimators when there exist temporary modelling errors and numerical errors. In Figure(2), the estimates of MVF smoother and filter are

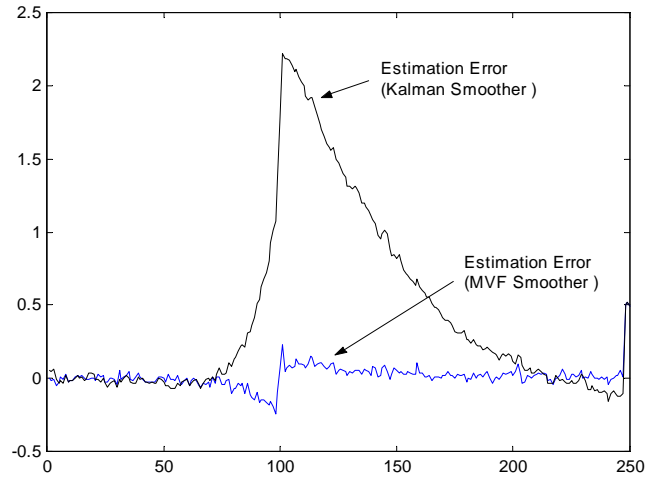


Fig. 1. Estimation errors of MVF smoother and Fixed-lag Kalman smoother

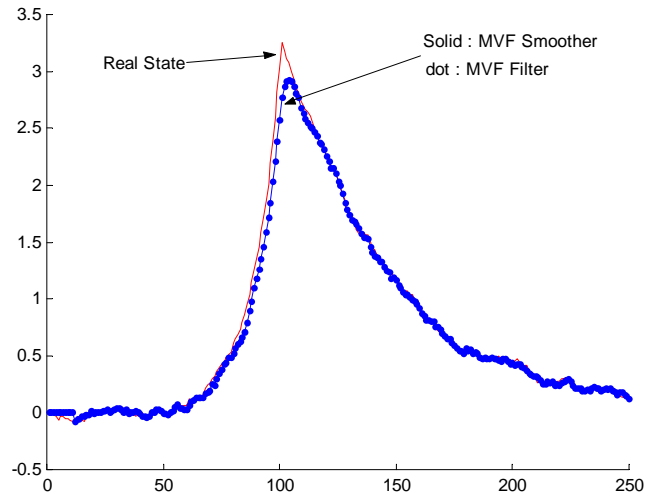


Fig. 2. Estimates of MVF smoother and MVF filter ($h = 0$)

compared when the delay factor in MVF smoother is equal to zero. This means that the future data values are not used to estimate the state. Therefore, MVF filter is the special forms of MVF smoother when no future data values are used to estimate the state.

4. Conclusion

In this paper, the finite impulse response (FIR) smoother is proposed for discrete-time systems. The proposed smoother is chosen to optimize the minimum variance performance criterion with unbiased constraint. They are designed with linearity, unbiasedness, FIR structure, and independence of the initial state information.

The approaches of MVF smoother are logical and systematic, while the existing result has heuristic assumption, such as infinite covariance of the initial state. Since there are few result and theoretical approaches on FIR smoother for model

based signal, this approaches have a great significance. Additionally, it is meaningful that the proposed MVF smoother is based on the general system that may have the singular system matrix and has both system and measurement noises. Thorough simulation studies, it is shown that the MVF filter is the special forms of MVF smoother when no future data values are used to estimate the signal. It means that the proposed MVF smoother is generalized form of MVF filter. The result of comparison between the proposed MVF smoother and fixed-lag Kalman smoother is also provided by simulation. It shows that the proposed MVF smoother is very useful for control problems than IIR type estimators when there are temporary modelling errors and numerical errors.

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