# Intelligent Gain and Boundary Layer Based Sliding Mode Control for Robotic Systems with Unknown Uncertainties

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**Abstract:** This paper proposes a intelligent gain and boundary layer based sliding mode control (SMC) method for robotic systems with unknown model uncertainties. For intelligent gain and boundary layer, we employ the self recurrent wavelet neural network (SRWNN) which has the properties such as a simple structure and fast convergence. In our control structure, the SRWNNs are used for estimating the width of boundary layer, uncertainty bound, and nonlinear terms of robotic systems. The adaptation laws for all parameters of SRWNNs and reconstruction error bounds are derived from the Lyapunov stability theorem, which are used for an on-line control of robotic systems with unknown uncertainties. Accordingly, the proposed method can overcome the chattering phenomena in the control effort and has the robustness regardless of unknown uncertainties. Finally, simulation results for the three-link manipulator, one of the robotic systems, are included to illustrate the effectiveness of the proposed method.

**Keywords:** Self recurrent wavelet neural network, Sliding mode control, Unknown uncertainty, Intelligent gain/boundary

## 1. Introduction

The sliding mode control (SMC) is known as one of simple and popular techniques for a robust control of robotic systems with uncertainties and external disturbances due to its simplicity, fast response and good performance [1]. However, the traditional SMC has two important drawbacks. First, the bounds of uncertainties and external disturbances of the plant must be known for solving control problems [2,3]. However, in real applications, since the parameter variations of the system are difficult to predict, and the external disturbances changed according to the environment are also difficult to know, the switching gain of the sliding phase in the traditional SMC law cannot be computed accurately. Second, the traditional SMC always suffers from chattering control input owing to its discontinuous switching control input and its delays in the sliding phase [2,3]. The chattering control input results in low control accuracy and high wear of moving mechanical parts.

On the other hand, recently wavelet neural network (WNN), which absorbs the advantages of high resolution of wavelets and learning of neural network, has been proposed to guarantee the fast convergence and is used for the identification and control of the nonlinear systems [5, 6]. However, the WNN does not require prior knowledge about the plant to be controlled due to its feedforward structure. Therefore, the WNN cannot adapt rapidly under the circumstances to change frequently the operating conditions and parameters of dynamics. To overcome these problems, the self recurrent wavelet neural network (SRWNN), which combines the properties of attractor dynamics of recurrent neural network and the fast convergence of WNN, has been proposed and used for estimating and controlling nonlinear systems [7]. Since the SRWNN has a mother wavelet layer composed of self-feedback neurons, it can capture the past information of the network and adapt rapidly to sudden changes of the control environment. Due to these properties, the structure of the SRWNN can be simpler than that of the WNN.

In this paper, the self recurrent wavelet neural network based sliding mode control (SRWNNSMC) method is presented to control robotic systems with unknown uncertainties. We first introduce the robot model which the model uncertainties are separated, then the sliding controller with the intelligent gain and boundary is designed. In our SRWNNSMC system, the bound of the uncertainty assumed to be unknown is used as the gain of the sliding phase with the boundary layer. For the intelligent gain and boundary layer, the SRWNNs are used to approximate the unknown gain and boundary layer in the sliding phase and the nonlinear term in the robot dynamics. Besides, the acceleration term in the sliding phase is added to accelerate the convergence of the operating point. To analyze the SRWNNSMC system and to learn weights of the SRWNN, we apply the adaptive technique based on the Lyapunov stability theory [8, 9] recently to be used chiefly. Accordingly, the adaptation laws for learning the weights of the SRWNN and these for the error compensator are induced out of consideration for the stability, robustness and performance of the SRWNNSMC system. Finally, we simulate the three-link manipulator, one of the robotic systems, to show the effectiveness of the suggested SRWNNSMC system.

## 2. Problem Formulation

### 2.1. Model of robot systems with uncertainties

The nominal model of a robot system having $n$ rigid joints can be expressed in the following Lagrange form:

$$\boldsymbol{M(q)\ddot{q} + C(q,\dot{q}) + G(q) + F(\dot{q}) = \tau}, \quad (1)$$

where $\boldsymbol{q}$, $\boldsymbol{\dot{q}}$, $\boldsymbol{\ddot{q}} \in R^n$ are the joint position, velocity, and acceleration respectively. $\boldsymbol{M(q)} \in R^{n\times n}$ is the inertia matrix, $\boldsymbol{C(q,\dot{q})} \in R^n$ denotes the Coriolis and centripetal forces, $\boldsymbol{G(q)} \in R^n$ is the gravity vector, $\boldsymbol{F(\dot{q})} \in R^n$ represents the friction term, and the control input torque is $\boldsymbol{\tau} \in R^n$.

