Analysis of Appropriate Parameters for Piezoelectric Ceramic Utilization by Using BVD Model

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Abstract: This paper presents an approach to evaluate the appropriate parameters for Piezoelectric ceramic utilization by adopting Impedance Method. Butterworth Van Dyke model (BVD) is considered to use as an equivalent circuit of Piezoelectric ceramic in case of no load. The experimental results from this model will be compared with the results from a circular Piezoelectric ceramic with 4.8 cm. diameter and 3 mm. thickness. The Thickness Mode vibration measured by Impedance Analyzer model 4192A can be analyzed from 1Hz to 13MHz for calculating and analyzing parameters at resonance frequency and anti-resonance frequency. These parameters are evaluated to design the efficient circuit for Piezoelectric ceramic utilization to obtain the optimal efficiency.

Keywords: Piezoelectric ceramic, Equivalent circuit, Electrical equivalent circuit, BVD

1. INTRODUCTION

At present, Piezoelectric ceramic is used in several machines such as ultrasonic cleaner and ultrasonic welder. To obtain the optimal efficiency, the equivalent circuit of Piezoelectric ceramic should be well analyzed. In this paper, the Butterworth Van Dyke model is considered. This model has 2 parallel branches as shown in Fig. 1. The First one consists of series of motional resistance (*R*) which represents the mechanical dissipation , inductor (*L)* which is related to the Piezoelectric motional **,** and capacitor (*C*). Another one consists of the capacitor corresponds to the electrostatic capacitance (C_o). The R, C, L and C_o parameters are calculated by adopting various methods such as IEEE Standard [2], [3], Smits [7], Sherrit [8] and Xu [10]. Smits's and Sherrit's consider those parameters as complex numbers whilst Xu's is similar to Smits's. The impedance of Piezoelectric ceramic is measured under the frequency that covers the resonance frequency (f_r), anti-resonance frequency (f_a) and mechanical parameters (elastic, piezoelectric constant and coupling coefficient). However, it is complicated to calculate due to many parameters required. As a result, this paper presents the calculation of BVD parameters by considering the frequency response of *R*, *C*, *L* and C_o in BVD circuit and then the Piezoelectric ceramic applied in this research will be utilized in an ultrasonic cleaner.

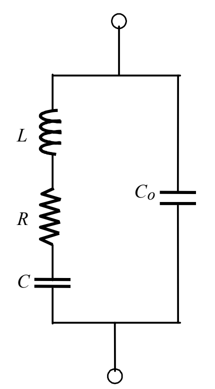

Fig.1 The Butterworth Van Dyke (BVD) model

2. THEORY

Fig. 1 is illustrated the equivalent circuit of no-load Piezoelectric ceramic BVD in thickness mode. *R*, *C*, *L* and C_o are considered as constant parameters and independent to the frequency and impedance as shown in Eq. (1).

$$Z = \frac{1}{j\omega C_o} // \left(R + j\omega L - \frac{j}{\omega C} \right)$$

$$Z = \frac{R + j\omega L - {}^{j}\!/_{\omega C}}{1 - \omega^2 C_o L + {}^{C_o}\!/_{C} + j\omega R C_o} \quad (1)$$

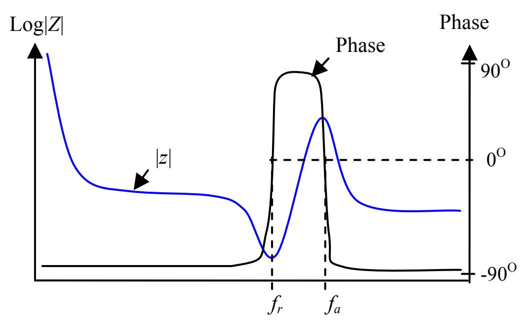

Fig.2 The frequency response characteristic of piezoelectric ceramic

From Fig. 2, the characteristics of Piezoelectric ceramic in various frequencies are as follow:

2.1) At Resonance Frequency, the reactance of *R-L-C* branch is zero, thus:

$$\omega_r L = \frac{1}{\omega_r C}$$

$$LC = \frac{1}{\omega_r^2} \quad (2)$$

Where $\omega_r = 2\pi f_r$ is the angle velocity at resonance frequency.

2.2) At Anti-resonance Frequency:

$$\omega_a^2 C_o L = 1 + {C_o}/{C} \ , \omega_a = \sqrt{\frac{C + C_o}{C_o C L}}$$

$$C_o L = \left(1 + \frac{C_o}{C}\right) \Big/ \omega_a^2 \qquad (3)$$

Where ω_a is the angle velocity at anti-resonance frequency (= $2\pi f_a$). Divide Eq. (3) by Eq. (2):

$$\frac{C_o}{C} = \frac{\omega_r^2}{\omega_a^2 - \omega_r^2} \qquad (4)$$

From Eq. (2), (3) and (4):

$$C_o L = \frac{1}{\omega_a^2 - \omega_r^2} \qquad (5)$$

Calculate Eq. (1) by using $\frac{C_o}{C}$ and LC_O from Eq. (4) and (5):

$$Z = \frac{R + j\omega L - {j}/{\omega C}}{\left(\omega_a^2 - \omega^2 / \omega_a^2 - \omega_r^2\right) + j\omega R C_o} \qquad (6)$$

