Dynamic Anti-Windup for Robot Systems with Friction

S.-S. Yoon*, Y. Yamada*, J.-K. Park**, and T.-W. Yoon***

*Intelligent Systems Institute, AIST, Tsukuba-shi, Ibaraki 305-8568, Japan
(Tel: +81-29-861-3311; Fax: +81-29-861-5942 ; Email:yoon@cello.korea.ac.kr, yamada-yoji@aist.go.jp)
**School of Information and Communication Engineering, Sungkyunkwan University, Suwon 440-746, Korea
(Tel: +82-31-290-7138; Fax: +82-31-290-7231; Email:pjk@yurim.skku.ac.kr)
***School of Electrical Engineering, Korea University, Seoul 136-701, Korea
(Tel: +82-2-3290-3240; Fax: +82-2-928-8909; Email:twy@cello.korea.ac.kr)

Abstract: Though several previous anti-windup techniques have been proposed, they are limited to linear systems or friction is not considered. Thus this paper proposes a compensation scheme for input-constrained robot systems with friction to cope with the windup phenomenon and shows its effectiveness by simulations. Given a feedback linearizing controller for a robot system designed without considering its input constraint, an additional dynamic compensator is proposed to account for the constraint. The dynamic anti-windup is based on the minimization of a reasonable performance index, and properties of the resulting closed-loop are presented.

Keywords: anti-windup, robot, saturation, friction, nonlinear.

1. INTRODUCTION

Actuator saturation causes a nonlinear problem that needs to be dealt with in almost all practical control systems. Feedback loops are broken when the actuators saturate. Performance deterioration and even instability may result especially if the plants or controllers are unstable. A general term for these phenomena is referred to as windup, and compensation for preventing this windup is called anti-windup [2]. Recently, a rigorous definition of the anti-windup is presented on the basis of an L_2 criterion, and it is shown that all static linear observer-based compensation schemes satisfy the definition at least locally [10]. Generally, the following strategy is adopted for anti-windup: design a controller ignoring the saturation, and then add an appropriate compensator to account for the saturation. This is referred to as the two-step design technique. However, anti-windup design at the second phase is often carried out in an ad hoc manner. It is therefore necessary to have a systematic anti-windup design method, which is based on optimization involving errors between constrained and unconstrained outputs (i.e. with and without saturation). In major anti-windup techniques, there are observer-type technique [1], conditioning technique [7], conventional anti-windup [5], unified framework by coprime factorization [12], finite gain technique [19], [10], and optimization based methods [17], [16].
Recently in nonlinear control methods, feedback linearization [9], [18], backstepping stabilization technique [13], and sliding mode control [18] have attracted researchers' attention. Among the control schemes for nonlinear systems, this paper deals with a feeback linearizing control scheme for nonlinear robots. In anti-windup schemes for input-constrained feeback linearizable sytems, there are a method using linear MPC (model predictive control) and constraint mapping [14], one using internal model control and constraint mapping, [6], observer-type anti-windup [11], anti-windup uniting local and global controllers for Euler-Lagrange systems [15]. These schemes do not use an explicit performance index related to anti-windup.
Though several previous anti-windup techniques have been proposed, they are limited to linear systems or a Coulomb friction term is not considered in their nonlinear models. As a robot system with the Coulomb friction term may cause a larger windup problem compared with a system without it, anti-windup compensation is more required. Thus this paper proposes a compensation scheme for input-constrained robot systems with friction to cope with the windup phenomenon, which is extended from the results for constrained linear systems in [17]. Given a feedback linearizing controller for a robot system designed without considering its input constraint, an additional dynamic compensator is proposed to account for the constraint. To design a compensator, an error model between the controller state variables of systems with and without saturating actuators is derived. Then using the Parseval's theorem and Laplace transform, an optimal dynamic anti-windup solution to minimize a reasonable performance index is derived. In the proposed method, the controller state is maintained as in the unsaturated system. Finally, properties and simulation results of the resulting closed-loop are presented.