However, for the model uncertainty and external disturbance $\boldsymbol{\tau_d}$, the actual model of a robot system may be different from the nominal model (1). Thus, the actual dynamics of the nominal model (1) can be expressed as

$$\boldsymbol{\bar{M}(q)\ddot{q} + \bar{C}(q,\dot{q}) + \bar{G}(q) + \bar{F}(\dot{q}) + \tau_d = \tau}, \quad (2)$$

where $\boldsymbol{\bar{M}(q)}$, $\boldsymbol{\bar{C}(q,\dot{q})}$, $\boldsymbol{\bar{G}(q)}$, and $\boldsymbol{\bar{F}(\dot{q})}$ are the actual values having the uncertainty in the nominal values $\boldsymbol{M(q)}$, $\boldsymbol{C(q,\dot{q})}$, $\boldsymbol{G(q)}$, and $\boldsymbol{F(\dot{q})}$, respectively.

**Assumption** 1: In this paper, suppose that the nominal value $\boldsymbol{M(q)}$ is only the known value for a given robot, but the nominal values $\boldsymbol{C(q,\dot{q})}$, $\boldsymbol{G(q)}$, and $\boldsymbol{F(\dot{q})}$, the actual values $\boldsymbol{\bar{M}(q)}$, $\boldsymbol{\bar{C}(q,\dot{q})}$, $\boldsymbol{\bar{G}(q)}$, $\boldsymbol{\bar{F}(\dot{q})}$, and the external disturbance $\tau_d$ are the unknown values.

We must express the model uncertainty separately with the nominal model according to Assumption 1. The actual robot dynamics (2) can be written in the following formulation using the nominal model [9]:

$$\begin{aligned}\boldsymbol{M(q)\ddot{q} + C(q,\dot{q}) + G(q) + F(\dot{q})} &+ \boldsymbol{\Xi(q,\dot{q},\tau)} \\ &= \boldsymbol{\tau}, \quad (3)\end{aligned}$$

where

$$\begin{aligned}\boldsymbol{\Xi(q,\dot{q},\tau)} \equiv &-\boldsymbol{M(q)\bar{M}^{-1}(q)\{\tau - \tau_d - \bar{C}(q,\dot{q})} \\ &-\boldsymbol{\bar{G}(q) - \bar{F}(\dot{q})\} + \{\tau - C(q,\dot{q})} \\ &-\boldsymbol{G(q) - F(\dot{q})\}} \quad (4)\end{aligned}$$

denotes the uncertainty of the robot system. The uncertainty term $\boldsymbol{\Xi(q,\dot{q},\tau)}$ cannot be evaluated directly by Assumption 1.

### 2.2. Design of the traditional SMC system

In this section, we discuss the traditional SMC design method for the robot model (3) with uncertainty and its problems. The actual model (3) of robot system with uncertainty can be written as follows:

$$\begin{aligned}\boldsymbol{\ddot{q}} = &\boldsymbol{M^{-1}(q)\{-C(q,\dot{q}) - G(q) - F(\dot{q}) + \tau\}} \\ &+ \boldsymbol{\Lambda(q,\dot{q},\tau)}, \quad (5)\end{aligned}$$

where $\boldsymbol{\Lambda(q,\dot{q},\tau) \equiv -M^{-1}(q)\Xi(q,\dot{q},\tau)}$ denotes the uncertainty term. Here, $\boldsymbol{\tau}$ is a function of $\boldsymbol{q}$, $\boldsymbol{\dot{q}}$ and $\boldsymbol{Q_d = (q_d, \dot{q}_d, \ddot{q}_d)}$ which denotes the reference position, velocity, and acceleration. Accordingly,

$$\begin{aligned}\boldsymbol{\ddot{q}} = &\boldsymbol{M^{-1}(q)\{-C(q,\dot{q}) - G(q) - F(\dot{q}) + \tau\}} \\ &+ \boldsymbol{\Lambda(q,\dot{q},Q_d)}, \quad (6)\end{aligned}$$

where the bound of the uncertainty terms $\Lambda_i(q,\dot{q},Q_d)$ is assumed to be given, that is,

$$|\Lambda_i(q,\dot{q},Q_d)| < \sigma_i, \quad (7)$$

where $i = 1, 2, \cdots, n$, $\sigma_i$ is a given positive constant. The objective of SMC is to get the joint position $\boldsymbol{q}$ to track a reference position $\boldsymbol{q_d}$. To solve this control problem, in the multi-input case, we define the time-varying sliding surface vector $\boldsymbol{S(t)}$ as follows:

