A servo design method for MIMO Wiener systems with nonlinear uncertainty

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Abstract: This paper presents theory for stability analysis and design of a servo system for a MIMO Wiener system with nonlinear uncertainty. The Wiener system consists of a linear time-invariant system(LTI) in cascade with a static nonlinear part $\psi(y)$ at the output. We assume that the uncertain static nonlinear part is sector bounded and decoupled. In this research, we treat the static nonlinear part as multiplicative uncertainty by dividing the nonlinear part $\psi(y)$ into $\phi(y) := \psi(y) - y$ and y, and then we reduce this stabilizing problem to a Lur'e problem. As a result, we show that the servo system with no steady state error for step references can be constructed for the Wiener system.

Keywords: Wiener system, Lur'e system, Servo design, Popov criterion

1. Introduction

Although many control system design methods have been proposed for nonlinear systems. There are several technical problems to be solved in their actual applications. The Wiener system is not only a simple but also a practical expressions of nonlinear systems. This is because various identification methods have been proposed for Wiener systems[1][2][6] on one hand, and on the other hand many linear control design methods can be applied to this system. Especially, in the area of process control, many examples of system identification and control system design using Wiener system have been reported[3]. To the best of our knowledge, however, servo system designs for Wiener systems have not been presented so much.

In this paper, we consider the stability analysis and design problems of a servo system for the MIMO Wiener system with nonlinear uncertainty. Our approach here is to reduce this stabilizing problem to a Lur'e problem. For stability analysis of Lur'e system, the Circle criterion and Popov criterion are well-known as useful tools[5], and they can be stated briefly that, if the dynamic linear part is strictly positive real, absolute stability is guaranteed, which will be used in this research. In Section 4, we show that, even if the dynamic linear part is "positive real", absolute stability of the system is guaranteed, thereby relaxing the sector conditions. In Section 5, we introduce a design method which enables to design a specified decoupled transfer function. In Section 6, we show the absolute stability of the servo system when a step reference signal is added to the system. In Section 7, the evaluation of the controller performance is presented via simulation. Finally, in Section 8, some concluding remarks are presented.

2. Preliminary

2.1. De nition

We define some basic notions for the static nonlinearity as well as an LTI system.

Definition 1: A memoryless function $\phi : [0, \infty) \times R^m \to R^m$ is said to belong to the the sector $[K_1, K_2]$ with $K = K_2 - K_1 = K^T > 0$(See Fig. 1) if

$$[\phi(t,y) - K_1 y]^T [\phi(t,y) - K_2 y] \leq 0$$

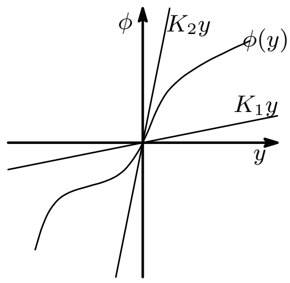

Fig. 1. Sector restriction

Definition 2: A $p \times p$ proper rational transfer function matrix $\quad(s)$ is called positive real if $\quad(s)$ satisfies the following conditions.

- The poles of all elements of $\quad(s)$ are in $\mathrm{Re}[s] \leq 0$
- For all real ω for which $j\omega$ is not a pole of any element of $\quad(s)$, the matrix $\quad(j\omega) + \quad^T(-j\omega)$ is positive semidefinite.
- Any pure imaginary pole $j\omega$ of any element of $\quad(s)$ is a simple pole and the residue matrix $\lim_{s \to j\omega}(s - j\omega) \quad(s)$ is Hermitian and positive semidefinite.

The transfer function $\quad(s)$ is called strictly positive real if $\quad(s - \varepsilon)$ is positive real for some $\varepsilon > 0$.

2.2. Lur'e system

Consider the feedback connection of Fig. 2. We assume that the external input r is 0 and study the behavior of the unforced system, represented by

$$\dot{x} = Ax + Bu \tag{1a}$$

$$y = Cx + Du \tag{1b}$$

$$u_i = -\phi_i(y_i), \ 1 \leq i \leq p \tag{1c}$$

where $x \in R^n$, $u, y \in R^p$, (A, B) is controllable, (C, A) is observable, and $\phi : [0, \infty) \times R^m \to R^m$ is a locally Lipschitz memoryless nonlinearity that belongs to the sector $[0, k_i]$. We assume that the transfer function $\quad (s) = C(sI - A)^{-1}B$ is strictly proper and ϕ is time-invariant and decoupled, that is, $\phi_i(y) = \phi_i(y_i)$.

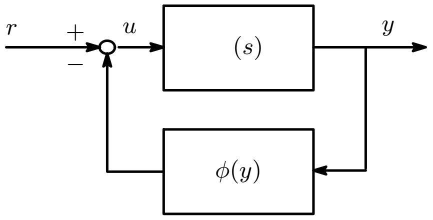

Fig. 2. Lur'e system

3. Problem Formulation

We consider the system described by Fig. 3.