2. PROBLEM FORMULATION

Consider a nonlinear robot with a constrained input and a friction term as follows:

$$M_R(q)\ddot{q} + C_R(q,\dot{q})\dot{q} + G_R(q) + d = \mathrm{sat}(u) - F_R(q,\dot{q}) \quad (1)$$

where $(q,\dot{q})$ represents the state variables for the robot, $M_R(q)$ is the generalized inertial matrix, $C_R(q,\dot{q})\dot{q}$ represents the generalized centrifugal and Coriolis terms, $G_R(q)$ is the vector of gravitational forces, u is the vector of the control inputs (typically consisting of forces and torques), d is an unknown disturbance input, and $\mathrm{sat}(\cdot)$ is the vectorized saturation function, and $F_R(q,\dot{q}) = [F_1,\ldots,F_m]^T$ are the friction forces which can be represented by the LuGre model [4] acting independently in each joint as

$$\begin{aligned} F_j &= \sigma_{0_j} z_j + \sigma_{1_j} \dot{z}_j + \sigma_{2_j} \dot{q}_j \\ \dot{z}_j &= -\alpha_j(q_j, \dot{q}_j)|\dot{q}_j| z_j + \dot{q}_j \end{aligned} \tag{2}$$

where $j = 1, \ldots, m$, and z_j denotes the average deflection of bristles, which is not measurable, $\sigma_{0_j}, \sigma_{1_j}, \sigma_{2_j}$ are friction force parameters that can be physically explained as the stiffness of bristles, damping coefficient, and viscous coefficient, and the nonlinear characteristic function $\alpha_j(q_j, \dot{q}_j)$ is a finite positive funciton whose one parametriztion to the Stribeck effect is given by

$$\alpha_j(\dot{q}_j) = \frac{\sigma_{0_j}}{f_{c_j} + (f_{s_j} - f_{c_j})e^{-(\dot{q}_j/\dot{q}_{s_j})^2}} \tag{3}$$

where f_{c_j} is the Coulomb friction level, f_{s_j} is the level of the stiction force and $\dot{q}_{s_j}$ is the constant Stribeck velocity. The LuGre-model can describe all the dynamic effects of friction such as the pre-sliding displacement, the frictional lag, the Stribeck effect [20]. In this paper, for a steady-state motion of the bristles the following simple measurable model is considered as a starting point towards the model (2)

$$F_j(\dot{q}_j) = f_{c_j} \mathrm{sgn}(\dot{q}_j) + \sigma_{2_j} \dot{q}_j \tag{4}$$

which contains the Coulomb and viscous friction terms. The dynamics (1) satisfies some important physical properties as follows:

Property 1. $M_R(q)$ is a positive definite symemetric matrix bounded by $m_1 I \leq M_R(q) \leq m_2 I$, where m_1, m_2 are positive constants.

Property 2. The unknown disturbance satisfies $\|d\| \leq d_M$, with d_M is a positive constant. ($\|\cdot\|$ is the Euclidean norm of the corresponding vector.)

The robot dynamics (1) can be rewritten into a normal form [9] as follows:

$$\dot{\psi} = A_p \psi + B_p(\alpha - \beta - M_R^{-1} d + M_R^{-1} \mathrm{sat}(u)) \tag{5}$$

where

$$\alpha = M_R^{-1}(q)(-C_R(q,\dot{q})\dot{q} - G_R(q) - F_R(q,\dot{q}))$$

$$\psi = \begin{bmatrix} q_1 - q_{1d} \\ \dot{q}_1 - \dot{q}_{1d} \\ \vdots \\ q_m - q_{md} \\ \dot{q}_m - \dot{q}_{md} \end{bmatrix}, \ \beta = \begin{bmatrix} \ddot{q}_{1d} \\ \vdots \\ \ddot{q}_{md} \end{bmatrix} \tag{6}$$

$$A_p = \begin{bmatrix} 0 & 1 & & & \\ 0 & 0 & & & \\ & & \ddots & & \\ & & & 0 & 1 \\ & & & 0 & 0 \end{bmatrix}, \ B_p = \begin{bmatrix} 0 & & \\ 1 & & \\ & \ddots & \\ & & 0 \\ & & 1 \end{bmatrix} \tag{7}$$

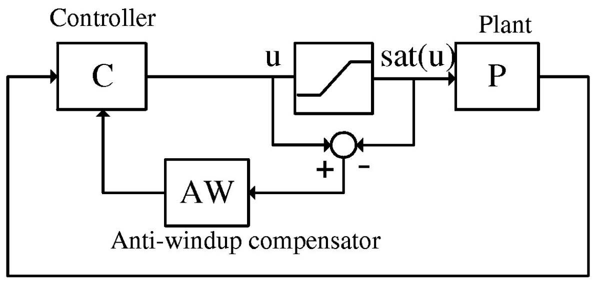

Fig. 1. Structure of the proposed anti-windup compensation.

where $q_{jd}, \ j = 1, \ldots, m$ are desired trajectories for the outputs q_j, respectively. Consider a feedback linearizing controller with an anti-windup compensation term ξ as shown in Fig. 1.

$$\dot{x}_c = A_c x_c + B_c \psi - \xi \tag{8}$$

$$u = M_R(-\alpha + \beta + C_c x_c + D_c \psi) \tag{9}$$

where, A_c, B_c, C_c, D_c are matrices with appropriate dimensions designed under the assumption without saturation, x_c are the state variables of the controller. The criterion to design the anti-windup compensator with the structure is to make the output of the constrained control system close to the corresponding output of the control system without saturation as possible.