$$\boldsymbol{S(t) = \dot{E} + P_1 E + P_2 \int_0^t E d\tau}, \quad (8)$$

where $\boldsymbol{E = q - q_d}$ is the control error, $\boldsymbol{P_1} = diag[p_{1,i}]$, and $\boldsymbol{P_2} = diag[p_{2,i}]$. Here $p_{1,i} > 0$ and $p_{2,i} > 0$ are constants, which are given for $\boldsymbol{S(t) = 0}$; $diag[\cdot]$ denotes a diagonal matrix. $\boldsymbol{S(t) = 0}$ means that the resultant system is stable. If the states are outside the sliding surface, to drive the states to the sliding surface, we need the sliding condition as follows:

$$\begin{aligned}\boldsymbol{S^T(t)\dot{S}(t)} &= \sum_{i=1}^{n} S_i(t)\dot{S}_i(t) \\ &< -\sum_{i=1}^{n} \mu_i |S_i(t)|, \qquad \text{if} \quad \boldsymbol{S(t) \neq 0}. \quad (9)\end{aligned}$$

where $S_i(t)$ is the $i$-th component of $\boldsymbol{S(t)}$ and the constants $\mu_i$ are strictly positive. We take the derivative of (8), and using (6), obtain

$$\begin{aligned}\boldsymbol{\dot{S}(t)} &= \boldsymbol{\ddot{q} - \ddot{q}_d + P_1\dot{E} + P_2 E} \\ &= \boldsymbol{M^{-1}(q)\{-C(q,\dot{q}) - G(q) - F(\dot{q}) + \tau\}} \\ &\quad + \boldsymbol{\Lambda(q,\dot{q},Q_d) - \ddot{q}_d + P_1\dot{E} + P_2 E}, \quad (10)\end{aligned}$$

Then, the total SMC law is assumed to take the following form:

$$\boldsymbol{\tau} = \boldsymbol{\tau_r} + \boldsymbol{\tau_c}, \tag{11}$$

where

$$\begin{aligned}\boldsymbol{\tau_r} =& \boldsymbol{C(q,\dot{q})} + \boldsymbol{G(q)} + \boldsymbol{F(\dot{q})} \\ &+ \boldsymbol{M(q)(\ddot{q}_d - P_1\dot{E} - P_2E - \sigma)}, \end{aligned}\tag{12}$$

$$\boldsymbol{\tau_c} = -\boldsymbol{M(q)L}\mathrm{sgn}(\boldsymbol{S(t)}) \tag{13}$$

in which $\boldsymbol{\sigma} = [\sigma_i]$, $\boldsymbol{L} = diag[L_i]$; $L_i > 0$ are constants, and sgn$(\cdot)$ is a sign function. The reaching control input $\boldsymbol{\tau_r}$ is used to cancel the nonlinear term and to specify the desired system performance. And the sliding control input $\boldsymbol{\tau_c}$ is used to keep the controlled system dynamics in the sliding surface and to guarantee the convergence of the state trajectory. Substituting (10) into (9), and using (11)∼(13), the sliding condition can be written as follows:

$$\begin{aligned}\boldsymbol{S^T(t)\dot{S}(t)} \le& \boldsymbol{S^T(t)}[\boldsymbol{M^{-1}(q)}\{-\boldsymbol{C(q,\dot{q})} - \boldsymbol{G(q)} \\ &- \boldsymbol{F(\dot{q})} + \boldsymbol{\tau}\} + \boldsymbol{\sigma} - \boldsymbol{\ddot{q}_d} + \boldsymbol{P_1\dot{E}} + \boldsymbol{P_2E}] \\ =& -\sum_{i=1}^{n} L_i|S_i(t)|,\end{aligned}$$

so that, letting $L_i = \mu_i$, the sliding condition (9) is satisfied throughout the whole control period and the stability of controlled system is guaranteed. But the traditional SMC method has the following problems.

**Problem** 1: In real robot system, since the parameter variations ($\sigma_i$), that is, the uncertainty are difficult to measure, and the exact value of the external disturbance is also difficult to know in advance, we cannot use the traditional SMC technique for the robust control of robotic systems with unknown uncertainties and disturbances.

**Problem** 2: In the traditional SMC technique, the sign function (sgn$(\cdot)$) of sliding control input $\tau_c$ will result in chattering phenomena, exciting unstable system dynamics and easy damage of mechanism, in the control efforts. For attenuating the chattering control input, the boundary layer method [4] is used commonly. Indeed, the control signal is smoother than the original one without using a boundary layer. However, the boundary layer method has a drawback that the width of boundary layer is difficult to choose optimally. A thin boundary layer for obtaining extreme tracking accuracy has risks exciting a high-frequency control input and chattering phenomena. And also, a thick boundary layer results in the large steady-state error and does not ensure the convergence of the state trajectory of the system to the sliding surface.