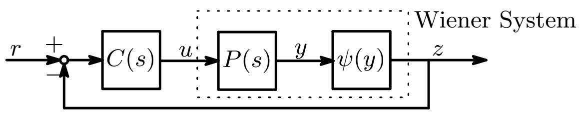

Fig. 3. Feedback control system for Wiener system

Part of the figure enclosed by the dotted line is the Wiener system of our concern, where $P(s)$ is the dynamic linear part, ψ is the uncertain static nonlinear part and $C(s)$ is a controller. The dynamic linear part $P(s)$ is described by the following equation:

$$\dot{x} = Ax + Bu \tag{2a}$$

$$y = Cx \tag{2b}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $y \in R^m$ is the output of the dynamic linear part, A is Hurwitz, (A, B) is controllable and (C, A) is observable.

We assume that the uncertain static nonlinear part ψ satisfies Assumption 1.

Assumption 1:

1) $\psi(y) = [\psi_1(y_1), \cdots, \psi_m(y_m)]^T$
2) ψ is sector bounded ($\psi_i \in [0, \infty]$)
3) $\psi(y) = 0$ if and only if $y = 0$

Under this assumption, we show that the closed-loop stability criterion and the servo system with no steady state error for step references r can be constructed for the Wiener system with the nonlinear uncertainty ψ.

4. Stability analysis

We transform the system described by Fig. 3 equivalently to the system described by Fig. 4 where $H(s) = P(s)C(s)$. To do this transform, we treat the static nonlinear part ψ as multiplicative uncertainty $\phi := \psi(y) - y$ and then stabilize the closed-loop system using a standard linear control theory. Here we consider an observer-based linear control system , and stabilize the closed-loop system by using the free parameter of Youla Parameterization. To be more specific, in view of the fact that the static nonlinear part treated as multiplicative uncertainty and the dynamic linear part constitutes

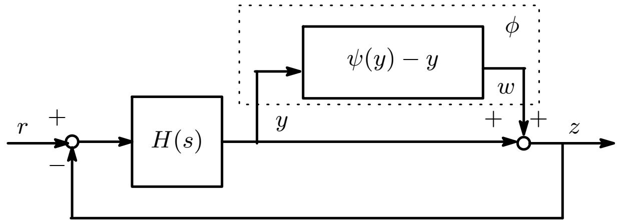

Fig. 4. Transformation of Fig. 3

a closed-loop system, we reduce this stabilizing problem to a Lur'e problem(Fig. 2).

The following lemma called Popov criterion is well-known as a stability criterion for Lur'e system(See [5]).

Lemma 1: Consider a special case of the system (1), given by

$$\dot{x} = A_L x + B_L u \tag{3a}$$

$$y = C_L x \tag{3b}$$

$$u_i = -\phi_i(y_i),\ 1 \le i \le p \tag{3c}$$

where $x \in R^n$, $u, y \in R^p$, (A, B) is controllable, (C, A) is observable, and $\phi : [0, \infty) \times R^p \to R^p$ is a locally Lipschitz memoryless nonlinearity that belongs to the sector $[0, \infty]$. The system (3) is absolutely stable if, for $1 \le i \le p$, $\phi_i \in [0, k_i]$, $0 < k_i \le \infty$, and there exists a constant $\gamma_i \ge 0$, with $(1 + {}_k\gamma_i) \ne 0$ for every eigenvalue ${}_k$ of A, such that $M + (1 + s\Gamma s) \quad (s)$ is strictly positive real, where $\Gamma = \mathrm{diag}(\gamma_1, \cdots, \gamma_p)$ and $M = \mathrm{diag}(\frac{1}{k_1}, \cdots, \frac{1}{k_p})$.

4.1. Extension of Popov Criterion

In this section, we extend Popov criterion to positive real lemma under the following assumption.

Assumption 2: $\phi(y) = 0$ if and if only $y = 0$

Lemma 2: Consider the autonomous system

$$\dot{x} = f(x)$$

where $\bar{x} = 0$ is an equilibrium point; that is, $f(0) = 0$. Let $V : R^n \to R$ be a continuously differentiable, radially unbounded, positive definite function such that $\dot{V} \le 0$ for all $x \in R^n$. Let $S = \{x \in R^n | \dot{V}(x) = 0\}$ and suppose that no solution can stay identically in S, other than the trivial solution $x(t) \equiv 0$. Then, the origin is globally asymptotically stable.

Using Lemma 2 and Assumption 1, we obtain the following theorem.