In the robot controller, specially the gravitational and Coulomb friction terms have an effect of narrowing the upper and lower marginal ranges towards saturation limit which are defined by

$$\begin{aligned} &upper - margin_j := u_{j_{\max}} - \max_q\{G_{R_j}(q)\} - f_{c_j}, \\ &lower - margin_j := |u_{j_{\min}} - \min_q\{G_{R_j}(q)\} + f_{c_j}| \end{aligned} \tag{10}$$

where $j = 1, \ldots, m$, and $u_{j_{\max}}, \ u_{j_{\min}}$ are the upper and lower limits of the saturation components. Thus the saturation may easily occur in the robot systems with Coulomb friction. Furthermore, in the operational condition pressed by a wall or a human arm as an external disturbance, the margin decreases more and also saturation may occur for a long time in the following over-range conditions:

$$\begin{aligned} &d_M + \max_q\{G_{R_j}(q)\} + f_{c_j} > u_{j_{\max}}, \\ &-d_M + \min_q\{G_{R_j}(q)\} - f_{c_j} < u_{j_{\min}} \end{aligned} \tag{11}$$

which are used in simulations of this paper. In such a case, unstable control action such as integration results in large error and system instability. By the way, to increase the limit of robot actuators for escaping the saturation is not desirable against the design of a light-weight robot. Thus, to have a light robot and stable operation, anti-windup is indispensable.

3. DERIVATION OF A DYNAMIC ANTI-WINDUP COMPENSATOR

In [3], [7] and [17], when inputs saturate, the reason of performance degradation is considered as the difference of the controller state variables between the constrained system and unconstrained system. This difference causes a distorted control signal, which finally leads to the performance degradation of the closed-loop system. For the reason, in this paper a compensator is designed to minimize the following explicit performance index:

$$J = \int_0^\infty \|\bar{x}_c(t) - x_c(t)\|^2 dt \tag{12}$$

where $\bar{x}_c$ and x_c be the controller states in the absence of and in the presence of saturating actuators, respectively, and the symbol " $^-$ " means that the corresponding variable operates in the absence of satuating acuators. To derive an optimal compensator, an error model between the controller states of the constrained closed-loop with anti-windup and the unconstrained closed-loop without input saturation is firstly derived. To this end, the unconstrained closed-loop system is derived from (5)-(9) as follows:

$$\dot{\bar{x}}_c = A_c\bar{x}_c + B_c\bar{\psi} \tag{13}$$

$$\dot{\bar{\psi}} = (A_p + B_pD_c)\bar{\psi} + B_pC_c\bar{x}_c - B_pM_R^{-1}(\bar{q})d \tag{14}$$

And, the constrained closed-loop system is derived:

$$\dot{x}_c = A_cx_c + B_c\psi - \xi \tag{15}$$

$$\begin{aligned}\dot{\psi} &= (A_p + B_pD_c)\psi + B_pC_cx_c - M_R^{-1}(q)(u - \mathrm{sat}(u)) \\ &\quad -B_pM_R^{-1}(q)d.\end{aligned} \tag{16}$$

In the following theorem, we present an optimal solution to minimize the performance index (12). For the solution and stability analysis, the following assumptions are necessary.

Assumption 1. The matrix $(A_p + B_pD_c)$ is Hurwitz.

Assumption 2. The matrix

$$\begin{bmatrix} A_c & B_c \\ B_pC_c & A_p + B_pD_c \end{bmatrix} \tag{17}$$

is Hurwitz.

Here Assumption 1 can be easily checked before designing a compensator and is not a restricted condition as explained in [8]. But, if the system does not satisfy Assumption 1, one needs to adjust or redesign the controller. The matrix in Assumption 2 is always designed to be stable for the stability of the unconstrained closed-loop system (13), (14).