## 3. Robust SRWNN based SMC System for Robotic Systems

In order to solving Problems 1 and 2 of the traditional SMC technique, we now introduce the design method of SMC system based on the SRWNN for robotic systems.

### 3.1. SRWNN structure

A schematic diagram of the SRWNN structure is shown in [7], which has $N_i$ inputs, one output, and $N_i \times N_w$ mother wavelets. The SRWNN structure consists of four layers: a input layer, a mother wavelet layer, a product layer, and a output layer. Each node of a mother wavelet layer has a mother wavelet and a self-feedback loop. In this paper, we select the first derivative of a Gaussian function, $\phi(x) = -x\exp(-\frac{1}{2}x^2)$ which has the universal approximation property [6] as a mother wavelet function. The nodes in a product layer are given by the product of the mother wavelets as follows:

$$\Phi_j(\mathrm{x}) = \prod_{k=1}^{N_i} \phi(z_{jk}), \quad \text{with} \quad z_{jk} = \frac{u_{jk} - m_{jk}}{d_{jk}},$$

where, $m_{jk}$ and $d_{jk}$ are the translation factor and the dilation factor of the wavelets, respectively. The subscript $jk$ indicates the $k$-th input term of the $j$-th wavelet. In addition, the inputs $u_{jk}$ of the wavelet nodes can be denoted by

$$u_{jk} = x_k + \phi_{jk}\mathrm{z}^{-1} \cdot \theta_{jk}, \tag{14}$$

where, $\theta_{jk}$ denotes the weight of the self-feedback loop, and $\mathrm{z}^{-1}$ is a time delay. The input of mother wavelet layer contains the memory term $\phi_{jk}\mathrm{z}^{-1}$, which can store the past information of the network. That is, the current dynamics of the system is conserved for the next sample step. Thus, even if the SRWNN has less mother wavelets than the WNN, the SRWNN can attract nicely the system with complex dynamics. Here, $\theta_{jk}$ is a factor to represent the rate of information storage. These aspects are the apparent dissimilar point between the WNN and the SRWNN. And also, the SRWNN is a generalization system of the WNN because the SRWNN structure is the same as the WNN structure when $\theta_{jk} = 0$.

The SRWNN output is a linear combination of consequences obtained from the output of the product layer. In addition, the output node accepts directly input values from the input layer. Therefore, the SRWNN output $y$ is composed by self-recurrent wavelets and parameters as follows:

$$y = \sum_{j=1}^{N_w} w_j \Phi_j(\mathrm{x}) + \sum_{k=1}^{N_i} a_k x_k, \tag{15}$$

where, $w_j$ is the connection weight between product nodes and output nodes, and $a_k$ is the connection weight between the input nodes and the output node. By using the direct term, the SRWNN has a number of advantages such as a direct linear feedthrough network, including initialization of network parameters based on process knowledge and enhanced extrapolation outside of examples the learning data sets [10]. In this paper, five weights $a_k$, $m_{jk}$, $d_{jk}$, $\theta_{jk}$, and $w_j$ of the SRWNN are trained by the adaptation laws induced from the Lyapunov stability in the following section.

### 3.2. SRWNN based SMC system

For the SRWNNSMC system design, we reformulate the derivative (10) of the sliding surface as

$$\begin{aligned}\dot{\boldsymbol{S}}(\boldsymbol{t}) &= \boldsymbol{M}^{-1}(\boldsymbol{q})\{-\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) - \boldsymbol{G}(\boldsymbol{q}) - \boldsymbol{F}(\dot{\boldsymbol{q}}) + \boldsymbol{\tau}\} \\ &\quad + \boldsymbol{\Lambda}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{Q_d}) - \ddot{\boldsymbol{q}}_d + \boldsymbol{P_1}\dot{\boldsymbol{E}} + \boldsymbol{P_2}\boldsymbol{E} \\ &= \boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{\tau} + \boldsymbol{\Omega}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{\Lambda}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{Q_d}) \\ &\quad - \ddot{\boldsymbol{q}}_d + \boldsymbol{P_1}\dot{\boldsymbol{E}} + \boldsymbol{P_2}\boldsymbol{E},\end{aligned} \tag{16}$$

where

$$\boldsymbol{\Omega}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{M}^{-1}(\boldsymbol{q})\{-\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) - \boldsymbol{G}(\boldsymbol{q}) - \boldsymbol{F}(\dot{\boldsymbol{q}})\} \tag{17}$$

