Derivative State Constrained Optimal H_2 Integral Controller and its Application to Crane System

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Abstract: Study in this paper concerns the optimal H_2 integral servo problems for linear crane model systems via the constraints of the derivatives of state variables added to the standard constraints. It is shown in the paper that the derivative state constrained optimal H_2 integral servo problems can be reduced to the standard optimal H_2 control problem. The main subject of the paper is to apply the results of derivative state constrained H_2 integral servo theorem in crane system. The effect of our proposed controller with respect to mitigate an under damping for crane model system is also verified.

Keywords: Optimal H_2 Integral Servo Problem, Derivative state constraints, Crane system.

1. INTRODUCTION

The problem of how to control an under damping response of controlled system has been one of the fundamental problem in practical control engineering. The optimal servo control method minimizing a given performance index is known as a method for tracking desired position of trolley of crane system and stabilizing swing of its load. However, it is difficult to select the weighting matrices of performance index to mitigate an under damping response of swing of crane system. The integral servo problem is initiated by H. W. Smith and E. J. Davison [1], in which they proposed dual approaches, prototype affine and differential transformations, and gave some suggestions on measurement feedback schemes. However, successive researches are restricted only on the affine transformation approach that introduces integrators deductively and employs exclusively the state feedback [2-4]. The optimal H_2 integral control yields a zero steady-state tracking error for a disturbance which equals to both the dimension of the disturbance input and the dimension of the reference output. The regulator problem is formulated as the optimal control for oscillatory system such as minimizing a performance criterion involving time derivatives of state vector as well as usual system two-norm [5-6].

In this paper, we derive the optimal H_2 integral servo controller which stabilizes an oscillatory system such that the optimal control law is more effective to control an under damping steady-state tracking error by H_2 control framework. The main subject of the paper is to apply the results of derivative state constrained H_2 integral servo theorem in the crane system. The verification of the effect of our proposed controller with respect to mitigate an under damping for crane model system is also shown in the paper.

2. H_2 INTEGRAL SERVO PROBLEMS

2.1 Problem Statement

Given the controlled plant dynamics:

$$\begin{aligned}\frac{d}{dt}x(t) &= Ax(t) + B_1 w(t) + B_2 u(t)\\ y(t) &= C_2 x(t) + D_{12} w(t)\end{aligned} \tag{1}$$

where $x(t)$, $w(t)$, $u(t)$ and $y(t)$ denote the state vector, the step disturbance vector, the input vector and the output vector, respectively. Let $r(t)$ denote the reference step vector. For the error vector $e(t)$, its integral $x_I(t)$ are defined as follow:

$$\frac{d}{dt}x_I(t) = e(t) = r(t) - y(t) \tag{2}$$

In oder to suppress unwanted oscillations in servo problem, the derivative state constraint is introduced as in the following equations:

$$\frac{d}{dt}\begin{bmatrix}\dot{x}(t)\\ \dot{x}_I(t)\end{bmatrix} = \begin{bmatrix}A & 0\\ -C_2 & 0\end{bmatrix}\begin{bmatrix}\dot{x}(t)\\ \dot{x}_I(t)\end{bmatrix} + \left[\begin{array}{cc|cc|cc}\overline{B}_1 & 0 & 0 & 0 & 0 & 0\\ 0 & \overline{B}_{1I} & 0 & 0 & 0 & 0\end{array}\right]\dot{w}(t) + \begin{bmatrix}B_2\\ 0\end{bmatrix}\dot{u}(t) \tag{3}$$

$$\dot{z}(t) = \left[\begin{array}{cc}\overline{C}_1 & 0\\ 0 & \overline{C}_{1I}\\ \hline 0 & 0\\ 0 & 0\\ \hline 0 & 0\end{array}\right]\begin{bmatrix}\dot{x}(t)\\ \dot{x}_I(t)\end{bmatrix} + \left[\begin{array}{cc|cc|cc}0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ \hline 0 & 0 & \overline{D}_{11} & 0 & 0 & 0\\ 0 & 0 & 0 & \overline{D}_{11I} & 0 & 0\\ \hline 0 & 0 & 0 & 0 & 0 & 0\end{array}\right]\dot{w}(t) + \left[\begin{array}{c}0\\ 0\\ \hline 0\\ 0\\ \hline \overline{D}_{12}\end{array}\right]\dot{u}(t) \tag{4}$$