Theorem 1: Consider the nonlinear robot control system (1), (5)-(9) safisfying Assumptions 1 and 2. If the inertia matrix M_R is contant or the disturbance d is free, the dynamical compensator that makes the performance index (12) be zero is determined by

$$\begin{aligned}\dot{x}_{aw} &= (A_p + B_pD_c)x_{aw} + B_pM_R^{-1}(q)(u - \mathrm{sat}(u)) \\ \xi &= -B_cx_{aw}.\end{aligned} \tag{18}$$

Proof. By the Parseval's theorem, the performance index in (12) can be rewritten as:

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \|\widehat{\bar{x}}_c(s) - \widehat{x}_c(s)\|^2 ds \tag{19}$$

where "$\widehat{\ }$" denotes the Laplace transform of corresponding variables. And, from the Laplace tranform of (13)-(16), we obtain

$$\begin{aligned}&\widehat{\bar{x}}_c(s) - \widehat{x}_c(s) = \\ &\Theta_1^{-1}(s)\{[\bar{x}_c(0) - x_c(0)] \\ &\qquad + B_c\Theta_2^{-1}(s)[\bar{\psi}(0) - \psi(0)] \\ &\qquad + \{\widehat{\xi} + B_c\Theta_2^{-1}(s)B_p\widehat{\xi}_0\} \\ &\qquad + B_c\Theta_2^{-1}(s)(-B_p)[\widehat{M_R^{-1}(\bar{q})d}(s) - \widehat{M_R^{-1}(q)d}(s)]\}\end{aligned} \tag{20}$$

where

$$\Theta_1(s) = \{(sI - A_c) - B_c[sI - (A_p + B_pD_c)]^{-1}B_pC_c\} \tag{21}$$

$$\Theta_2(s) = [sI - (A_p + B_pD_c)] \tag{22}$$

$$\xi_0 := M_R^{-1}(u - \mathrm{sat}(u)) \tag{23}$$

and the initial values can be set as the following conditions:

$$x_{aw}(0) = 0,\ x_c(0) = \bar{x}_c(0),\ \psi(0) = \bar{\psi}(0). \tag{24}$$

And, if the inertia matrix is contant or the disturbance is free, $[M_R^{-1}(\bar{q})d - M_R^{-1}(q)d]$ becomes zero. In the error model (20), $\Theta_2(s)$ is nonsingular $\forall Re[s] \geq 0$ from Assumption 1. The nonsingularity of $\Theta_1(s)$ can be proved from the nonsingularity of $\Theta_2(s)$ and Assumption 2 as in [17] which is a linear version of this paper. Finally, the Laplace transfrom of the compensation term that makes the performance index (12) be zero is obtained as

$$\begin{aligned}\widehat{\xi} &= -B_c\Theta_2^{-1}(s)B_p\widehat{\xi}_0 \\ &= -B_c[sI - (A_p + B_pD_c)]^{-1}B_p\widehat{\xi}_0\end{aligned} \tag{25}$$

and the compensator is determined to (18). □

Remark 1. If the inertia matrix M_R is not contant and the disturbance d is not free, the error equation (20) in case with the compensator (18) becomes

$$\begin{aligned}\widehat{\bar{x}}_c(s) - \widehat{x}_c(s) = &\ \Theta_1^{-1}(s)B_c\Theta_2^{-1}(s)(-B_p) \\ &\times[\widehat{M_R^{-1}(\bar{q})d}(s) - \widehat{M_R^{-1}(q)d}(s)].\end{aligned} \tag{26}$$

And the error $\bar{x}_c - x_c$ is bounded from Properties 1, 2, and the stability of $\Theta_1^{-1}(s)$ and $\Theta_2^{-1}(s)$ in the proof of Theorem 1.

Remark 2. The solution can be obtained regarless of the existence condition (14) in [15] which may be broken for robot systems with Coulomb friction and a disturbance resulting in control with large magnitude.