denotes the nonlinear term of the robotic system, and the bound of the uncertainty is assumed as a unknown positive constant. In our control structure, the SRWNN system $\hat{\boldsymbol{\Omega}}(\boldsymbol{q},\dot{\boldsymbol{q}})$ will be employed to approximate the nonlinear term $\boldsymbol{\Omega}(\boldsymbol{q},\dot{\boldsymbol{q}})$ to a sufficient degree of accuracy. Thus, the nonlinear term $\boldsymbol{\Omega}(\boldsymbol{q},\dot{\boldsymbol{q}})$ can be described by the optimal SRWNN plus a reconstruction error vector $\boldsymbol{\varepsilon_1}$ as follows:

$$\boldsymbol{\Omega}(\boldsymbol{x}) = \hat{\boldsymbol{\Omega}}(\boldsymbol{x}|\hat{\boldsymbol{A}}) + [\boldsymbol{\Omega}^*(\boldsymbol{x}|\boldsymbol{A}^*) - \hat{\boldsymbol{\Omega}}(\boldsymbol{x}|\hat{\boldsymbol{A}})] + \boldsymbol{\varepsilon_1}, \tag{18}$$

where $\boldsymbol{x} = (\boldsymbol{q},\dot{\boldsymbol{q}})$, $\hat{\boldsymbol{A}} = [\hat{A}_i]$; $\hat{A}_i$ are the collections of the estimated weighting parameters of the SRWNN defined in Section 3.1.and $\boldsymbol{A}^*$ is the optimal weighting vector that achieves the minimum reconstruction error. Then, taking the Taylor series expansion of $\boldsymbol{\Omega}^*(\boldsymbol{x}|\boldsymbol{A}^*)$ around $\hat{\boldsymbol{A}}$, it can be obtained that

$$\begin{aligned}&\boldsymbol{\Omega}^*(\boldsymbol{x}|\boldsymbol{A}^*) - \hat{\boldsymbol{\Omega}}(\boldsymbol{x}|\hat{\boldsymbol{A}}) \\ &\quad = \tilde{\boldsymbol{A}}^{\boldsymbol{T}}\left[\frac{\partial\hat{\boldsymbol{\Omega}}(\boldsymbol{x}|\hat{\boldsymbol{A}})}{\partial\hat{\boldsymbol{A}}}\right] + \boldsymbol{H_1}(\boldsymbol{A}^*,\hat{\boldsymbol{A}}),\end{aligned} \tag{19}$$

where $\tilde{\boldsymbol{A}} = \boldsymbol{A}^* - \hat{\boldsymbol{A}}$, and $\boldsymbol{H_1}$ is a high-order term. Substituting (19) into (18), we obtain

$$\boldsymbol{\Omega}(\boldsymbol{x}) = \hat{\boldsymbol{\Omega}}(\boldsymbol{x}|\hat{\boldsymbol{A}}) + \tilde{\boldsymbol{A}}^{\boldsymbol{T}}\left[\frac{\partial\hat{\boldsymbol{\Omega}}(\boldsymbol{x}|\hat{\boldsymbol{A}})}{\partial\hat{\boldsymbol{A}}}\right] + \boldsymbol{\alpha}, \tag{20}$$

where each magnitude of element of the vector $\boldsymbol{\alpha} = \boldsymbol{H_1} + \boldsymbol{\varepsilon_1}$ is assumed to be bounded by $|\alpha_i| \leq \delta_i$ $(i = 1, 2, \cdots, n)$. Based on the facts that the nonlinear term $\boldsymbol{\Omega}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is approximated by the adaptive SRWNN system, we propose the reaching control input $\boldsymbol{\tau_r}$ as follows:

$$\boldsymbol{\tau_r} = \boldsymbol{M}(\boldsymbol{q})\{-\hat{\boldsymbol{\Omega}}(\boldsymbol{x}|\hat{\boldsymbol{A}}) + \ddot{\boldsymbol{q}}_d - \boldsymbol{P_1}\dot{\boldsymbol{E}} - \boldsymbol{P_2}\boldsymbol{E}\} + \boldsymbol{\tau_{e1}}, \tag{21}$$

where $\boldsymbol{\tau_{e1}} = -\boldsymbol{M}(\boldsymbol{q})\hat{\boldsymbol{\delta}}$ in which $\hat{\boldsymbol{\delta}}$ is the estimated vector of $\boldsymbol{\delta}$ is designed to compensate the approximation error $\boldsymbol{\varepsilon_1}$ and the high-order term $\boldsymbol{H_1}$ of the SRWNN.