Phase of impedance is:

$$Z(\theta) = tan^{-1}\left(\frac{Z_{Im}}{Z_{Real}}\right) \qquad (7)$$

Where Z_{Real} is the real number of impedance
Z_{Im} is the imaginary number of impedance

From $Z(\theta)$ graph in Fig. (2), if the frequency is greatly lower than resonance frequency, $Z(\theta) \sim -90^o$, then:

$$Z_{Im} \sim {1}/{\omega C} \ \text{or} \ \omega L << {1}/{\omega C} \ \text{and} \ RC_O \approx 0 \qquad (8)$$

If ω is greatly lower than resonance, then:

$$Z \approx \frac{\omega_a^2 - \omega_r^2}{\omega_a^2 - \omega^2}\left(R - {j}/{\omega C}\right) \qquad (9)$$

At resonance frequency $(\omega = \omega_r)$, the reactance is zero $(j\omega L - {j}/{\omega C} = 0)$. Thus the impedance at resonance point is the resistance or R only as proven in Eq. (10) where $LC = 1/\omega_r^2$:

$$Z = \frac{R}{1 + j\omega R C_O}$$

$$Z \approx R \qquad (10)$$

From Eq. (8), if $j\omega_r R C_O \approx 0$, then:

$$\omega = \omega_r \qquad (11)$$

From Eq. (10), it is found that when the impedance measured at the low frequency is calculated in Eq. (8) and ω is the angle velocity at the considered frequency of $|Z|$ and R is $|Z|$ at resonance frequency, C can be calculated by using Eq. (12):

$$C = \frac{1}{\omega\sqrt{\left(\frac{\omega_a^2 - \omega^2}{\omega_a^2 - \omega_r^2}\right)^2 |Z|^2 - R^2}} \qquad (12)$$

Finally, C_O and L can be calculated by applying C into Eq. (4) and (5)

3. EXPERIMENT

The Piezoelectric ceramic with 4.8 cm. diameter and 3 mm. thickness is electroded by silver paste at the top and the bottom and then connected to the system as shown in Fig. 4. The HP Vee program is applied to control the Impedance Analyzer Model HP4192A via HP-IB interface and collect the impedance at 100 Hz - 1.3 MHz. Then, the resonance frequency and the impedance at resonance frequency and lower resonance frequency will be applied to calculate R, L and C_o using Eq. (10), (4) and (5). The calculated value will be evaluated and compared with the value calculated by using Eq. (1). In this paper, only the fundamental resonance is considered regardless the resonance frequency at harmonic period.

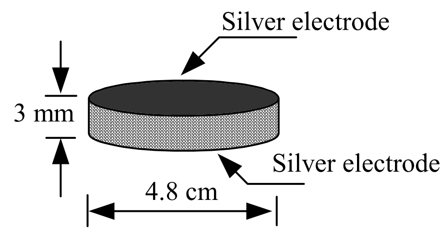

Fig.3 Physical of Piezoelectric ceramic sample

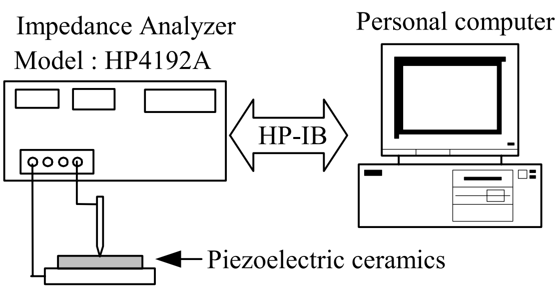

Fig.4 Impedance of Piezoelectric ceramic measurement diagram

4. EXPERIMENTAL RESULTS

From experiment and measurement of Piezoelectric ceramic impedance, the calculated parameters from BVD model are as shown in Table 1. Fig. 5 and 6 are illustrated the comparison of size ($|Z|m$) and phase (PhaseMeas) of the impedance at 100 Hz - 100 kHz measured by Impedance Analyzer Model HP4192A and the calculated impedance ($|Z|cal$ and PhaseCal). The impedance at 1 kHz is selected to calculated in Eq. (12). To obtain the appropriate value of C, the impedance at lower resonance frequency is suggested such as 100 Hz, 1 kHz, 2 kHz, 3 kHz, 6 kHz and 14 kHz. The BDV parameters are as shown in Table 2 and the percentage of error (%error) is shown in Fig. 7.