4. AN INTERPRETATION OF THE COMPENSATOR AND STABILITY OF THE RESULTING SYSTEM

First, this section addresses an interpretation of the derived anti-windup state equation (18) as an observer. Deriving a state equation of $(\bar{\psi} - \psi)$ from (14) and (16) gives the following

$$\begin{aligned}\dot{\bar{\psi}} - \dot{\psi} &= (A_p + B_pD_c)(\bar{\psi} - \psi) + B_cC_c(\bar{x}_c - x_c) \\ &\quad +B_pM_R^{-1}(q)[u - \mathrm{sat}(u)] - B_p[M_R^{-1}(\bar{q}) - M_R^{-1}(q)]d\end{aligned} \tag{27}$$

While the compensator to a robot with a constant inertia matrix is applied or the disturbance is zero, $(M_R^{-1}(\bar{q}) - M_R^{-1}(q))d$ is a zero matrix and then $\bar{x}_c$ is equal to x_c from Theorem 1. Then the state equation (27) can be rewritten as

$$\dot{\bar{\psi}} - \dot{\psi} = (A_p + B_pD_c)(\bar{\psi} - \psi) + B_pM_R^{-1}(u - \mathrm{sat}(u)) \quad (28)$$

This equation has the same form of the state equation of the compensator (18) which can be interpreted as an observer of $(\bar{\psi} - \psi)$. After all, the compensator plays a role of filling the insufficient updating quantity of ψ due to the saturation into the state equation (8) of the controller as the following

$$\begin{aligned}\dot{x}_c &= A_cx_c + B_c\psi + B_cx_{aw} = A_cx_c + B_c\psi + B_c(\widetilde{\bar{\psi} - \psi}) \\ &\simeq A_cx_c + B_c\bar{\psi}\end{aligned} \quad (29)$$

where, "~" means the result observed by an observer of the corresponding signal.

If a nonlinear system is unstable, any controller cannot globally stabilize the constrained system. Thus in the following theorem the stability of the resultant system is analyzed on the assumption of BIBS (bounded-input-bounded-state) of the nonlinear robot plant. This stability is not an asymptotic property but a global property. In case of robot systems, velocity or force control systems can be assumed to be BIBS.

Theorem 2: Suppose that Assumptions 1 and 2 are satisfied. Then if the system (1) is bounded-input-bouded-state to the saturation input, the closed-loop system of (1), (5)-(9), and (18) satisfies the following properties:

$$\|x_c(t)\| < \infty, \|x_{aw}(t)\| < \infty,\ \forall t. \quad (30)$$

Proof. In (26), the disturbance injection $M_R^{-1}d$ is bounded by Properties 1 and 2. And since Assumptions 1 and 2 ensure that $\Theta_1^{-1}(s)$ and $\Theta_2^{-1}(s)$ are stable as in the proof of Theorem 1, x_c is boudend. Adding (16) and (18), we obtain:

$$\frac{d}{dt}(\psi + x_{aw}) = (A_p + B_pD_c)(\psi + x_{aw}) + BC_cx_c - B_pM_R^{-1}(q)d. \quad (31)$$

Since $(A_p + B_pD_c)$ is Hurwitz from Assumption 1, $(\psi + x_{aw})$ is bounded, which results in the boundedness of x_{aw} from the BIBS assumption of the system that ψ is bounded. □

5. SIMULATION RESULTS

In this section we show the necessity of an anti-windup scheme and the usefulness of our proposed anti-windup scheme through simulations under the condition (11).

5.1. Anti-windup results for position control

Consider a one-degree-of-freedom robot [21] consisting of a linear (not rotational) actuator with a constrained input and a nonlinear Coulomb friction term as follows:

$$M_R\ddot{q} + f_{dist} = \mathrm{sat}(u) - f_c\mathrm{sgn}(\dot{q}) - \sigma_2\dot{q} \quad (32)$$

where the mass M_R of the movable part is 6.67 Kg, the viscous damping coefficient σ_2 of the linear slider part is 48.54 Ns/m, u is the control force by the linear actuator with $u_{min} = -100$ N and $u_{max} = 100$ N being the lower and upper limits of the saturation, q is the position, the Coulomb friction coefficient f_c is 25 N, f_{dist} is a force disturbance. A nonlinear position controller with the proposed compensation term is applied to the robot as the following:

$$\dot{x}_c = 0x_c + (q - q_d) - \xi \quad (33)$$

$$\begin{aligned}u &= -\sigma_2\dot{q} + M_R\ddot{q}_d + f_c\mathrm{sgn}(\dot{q}) \\ &\quad -K_Ix_c - K_P(q - q_d) - K_D(\dot{q} - \dot{q}_d)\end{aligned} \quad (34)$$

and the PID-type dynamic controller is given with $K_P = 1000,\ K_D = 500,\ K_I = 4000$. Then

$$\begin{aligned}A_c = 0,\ B_c &= \begin{bmatrix}1 & 0\end{bmatrix}, \\ C_c = -M_R^{-1}K_I,\ D_c &= \begin{bmatrix}-M_R^{-1}K_P & -M_R^{-1}K_D\end{bmatrix}.\end{aligned} \quad (35)$$