Then, the sliding control input $\boldsymbol{\tau_c}$ is presented to guarantee the convergence of the states trajectory and to eliminate the chattering phenomenon as follows:

$$\boldsymbol{\tau_c} = -\boldsymbol{M}(\boldsymbol{q})\,(\boldsymbol{\Upsilon} + \boldsymbol{K})\,\boldsymbol{S}(\boldsymbol{t}),$$

where $\boldsymbol{\Upsilon} = diag[\sigma_i/(|S_i(t)|+R_i)]$; $\sigma_i/[|S_i(t)|+R_i] > 0$ $(i = 1, 2, \cdots, n)$. And also, $R_i$ is defined as the width of the boundary layer and is the small positive constant. The second term $-\boldsymbol{M}(\boldsymbol{q})\boldsymbol{K}\boldsymbol{S}(\boldsymbol{t})$ is used to accelerate the convergence of the operating point; $\boldsymbol{K} = diag[K_i]$ and $K_i > 0$ is a constant. For the unknown uncertainty bound and the width of the boundary layer, we adopt the SRWNN to estimate the diagonal terms of $\boldsymbol{\Upsilon}$. Thus, similar to (18) $\sim$ (20), $\boldsymbol{\Upsilon}$ is

$$\begin{aligned}\boldsymbol{\Upsilon} &= \boldsymbol{\Upsilon}^*(\boldsymbol{S}|\boldsymbol{B}^*) + \boldsymbol{\varepsilon_2} \\ &= \hat{\boldsymbol{\Upsilon}}(\boldsymbol{S}|\hat{\boldsymbol{B}}) + [\boldsymbol{\Upsilon}^*(\boldsymbol{S}|\boldsymbol{B}^*) - \hat{\boldsymbol{\Upsilon}}(\boldsymbol{S}|\hat{\boldsymbol{B}})] + \boldsymbol{\varepsilon_2} \\ &= \hat{\boldsymbol{\Upsilon}}(\boldsymbol{S}|\hat{\boldsymbol{B}}) + \tilde{\boldsymbol{B}}^{\boldsymbol{T}}\left[\frac{\partial\hat{\boldsymbol{\Upsilon}}(\boldsymbol{S}|\hat{\boldsymbol{B}})}{\partial\hat{\boldsymbol{B}}}\right] + \boldsymbol{H_2}(\boldsymbol{B}^*,\hat{\boldsymbol{B}}) + \boldsymbol{\varepsilon_2} \\ &= \hat{\boldsymbol{\Upsilon}}(\boldsymbol{S}|\hat{\boldsymbol{B}}) + \tilde{\boldsymbol{B}}^{\boldsymbol{T}}\left[\frac{\partial\hat{\boldsymbol{\Upsilon}}(\boldsymbol{S}|\hat{\boldsymbol{B}})}{\partial\hat{\boldsymbol{B}}}\right] + \boldsymbol{\beta},\end{aligned} \tag{22}$$

where $\tilde{\boldsymbol{B}} = \boldsymbol{B}^* - \hat{\boldsymbol{B}}$, each element of the diagonal matrix $\boldsymbol{\beta} = \boldsymbol{H_2} + \boldsymbol{\varepsilon_2}$ is assumed to be bounded by $|\beta_i| \leq \zeta_i$; $\boldsymbol{H_2}$ and $\boldsymbol{\varepsilon_2}$ are a high-order term and a reconstruction error of the SRWNN, respectively. Therefore, the sliding control law is redefined as

$$\boldsymbol{\tau_c} = -\boldsymbol{M}(\boldsymbol{q})\left(\hat{\boldsymbol{\Upsilon}}(\boldsymbol{S}|\hat{\boldsymbol{B}}) + \hat{\boldsymbol{\zeta}} + \boldsymbol{K}\right)\boldsymbol{S}(\boldsymbol{t}), \tag{23}$$

where $\hat{\boldsymbol{\Upsilon}} = diag[\hat{\Upsilon}_i]$, the second term $-\boldsymbol{M}(\boldsymbol{q})\hat{\boldsymbol{\zeta}}\boldsymbol{S}(\boldsymbol{t})$ is added to compensate the approximation error $\boldsymbol{\varepsilon_2}$ and the high-order term $\boldsymbol{H_2}$ of the SRWNN; $\hat{\boldsymbol{\zeta}} = diag[\hat{\zeta}_i]$ is the estimated diagonal matrix of $\boldsymbol{\zeta}$. Accordingly, from (21) and (23), the total control input is proposed as follows:

$$\begin{aligned}\boldsymbol{\tau} &= \boldsymbol{\tau_r} + \boldsymbol{\tau_c} \\ &= \boldsymbol{M}(\boldsymbol{q})\Big\{-\hat{\boldsymbol{\Omega}}(\boldsymbol{x}|\hat{\boldsymbol{A}}) - \hat{\boldsymbol{\delta}} + \ddot{\boldsymbol{q}}_d - \boldsymbol{P_1}\dot{\boldsymbol{E}} - \boldsymbol{P_2}\boldsymbol{E} \\ &\quad - \left(\hat{\boldsymbol{\Upsilon}}(\boldsymbol{S}|\hat{\boldsymbol{B}}) + \hat{\boldsymbol{\zeta}} + \boldsymbol{K}\right)\boldsymbol{S}(\boldsymbol{t})\Big\}.\end{aligned} \tag{24}$$