Table 1 The Piezoelectric ceramic properties

Piezoelectric ceramic properties	
Fundamental resonance frequency (f_r)	60.329 kHz
Fundamental anti-resonance frequency (f_a)	68.971 kHz
Impedance at f_r	2.847 Ω
Impedance at 1kHz	41.040 kΩ
Impedance at 2 kHz	20.540 kΩ
Impedance at 3 kHz	11.245 kΩ
Impedance at 6 kHz	6.526 kΩ
Impedance at 14 kHz	2.863 kΩ

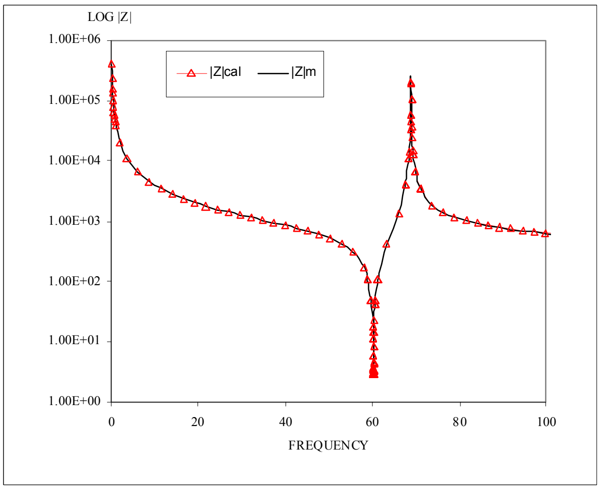

Fig.5 Gain impedance of Piezoelectric ceramic measurement at various frequency

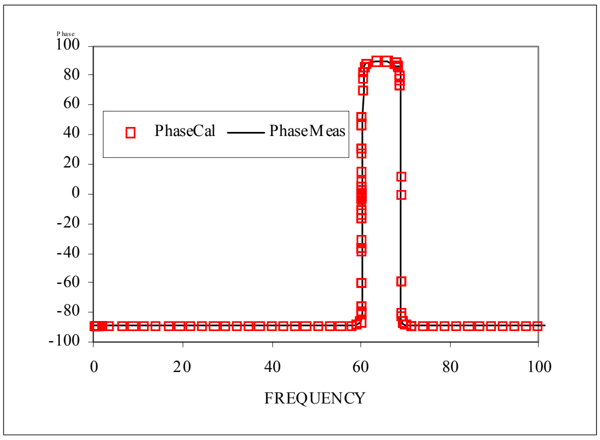

Fig.6 Phase impedance of Piezoelectric ceramic at various frequency

Table 2 Calculated parameters of Butterworth Van Dyke circuit at 100 Hz, 1 kHz, 2 kHz, 3 kHz, 6 kHz, 14 kHz

Parameters	Frequency					
	100 (Hz)	1 (kHz)	2 (kHz)	3 (kHz)	6 (kHz)	14 (kHz)
R (Ω)	2.847	2.847	2.847	2.847	2.847	2.847
C (pF)	91.22	91.16	91.17	92.69	91.33	97.39
C_O (nF)	2.9715	2.9693	2.9697	3.019	3.037	3.172
L (mH)	7.63	7.64	7.64	7.51	7.47	7.15

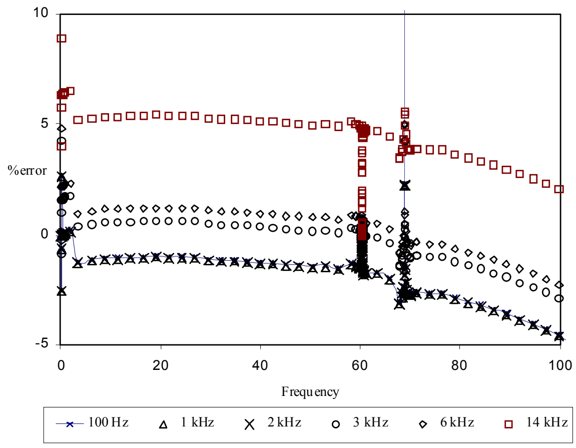

Fig.7 Error percentage of calculated impedance at various frequency (100 Hz, 1 kHz, 2 kHz, 3 kHz, 6 kHz and 14 kHz)

5. CONCLUSION

In conclusion, from the calculation of size ($|Z|$) and phase ($Z(\theta)$), it is found that the impedance in the greatly low frequency or in the range (f_r - f_a) is close to the impedance measured by HP4192A. If the low frequency such as 100 Hz, 1 kHz and 2 kHz, is used to calculate C, then the error percentage of $|Z|$ is about -1% as well as the use of impedance and frequency at 3 kHz that has the error percentage lower than

1%. However, if the impedance at 14 kHz is calculated, then the error percentage is about 5% because the impedance is not as estimated as in Eq. (8). ωL has more influence upon the impedance than $1/\omega C$ does. Therefore, the impedance chosen to calculate should be less than $f << (f_a^2 - f_r^2)$. From the consideration of error percentage, it is found that the selected impedance at 100 Hz, 1 kHz and 2 kHz provide the similar results. Normally, the Dielectric Constant measurement is operated at 1 kHz [1] with 20-25% error at anti-resonance frequency. In determining the frequency as the resonance and anti-resonance frequency with minimum and maximum impedance respectively, the error in calculation will be occurred because those frequencies are not in the same point based on IEEE standard [2]. In this paper, the Piezoelectric ceramic applied to the ultrasonic cleaner is used the frequency within the resonance frequency range only. Hence, the impedance error is acceptable to apply in electronic design.

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