Thus $(A_p + B_pD_c)$ is Hurwitz. Now we apply the proposed dynamic compensation method to improve the performance. The parameters are calculated as:

$$\begin{aligned}A_{aw} &= \begin{bmatrix}0 & 1 \\ -150 & -75\end{bmatrix},\ B_{aw} = \begin{bmatrix}0 \\ 1\end{bmatrix}, \\ C_{aw} &= \begin{bmatrix}-1 & 0\end{bmatrix},\ D_{aw} = 0,\ K_{aw} = 0.15.\end{aligned} \quad (36)$$

Here the desired trajectory q_d is given by:

$$\widehat{q}_d(s) = \frac{130}{s^2 + 11.4s + 130}\widehat{r}(s) \quad (37)$$

where $r(t)$ is 0.1 m. And the disturbance is 98 N if $1 \le t < 4$ and otherwise 0, which can be interpreted as the situation pushed by a human arm. In case without compensation as shown in Fig. 2, the state of controller increases proportionally to the time during the saturation and the output has a big overshoot after 4 s from the reference (the dotted line), which implies the neccessity of anti-windup compensation in the condition (11). On the other hand in case with the proposed compensator the small overshoot in the output after 4 s with small maginitude of the controller state is achieved as shown in Fig. 3, which shows the usefulness of the proposed anti-windup compensator.

5.2. Anti-windup results for force control

Consider a robot with a constrained input contacting a compliant wall as follows:

$$M_R\ddot{q} + f_{dist} = \mathrm{sat}(u) - f_c\mathrm{sgn}(\dot{q}) - \sigma_2\dot{q} - f_e \quad (38)$$

where f_e is a force reflected by a compliant wall, $f_e = k_eq$, the stiffness k_e of the wall is 1000 N/m, $M_R = 6.67$ Kg, $f_c = 25$ N, $\sigma_2 = 48.54$ Ns/m, $u_{min} = -100$ N, and $u_{max} = 100$ N. The robot model can be rewritten as

$$M_Rk_e^{-1}\ddot{f}_e + f_{dist} = \mathrm{sat}(u) - f_c\mathrm{sgn}(k_e^{-1}\dot{f}_e) - \sigma_2k_e^{-1}\dot{f}_e - f_e. \quad (39)$$

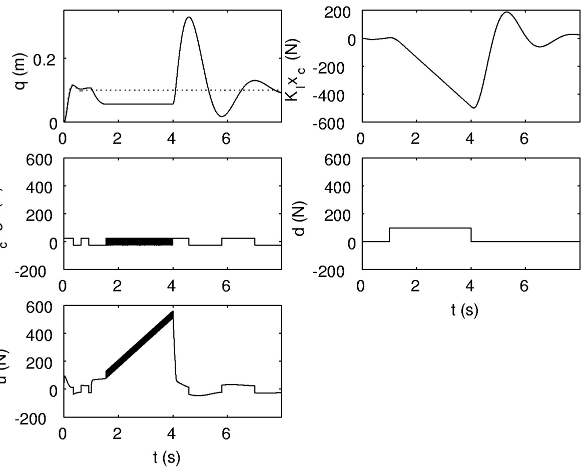

Fig. 2. Position control results without compensation.

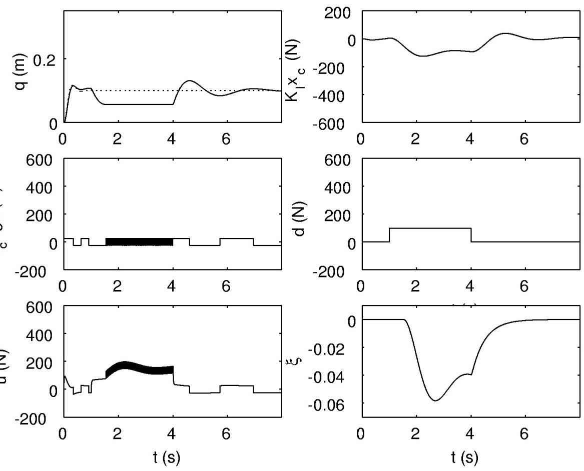

Fig. 3. Position control results with the proposed compensation.