**Theorem 1:** Assume that the robot system (6) with unknown model uncertainty is controlled by the SRWNNSMC laws (24). If the adjustable parameters of the SRWNN and the error compensation vectors are tuned by the following adaptation rules:

$$\dot{\hat{A}}_i = \lambda_{1,i}\left[\frac{\partial\hat{\Omega}_i(x_i|\hat{A}_i)}{\partial\hat{A}_i}\right]|S_i(t)|, \tag{25}$$

$$\dot{\hat{B}}_i = \lambda_{2,i}\left[\frac{\partial\hat{\Upsilon}_i(S_i|\hat{B}_i)}{\partial\hat{B}_i}\right]S_i^2(t), \tag{26}$$

$$\dot{\hat{\delta}}_i(t) = \lambda_{3,i}|S_i(t)|, \tag{27}$$

$$\dot{\hat{\zeta}}_i(t) = \lambda_{4,i}S_i^2(t), \tag{28}$$

where $i = 1, 2, \cdots, n$, $\lambda_{1,i}$, $\lambda_{2,i}$, $\lambda_{3,i}$, and $\lambda_{4,i}$ are positive tuning gains, then the asymptotic stability of the controlled system can be guaranteed.

**Remark** 1: In Theorem 1, $\left[\partial\hat{\Omega}_i(x_i|\hat{A}_i)/\partial\hat{A}_i\right]$ and $\left[\partial\hat{\Upsilon}_i(S_i|\hat{B}_i)/\partial\hat{B}_i\right]$ can be computed using the backpropagation algorithm and are shown in [7].

**Remark** 2: Since the sign function of the sliding control input in the traditional SMC technique is substituted by the SRWNN, there are no chattering phenomena in the control efforts of the SRWNNSMC system.

## 4. Computer Simulation

In this section, to illustrate the effectiveness of the suggested SRWNNSMC system, the three-link manipulator with unknown uncertainties, one of the robotic systems, is simulated using a fourth-order Runge-Kutta algorithm with a step size of 0.01 s. The matrices $M(q)$, $C(q, \dot{q})$, and $G(q)$ in the dynamics (1) of the three-link manipulator are given by [9]. In this simulation, the initial positions are set to $q_1(0) = q_2(0) = q_3(0) = 0$ and the link masses $m_i$s and the link lengths $a_i$s are assumed to be uncertain. Table 1 shows the parameters of the robot system used in this simulation. In Table 1, we assume that the nominal values only are known. The parameters of the proposed control system are given in the following:

$$\begin{aligned}
\boldsymbol{P_1} &= diag[60 \quad 60 \quad 100]\\
\boldsymbol{P_2} &= diag[0.5 \quad 6 \quad 0.1]\\
\boldsymbol{K} &= diag[80 \quad 60 \quad 140]\\
\boldsymbol{\lambda_1} &= diag[0.002 \quad 0.002 \quad 0.002]\\
\boldsymbol{\lambda_2} &= diag[0.0001 \quad 0.00001 \quad 0.00001]\\
\boldsymbol{\lambda_3} &= diag[0.2 \quad 0.2 \quad 1.0]\\
\boldsymbol{\lambda_4} &= diag[0.1 \quad 0.001 \quad 0.08].
\end{aligned}$$

In addition, the external disturbance $\boldsymbol{\tau_d} = [\tau_{d1} \ \tau_{d1} \ \tau_{d1}]^T$ given by

$$\begin{aligned}
\tau_{d1} &= 0.4\sin(2t)\\
\tau_{d2} &= 0.3\cos(2t)\\
\tau_{d3} &= 0.5\sin(3t)
\end{aligned}$$

is assumed to influence the robot as in (2). The desired trajectory is considered as

$$\begin{aligned}
q_{d1}(t) &= 0.3\cos\left(1.5t + \frac{\pi}{3}\right)\\
q_{d2}(t) &= 0.1\cos(1.5t)\\
q_{d3}(t) &= 0.2\cos(1.5t).
\end{aligned}$$