$$\begin{bmatrix} \dot{\bar{y}}(t) \\ \dot{y}_I(t) \end{bmatrix} = \begin{bmatrix} \bar{C}_2 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \left[\begin{array}{cc:cc:cc} 0 & 0 & 0 & 0 & \bar{D}_{21} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{D}_{21I} \end{array}\right] \dot{w}(t) \tag{5}$$

where

$$\dot{w}(t) = \left[\dot{w}_1(t) \quad \dot{w}_2(t) \;\vdots\; \dot{x}(t) \quad \dot{x}_I(t) \;\vdots\; \dot{w}_3(t) \quad \dot{w}_I(t)\right]^T \tag{6}$$

This system constitutes a singular problem. From FI and FC problems, the augmented generalized plant reduces to the following nonsingular plant;

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \breve{B}_1 \dot{w}(t) + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} u(t) \\ z(t) &= \breve{C}_1 \begin{bmatrix} x(t) \\ x_I(t) \end{bmatrix} + \breve{D}_{12} \dot{u}(t) \\ \begin{bmatrix} \dot{\bar{y}}(t) \\ \dot{y}_I(t) \end{bmatrix} &= \begin{bmatrix} \bar{C}_2 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \breve{D}_{21} \dot{w}(t) \end{aligned} \tag{7}$$

$w(t) = w_0 m(t)$, where *$m(t)$* stands for the unit step function, and the disturbance is taken in the sense of a hyperfunction or generalized function, and where

$$\breve{B}_1 = \left[\begin{array}{cc:c:cc} \bar{B}_{11} & 0 & 0 & A\bar{D}_{11} & 0 \\ 0 & \bar{B}_{11I} & 0 & -C_2\bar{D}_{11I} & 0 \end{array}\right] \tag{8}$$

$$\breve{C}_1 = \left[\begin{array}{cc} \bar{C}_{11} & 0 \\ 0 & \bar{C}_{11I} \\ \hdashline 0 & 0 \\ \hdashline \bar{D}_{11}A & 0 \\ -\bar{D}_{11I}C_2 & 0 \end{array}\right] \tag{9}$$

$$\breve{D}_{12} = \left[\begin{array}{c} 0 \\ 0 \\ \hdashline \bar{D}_{12} \\ \hdashline \bar{D}_{11}B_2 \\ 0 \end{array}\right] \tag{10}$$

$$\breve{D}_{21} = \left[\begin{array}{c:cc:cc} 0 & \bar{D}_{21} & & \bar{C}_2\bar{B}_{11} & 0 \\ 0 & & \bar{D}_{21I} & 0 & \bar{B}_{11I} \end{array}\right] \tag{11}$$

The integral servo controller is to be designed by deriving the H_2 controller to the above plant (7).

2.2 Solution

The solution to the derivative state constrained H_2 optimal control defined above is given by the loop shifting procedure.

Loop shifting

i) Singular value decomposition

There always exist unitary matrices $U_i, V_i,\ i = 1,2$ for the singular value decomposition of $\breve{D}_{12}$ and $\breve{D}_{21}$;

$$\breve{D}_{12} = U\begin{bmatrix} 0 \\ \Sigma_1 \end{bmatrix} V,\ \Sigma_1 = \begin{bmatrix} \sigma_{11} & & \\ & \ddots & \\ & & \sigma_{1r} \end{bmatrix},\ r = \dim(u) \tag{12}$$

$$\breve{D}_{21} = U\begin{bmatrix} 0 & \Sigma_2 \end{bmatrix} V_2^T, \Sigma_2 = \begin{bmatrix} \sigma_{11} & & \\ & \ddots & \\ & & \sigma_{1l} \end{bmatrix},\ l = \dim\begin{bmatrix} y(t) \\ y_I(t) \end{bmatrix} \tag{13}$$

where $\sum_i, i = 1,2$ are the diagonal singular value matrices. Using the results obtained above, input and output vectors and accordingly the generalized plant are transformed as follows.

ii) Variable transformation

The generalized plant can be obtained by using the following variable transformations defined by

$$\dot{w}(t) = V_2 \dot{\hat{w}}(t) \tag{14}$$

$$\dot{\hat{z}}(t) = U_1^T \dot{z}(t) \tag{15}$$

$$\dot{u}(t) = V_1 \Sigma_1^{-1} \dot{\hat{u}}(t) \tag{16}$$

$$\begin{bmatrix} \dot{y}(t) \\ \dot{y}_I(t) \end{bmatrix} = \Sigma_2^{-1} U_2^T \begin{bmatrix} y(t) \\ y_I(t) \end{bmatrix}. \tag{17}$$