A nonlinear force controller with the proposed anti-windup compensation term is applied to the robot plant as follows:

$$\dot{x}_c = 0x_c + (f_e - f_d) - \xi \tag{40}$$

$$\begin{aligned} u \;&= f_c \mathrm{sgn}(k_e^{-1}\dot{f}_e) + \sigma_2 k_e^{-1}\dot{f}_e + f_e + M_R k_e^{-1}\ddot{f}_d \\ &- K_P(f_e - f_d) - K_I x_c - K_D(\dot{f}_e - \dot{f}_d) \end{aligned} \tag{41}$$

and the PID-type dynamic controller is given with $K_P = 10,\ K_D = 5,\ K_I = 10$. Then

$$\begin{gathered} A_c = 0,\ B_c = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ C_c = -M_R^{-1} k_e K_I,\ D_c = \begin{bmatrix} -M_R^{-1} k_e K_P & -M_R^{-1} k_e K_D \end{bmatrix}. \end{gathered} \tag{42}$$

Thus the parameters of the proposed compensator are calculated as:

$$\begin{gathered} A_{aw} = \begin{bmatrix} 0 & 1 \\ -1500 & -750 \end{bmatrix},\ B_{aw} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ C_{aw} = \begin{bmatrix} -1 & 0 \end{bmatrix},\ D_{aw} = 0,\ K_{aw} = 150. \end{gathered} \tag{43}$$

Here the desired force f_d is given by:

$$\widehat{f}_d(s) = \frac{130}{s^2 + 11.4s + 130}\widehat{r}(s) \tag{44}$$

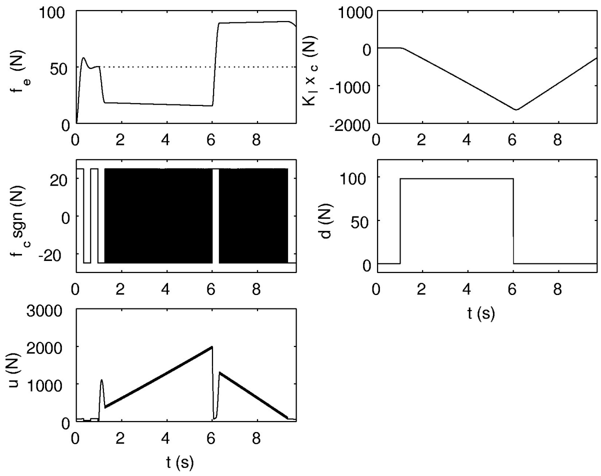

Fig. 4. Force control results without compensation.

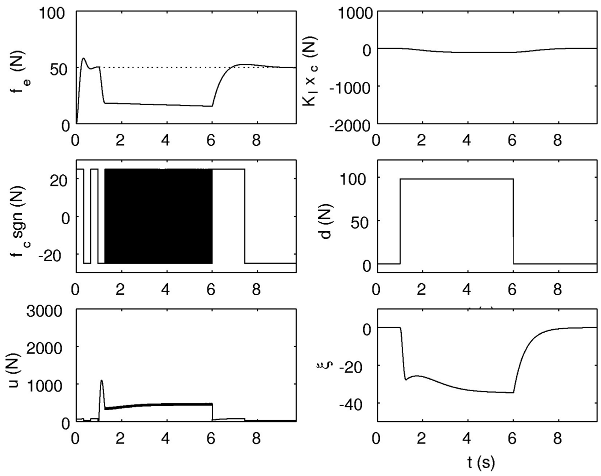

Fig. 5. Force control results with the proposed compensation.

where $r(t)$ is 50 N (the half of the limit). And the disturbance is 98 N if $1 \le t < 6$ and otherwise 0, which comes from the condition (11). In case without compensation as shown in Fig. 4, the state of controller increases proportionally to the time during the saturation and the output has a big overshoot after 6 s from the reference (the dotted line). On the other hand in case with the proposed compensator the small overshoot in the output after 6 s is achieved with small maginitude of the controller state as shown in Fig. 5, which shows the usefulness of the anti-windup scheme in the force control of the robot.

6. CONCLUSIONS

This paper proposes a new optimal dynamic anti-windup scheme for input-constrained nonlinear robot systems with friction. We first derive an error model between controller dynamics of the constrained closed-loop with anti-windup and the unconstrained closed-loop without input saturation. Then using the Parseval's theorem and Laplace transform, an optimal solution to minimize a performance index is derived. In the proposed method, the controller state is maintained as in the unsaturated system. An interpretation of the compensator as an observer and stability of the resulting closed-loop systems are also given and the usefulness of the proposed design method is illustrated by position and force control simulations in some over-range conditions related to disturbance, gravity, and Coulomb friction. The effects of model uncertainties need to be examined in the future.