To show the simplicity of the SRWNNs in the SRWNNSMC system, each SRWNN consists of two input, two mother wavelet, one product node, one output. The initial values of the SRWNNs parameters $a_k$, $m_{jk}$, $d_{jk}$, and $w_j$ are given randomly in the range of [-1 1], but $d_{jk} > 0$. And also, the initial values of $\theta_{jk}$ are given by 0. That is, there are no feedback units initially. The inaccurate initial tuning parameters of the SRWNNs are trained optimally by the online parameter tuning methodology. The tracking results of the SRWNNSMC system shown in Fig. 1 indicate that the suggested method can overcome unknown model uncertainties resulting from the robot dynamics and external disturbances. Figs. 2 and 3 show the tracking errors and control inputs of joint 1, 2, and 3, respectively. In Fig. 2, we can see that the tracking errors which occur at the starting point drop quickly in less than a few seconds. Moreover, in Fig. 3, the chattering phenomena, the important problem of the traditional SMC, disappears since the SRWNNSMC system possesses the advantages of the intelligent boundary layer via the powerful learning capability of SRWNNs.

## 5. Conclusion

The intelligent gain and boundary layer based SMC method for robotic systems with unknown uncertainties has been developed. The SRWNNs having simple structures have been used to approximate the nonlinear terms in the robotic model, the width of the boundary layer and model uncertainty bound used as a switching gain in the sliding phase. The adaptation laws for training weights of SRWNNs and these for error compensators have been induced from the Lyapunov stability theory, which have been used to guarantee the asymptotic stability of the proposed controller. And also, the acceleration term in the sliding phase has been added to accelerate the convergence to the inside of the sliding surface. Finally, a simulation has been performed to show that the SRWNNSMC

Table 1. Simulation parameters

| | Mass ($m_i$, Kg) | | Link ($a_i$, m) | | Moment of Inertia |
|---|---|---|---|---|---|
| | Nominal | Actual | Nominal | Actual | ($I_{oi}$, Kgm$^2$) |
| Joint 1 | 1.0 | 1.5 | 0.5 | 0.6 | $43.33\times10^{-3}$ |
| Joint 2 | 0.7 | 2.0 | 0.4 | 0.6 | $25.08\times10^{-3}$ |
| Joint 3 | 1.4 | 3.0 | 0.3 | 0.5 | $32.67\times10^{-3}$ |

system could be applied for robotic systems without the knowledge of the uncertainty bound and eliminate the chattering phenomenon in the control effort.

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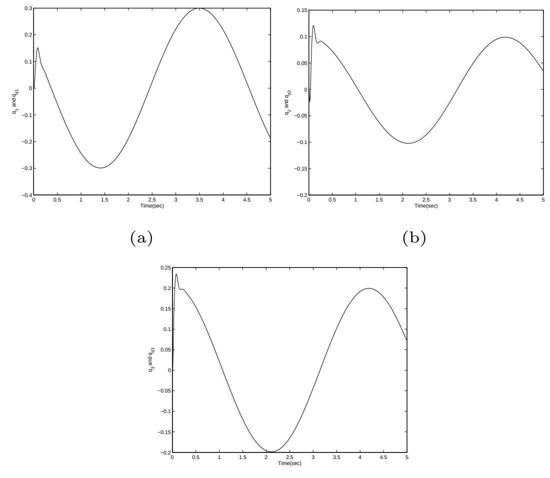


(a) (b) (c)

Fig. 1. Tracking results. (solid line: actual output, dotted line: desired trajectory) (a) Joint 1 (b) Joint 2 (c) Joint 3

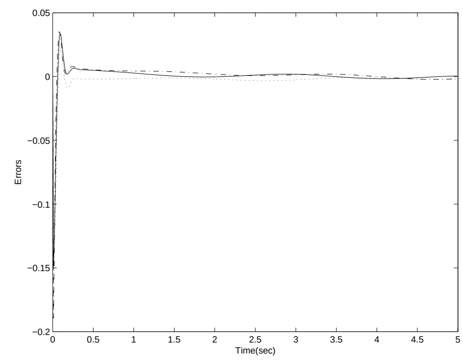


Fig. 2. Tracking errors. (solid line: Joint 1, dotted line: Joint 2, dash-dotted line: Joint 3)

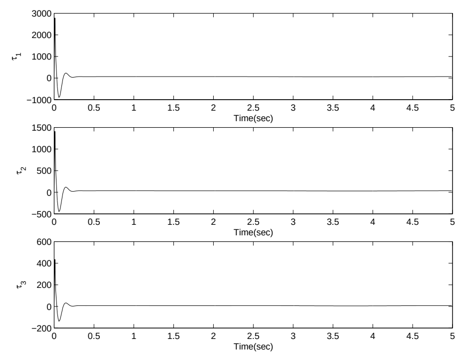


Fig. 3. Control inputs without the chattering phenomena.