Substituting Eq. (14) through Eq. (17) into the Eq. (7), then the generalized plant is obtained as

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \hat{B}_1 \dot{\hat{w}}(t) + \hat{B}_2 \dot{\hat{u}}(t) \\ \dot{\hat{z}}(t) &= \hat{C}_1 \begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \hat{D}_{12} \dot{u}(t) \\ \begin{bmatrix} \dot{\hat{y}}(t) \\ \dot{y}_I(t) \end{bmatrix} &= \hat{C}_2 \begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \hat{D}_{21} \dot{\hat{w}}(t) \end{aligned} \tag{18}$$

where the coefficient matrices are given as follows:

$$\begin{aligned} &\hat{B}_1 = \breve{B}_1 V_2,\ \ \hat{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix} V_1 \Sigma_1^{-1} \\ &\hat{C}_1 = U_1^T \breve{C}_1,\ \ \hat{C}_2 = \Sigma_2^{-1} U_2^T \begin{bmatrix} \bar{C}_2 & 0 \\ 0 & I \end{bmatrix} \\ &\hat{D}_{12} = U_1^T \breve{D}_{12} V_1 \Sigma_1^{-1} = \begin{bmatrix} 0 \\ I \end{bmatrix} \\ &\hat{D}_{21} = \Sigma_2^{-1} U_2^T \breve{D}_{21} V_2 = \begin{bmatrix} 0 & I \end{bmatrix} \end{aligned} \tag{19}$$

The generalized plant is now reduced to the standard form of the H_2 control problem. Suppose that the transformed generalized plant parameter matrices (19) satisfy the following relations:

A-1$_2$). $\begin{cases} \left(\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}, \hat{B}_2 \right) \text{ is controllable, and} \\ \left(\hat{C}_2 \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} \right) \text{ is observable.} \end{cases}$

A-2$_2$). $\begin{cases} \hat{D}_{12} \text{ has full column rank, and} \\ \hat{D}_{21} \text{ has full row rank.} \end{cases}$

A-3$_2$). $\begin{cases} \begin{bmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} - j\omega I & \begin{bmatrix} \hat{B}_2 \\ 0 \end{bmatrix} \\ \hat{C}_1 & \hat{D}_{12} \end{bmatrix} \\ \text{has full column rank } \forall\omega \in \Re, \text{ and} \\ \begin{bmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} - j\omega I & \hat{B}_1 \\ \hat{C}_2 & \hat{D}_{21} \end{bmatrix} \\ \text{has full row rank } \forall\omega \in \Re. \end{cases}$

Hamiltonian Matrices

Under the above assumptions, the optimal H_2 solution to the transformed generalized plant (18) is given as follows;

A couple of Hamiltonian matrices are constituted as

$$H_2 = \begin{bmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} - \hat{B}_2 \hat{D}_{12}^T \hat{C}_1 & -\hat{B}_2 \hat{B}_2^T \\ -\hat{C}_1^T \hat{C}_1 + \hat{C}_1^T \hat{D}_{12} \hat{D}_{12}^T \hat{C} & -\left\{ \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} - \hat{B}_2 \hat{D}_{12}^T \hat{C}_1 \right\}^T \end{bmatrix} \tag{20}$$

$$J_2 = \begin{bmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}^T - \hat{C}_2^T \hat{D}_{21} \hat{B}_1^T & -\hat{C}_2^T \hat{C}_2 \\ -\hat{B}_1 \hat{B}_1^T + \hat{B}_1 \hat{D}_{12} \hat{D}_{21}^T \hat{B}_1^T & -\left\{ \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}^T - \hat{C}_2^T \hat{D}_{21} \hat{B}_1^T \right\}^T \end{bmatrix} \tag{21}$$

Then, it is guaranteed that the solutions exist, which make

$\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \hat{B}_2 \hat{F}_2$ and $\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \hat{L}_2 \hat{C}_2$ stable.