ACKNOWLEDGMENTS

This work was supported by the Post-doctoral Fellowship Program of Korea Science & Engineering Foundation (KOSEF) and the SKIP project of Japan Organization for Small & Medium Enterprises and Regional Innovation (SMRJ).

References

[1] K. J. Åström and B. Wittenmark, *Computer Controlled Systems*, Prentice-Hall, 1984.

[2] K. J. Åström and B. Wittenmark, *Adaptive Control*, Addison-Wesley Publishing Co., 1995.

[3] P. J. Campo and M. Morari, "Robust control of process subject to saturation nonlinearities," *Computers & Chemical Engng.*, Vol. 14, No. 4/5, pp. 343–358, 1990.

[4] C. Canudas de Wit, H. Olsson, K. J. Åström, and P. Lischinsky, "A new model for control of systems with friction," *IEEE Trans. Auto. Control*, Vol. 40, No. 3, pp. 419-425, 1995.

[5] J. C. Doyle, R. S. Smith, and D. F. Enns, "Control of plants with input saturation nonlinearities," *Proceedings of Am. Control Conf.*, pp. 1034–1039, 1987.

[6] F. J. Doyle III, "An anti-windup input-output linearization scheme for SISO systems," *J. Process Control*, Vol. 39, pp. 213–220, 1999.

[7] R. Hanus, M. Kinnaert, and J. L. Henrote, "Conditioning techinique, a general anti-windup and bumpless transfer method," *Automatica*, Vol. 23, No. 6, pp. 729–739, 1987.

[8] P. Hippe and C. Wurmthaler, "Comments on "dynamic compensation method for multivariable control systems with saturating actuators"," *IEEE Trans. Auto. Control*, Vol. 41, No. 10, pp. 1549–1551, 1996.

[9] A. Isidori, *Nonlinear Control Systems*, Springer-Verlag, 1995.

[10] N. Kapoor, A. R. Teel, and P. Daoutidis, "An anti-windup design for linear systems with input saturation," *Automatica*, Vol. 34, No. 5, pp. 559–574, 1998.

[11] N. Kapoor and P. Daoutidis, "An observer-based anti-windup scheme for non-liner systems with input constraints," *Int. J. Control*, Vol. 72, No. 1, pp. 18–29, 1999.

[12] M. V. Kothare, P. J. Campo, and M. Morari, "A unified framework for the study of anti-windup design," *Automatica*, Vol. 30, No. 12, pp. 1869–1883, 1994.

[13] M. Krstić, I. Kanellakopoulos, and P. Kokotović, *Nonlinear and Adaptive Control Design*, John Willey & Sons, Inc., 1995.

[14] M. J. Kurtz and M. A. Henson, "Feedback linearizing control of discrete-time nonlinear systems with input constraints," *Int. J. Contr.*, Vol. 70, No. 4, pp. 603–616, 1998.

[15] F. Morabito, A. R. Teel, and L. Zaccarian, "Results on anti-windup design for Euler-Lagrange systems," *Proceedings of the 2002 IEEE Conference on Robotics & Automation*, pp. 3442–3447, Washington, DC, USA, May, 2002.

[16] S. Miyamoto and G. Vinnicombe, "Robust control of plants with saturation nonlinearity based on coprime factor representation," *Proceedings of 35th IEEE Conference on Decision and Control*, Vol. 3, pp. 2838–2840, Kobe, Japan, 1996.

[17] J.-K. Park and C.-H. Choi, "Dynamic compensation method for multivariable control systems with saturating actuators," *IEEE Trans. Automat. Contr.*, Vol. 40, No. 9, pp. 1635–1640, 1995.

[18] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, Inc., 1991.

[19] A. R. Teel and N. Kapoor "The L_2 anti-windup problem: its definition and solution," *Proceedings of European Control Conference (CD-ROM)*, Brussele, Belgium, 1997.

[20] J. Wang, S. S. Ge and T. H. Lee, "Adaptive NN control of robot manipulator with unknown dynamic friction," *Proceedings of the 4th Asian Conference on Robotics and its Applications (ACRA'01)*, pp. 160-165, Singapore, 6-8 June, 2001.

[21] Y. Yamada, H. Konosu, T. Morizono, and Y. Umetani, "Proposal of Skill-Assist for mounting operations in automobile assembly process," *Transactions of the Japan Society of Mechanical Engineers*, Vol. 68, No. 666, pp. 161–168, 2002.