From the couple of Riccati solutions,

$$X_2 = Ric(H_2) \geq 0 \tag{22}$$

$$Y_2 = Ric(J_2) \geq 0 \tag{23}$$

it is able to construct the following optimal solution to the generalized plant (18);

$$\hat{K}_{H_2}(s) = \left[\begin{array}{c|c} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \hat{B}_2 \hat{F}_2 + \hat{L}_2 \hat{C}_2 & -\hat{L}_2 \\ \hline \hat{F}_2 & 0 \end{array} \right] \tag{24}$$

where

$$\hat{F}_2 = -(\hat{B}_2^T X_2 + \hat{D}_{12}^T \hat{C}_1),$$
$$\hat{L}_2 = -(Y_2 \hat{C}_2^T + \hat{B}_1 \hat{D}_{21}^T) \tag{25}$$

2.3 Main Results

The theorem for the derivative state constrained H_2 integral servo problem is given as follows;

Theorem (Derivative State Constrained Optimal H_2 Integral Servo.)

The derivative state constrained H_2 integral servo controller for the controlled plant (24) is given as

$$K_{H_2}(s) = \left[\begin{array}{c|c} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} F_2 + L_2 \begin{bmatrix} \bar{C}_2 & 0 \\ 0 & I \end{bmatrix} & -L_2 \\ \hline F_2 & 0 \end{array} \right] \tag{26}$$

or its integral form

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} \hat{x}(t) \\ \hat{x}_1(t) \end{bmatrix} &= \hat{A}_2 \begin{bmatrix} \hat{x}(t) \\ \hat{x}_1(t) \end{bmatrix} - L_2 \begin{bmatrix} \bar{C}_2 \bar{y}(t) \\ \int_0^t e(t)dt \end{bmatrix} \\ u(t) &= F_2 \begin{bmatrix} \hat{x}(t) \\ \hat{x}_1(t) \end{bmatrix}; \end{aligned} \tag{27}$$

where

$$\left. \begin{aligned} \hat{A}_2 &= \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} V_1 \Sigma_1^{-1} \hat{F}_2 + \hat{L}_2 \Sigma_2^{-1} U_2^T \begin{bmatrix} \bar{C}_2 & 0 \\ 0 & I \end{bmatrix} \\ F_2 &= V_1 \Sigma_1^{-1} \hat{F}_2 = -V_1 \Sigma_1^{-2} V_1^T \left\{ \begin{bmatrix} B_2^T & 0 \end{bmatrix} X_2 + \breve{D}_{12}^T \breve{C}_1 \right\} \\ L_2 &= \hat{L}_2 \Sigma_2^{-1} U_2^T = -\left\{ Y_2 \begin{bmatrix} \bar{C}_2^T & 0 \\ 0 & I \end{bmatrix} + \breve{B}_1 \breve{D}_{21}^T \right\} U_2 \Sigma_2^{-2} U_2^T \end{aligned} \right\} \tag{28}$$

under the assumptions

A-1$'_2$). $\begin{cases} (A, B_2) \text{ is controllable,} \\ (C_2, A) \text{ is observable, and} \\ \text{rank} \begin{bmatrix} A & B_2 \\ C_2 & 0 \end{bmatrix} = \dim(x) + \dim(y). \end{cases}$

$$\text{A-}2_2'). \quad \begin{cases} \bar{D}_{12} \text{ has full column rank, and} \\ \begin{bmatrix} \bar{D}_{21} & 0 \\ 0 & \bar{D}_{21I} \end{bmatrix} \text{ has full row rank.} \end{cases}$$

$$\text{A-}3_2'). \quad \begin{cases} \begin{bmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} - j\omega I & \begin{bmatrix} B_2 \\ 0 \end{bmatrix} \\ \breve{C}_1 & \breve{D}_{12} \end{bmatrix} \\ \text{has full column rank } \forall \omega \in \Re, \text{ and} \\ \begin{bmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} - j\omega I & \breve{B}_1 \\ \begin{bmatrix} \bar{C}_2 & 0 \\ 0 & I \end{bmatrix} & \breve{D}_{21} \end{bmatrix} \\ \text{has full row rank } \forall \omega \in \Re. \end{cases}$$

□

Proof : The proof is neglected here.

3. ILLUSTRATIVE EXAMPLE

Consider the crane system shown in Fig.1 as the system to be controlled. Symbols *M, m, L* and *g* represent the mass of trolley, the mass of load, the rope length and gravitational acceleration respectively.

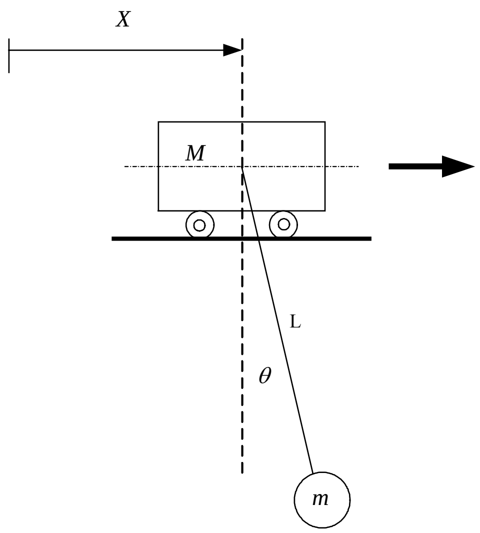

Fig.1 Crane system.

The state dynamic equation of the crane system can be represented by

$$\begin{aligned} \frac{d}{dt} x_p(t) &= f(x) + g_1(x) u(t) \\ y_p(t) &= h(x) \end{aligned} \tag{29}$$

where

$$f(x) = \begin{pmatrix} \dot{X}(t) \\ -\dfrac{mg}{M} \sin\theta \\ \dot{\theta} \\ -\dfrac{g \sin\theta}{L} + \dfrac{mg \sin\theta \cos\theta}{ML} \end{pmatrix}, \quad g_1(t) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\dfrac{\cos\theta}{L} \end{bmatrix},$$

$u(t) = \dfrac{F_X}{M}$, $h(t) = \begin{bmatrix} X(t) \\ \theta(t) \end{bmatrix}$. The state vector is defined by $x(t) = \begin{bmatrix} X(t) & \dot{X}(t) & \theta(t) & \dot{\theta}(t) \end{bmatrix}^T$ where $X(t)$ denotes the horizontal displacement of the trolley at time t, $\theta(t)$ denotes the angular rotation of the pendulum at time t and L represents the distance of the rope. The linearized state equation is given by

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & mg/M & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -g(1-m/M)/L & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1/L \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t) \end{aligned} \tag{30}$$

The numerical values of *M, m, L* and *g* used in the numerical crane model are shown in Table 1.

Table 1 Numerical values of the crane system.

M (kg)	m (kg)	L (m)	g (m/s^2)
120	12	1.6	9.8

The designing parameters in the generalized plant $\bar{B}_1$, $\bar{D}_{12}$, $\bar{C}_1$, $\bar{D}_{11}$ and $\bar{D}_{12}$ are chosen as:

$$\bar{C}_1 = \bar{B}_1^T = \begin{bmatrix} diag\begin{bmatrix} \sqrt{10^{qi}} & \sqrt{10^{qi}} & \sqrt{10^{qi}} & \sqrt{10^{qi}} & \sqrt{10^{qi}} \end{bmatrix} \\ \begin{bmatrix} 0_{5\times5} \end{bmatrix} \\ \begin{bmatrix} 0_{2\times5} \end{bmatrix} \end{bmatrix} \tag{31}$$

$$D_{11} = \begin{bmatrix} \begin{bmatrix} 0_{5\times5} \end{bmatrix} & \begin{bmatrix} 0_{5\times5} \end{bmatrix} & \begin{bmatrix} 0_{5\times2} \end{bmatrix} \\ \begin{bmatrix} 0_{5\times5} \end{bmatrix} & \bar{D}_{11} & \begin{bmatrix} 0_{5\times5} \end{bmatrix} \\ \begin{bmatrix} 0_{2\times5} \end{bmatrix} & \begin{bmatrix} 0_{2\times5} \end{bmatrix} & \begin{bmatrix} 0_{2\times2} \end{bmatrix} \end{bmatrix}$$

$$\bar{D}_{11} = diag\begin{bmatrix} \sqrt{e^{ni}} & \sqrt{e^{ni}} & \sqrt{e^{ni}} & \sqrt{e^{ni}} & \sqrt{e^{ni}} \end{bmatrix} \tag{32}$$

$$\bar{D}_{12} = \begin{bmatrix} \begin{bmatrix} 0_{5\times5} \end{bmatrix} \\ \begin{bmatrix} 0_{5\times5} \end{bmatrix} \\ \sqrt{0.1} \end{bmatrix} \tag{33}$$

$$\begin{bmatrix} \bar{D}_{21} & 0 \\ 0 & \bar{D}_{21I} \end{bmatrix} = \begin{bmatrix} \sqrt{0.01} & 0 \\ 0 & \sqrt{0.01} \end{bmatrix} \quad (34)$$

In order to be recovered from the slow time response, the prescribed degree of stability $\alpha = 0.5$, which was proposed by B.D.O Anderson and J. B. Moore [7], is employed. Figure 2 shows variation of the closed-loop poles of the crane system for the parameters varied from $ni = -6$ to $ni = -1$. It verifies that the smaller value of the parameter ni is, the smaller the imaginary part of the closed-loop poles become. Figure 3 and Figure 4 show the responses of the crane system controlled by H_2 integral servo controller with the initial state $x(0) = [0 \quad 0 \quad -0.25 \quad 0]^T$. Note that the under damping of the angular rotation suppressed within 5 seconds.

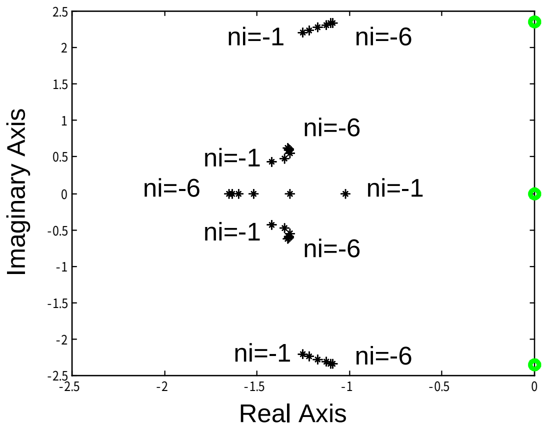

Fig.2 Closed-loop poles for
$qi = -1 \; and \; ni = -6, -5, -4, -3, -2 \; and \; -1.$

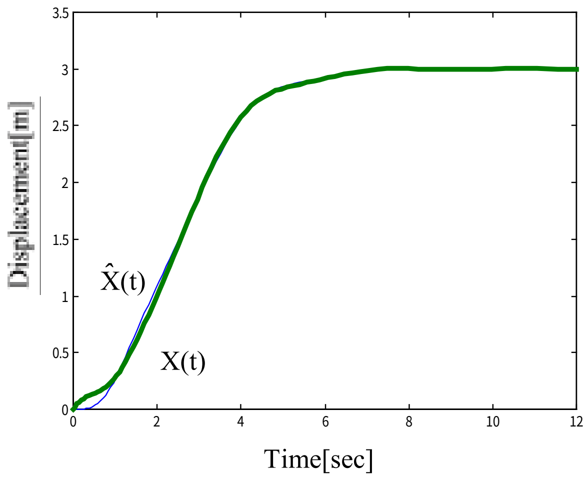

Fig.3 Closed-loop responses to the reference horizontal displacement $r = 3$ m.

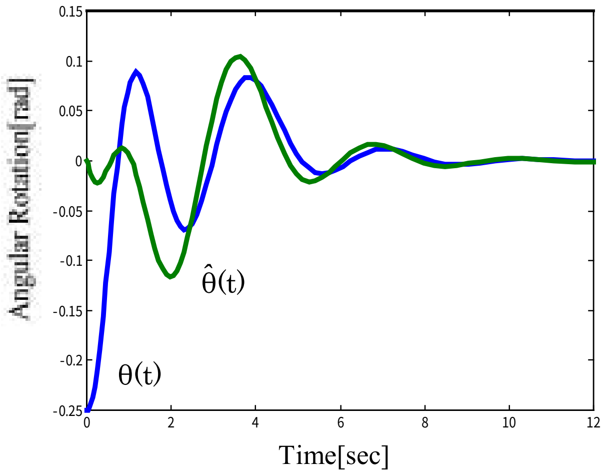

Fig.4 Closed-loop responses with initial angular rotation $\theta(0) = -0.25$ rad.

4. CONCLUSION

The derivative state constrained H_2 integral servo controller for oscillatory system has been proposed in this paper. It has been shown that the parameter *ni* could reduce the imaginary part of closed-loop poles than a popular parameter *qi* in term of performance cost as H_2 optimization in the general framework. It is recognized that the servo problem can be applied to the systems whose reference inputs as well as disturbances are all given by step functions. It has been shown in an illustrative example that the proposed schemes has applied to mitigate the under damping for crane model system.

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