Theorem 1: The system (3) is absolutely stable if, for $1 \le i \le p$, $\psi_i \in [0, \infty]$ and there exists a constant $\gamma_i \ge 0$, with $(1 + {}_k\gamma_i) \ne 0$ for every eigenvalue ${}_k$ of A , such that $(I + s\Gamma) \quad (s)$ is positive real, where $\Gamma = \mathrm{diag}(\gamma_1, \cdots, \gamma_p)$.

Proof: We prove this theorem in two steps. First, We search a Lyapunov function which is positive definite and radially unbounded, and prove that the derivative $\dot{V}$ is semi-negative definite function. Second, we prove that no solution can stay identically in S, other than the trivial solution $x(t) \equiv 0$ where $S = \{x \in R^n | \dot{V}(x) \equiv 0\}$.

(i) The loop transformation of Fig. 5 obviously results in a feedback connection of $\tilde{H}_1$ and $\tilde{H}_2$, where $\tilde{H}_1$ is a linear

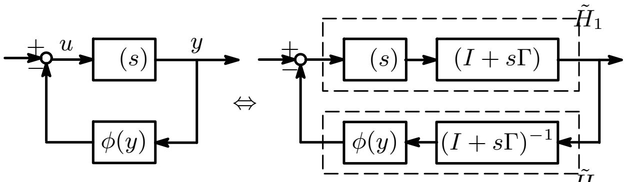

Fig. 5. Loop transformation

system whose transfer function can be expressed as

$$\begin{aligned}(I+s\Gamma)\ \ (s) &= (I+s\Gamma)C_L(sI-A_L)^{-1}B_L\\ &= C_L(sI-A_L)^{-1}B_L + \Gamma C_L s(sI-A_L)^{-1}B_L\\ &= C_L(sI-A_L)^{-1}B_L\\ &\quad + \Gamma C_L(sI-A_L+A_L)(sI-A_L)^{-1}B_L\\ &= (C_L+\Gamma C_L A_L)(sI-A_L)^{-1}B_L + \Gamma C_L B_L\end{aligned}$$

where $\{A_L, B_L, C_L\}$ is a minimal realization of $\ \ (s)$. Thus, $(I+s\Gamma)\ \ (s)$ can be realized by the state model $\{\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}\}$, where $\mathcal{A}=A_L$, $\mathcal{B}=B_L$, $\mathcal{C}=C_L+\Gamma C_L A_L$, $\mathcal{D}=\Gamma C_L B_L$. Let $\ _k$ be an eigenvalue of A_L and v_k be the associated eigenvector. Then

$$(C_L+\Gamma C_L A_L)v_k = (C_L+\Gamma C_L\ \ _k)v_k = (I+\ \ _k\Gamma)C_L v_k$$

The condition $(1+\ \ _k\gamma_i)\neq 0$ implies that $(\mathcal{A},\mathcal{C})$ is observable; hence, the realization $\{\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}\}$ is minimal. Thus, if $(I+s\Gamma)\ \ (s)$ is positive real, we can apply the positive real lemma to conclude that there exist matrices $P=P^T>0$, L , and, W such that

$$PA_L + A_L^T P = -L^T L \tag{4a}$$
$$PB_L = (C_L+\Gamma C_L A_L)^T - L^T W \tag{4b}$$
$$W^T W = \Gamma C_L B_L + B_L^T C_L^T \Gamma \tag{4c}$$

and $V=(\frac{1}{2})x^T P x$ is a storage function for $\tilde{H}_1$(See [5]). Thus, one storage function candidate for the transformed feedback connection of Fig. 5 is

$$V = (1/2)x^T P x + \sum_{i=1}^{p}\gamma_i\int_0^{y_i}\phi_i(\delta)d\delta$$

which is used later as a Lyapunov function candidate for the original feedback connection (3). Here, we note that

$$\sigma_{\min}(P)\,||x||^2 \le V(x)$$

thus, V is positive definite and radially unbounded. The derivative $\dot{V}$ is given by

$$\begin{aligned}\dot{V} &= \frac{1}{2}x^T P\dot{x} + \frac{1}{2}\dot{x}^T P x + \phi^T(y)\Gamma\dot{y}\\ &= \frac{1}{2}x^T(PA_L + A_L^T P)x + x^T PB_L u + \phi^T(y)\Gamma C_L(A_L x + B_L u)\end{aligned}$$

Using (4a) and (4b) yields

$$\begin{aligned}\dot{V} =& -\frac{1}{2}x^T L^T L x + x^T(C_L^T + A_L^T C_L^T\Gamma - L^T W)u\\ &+ \phi^T(y)\Gamma C_L A_L x + \phi^T(y)\Gamma C_L B_L u\\ =& -\frac{1}{2}x^T L^T L x + x^T C_L^T u + x^T A_L^T C_L^T \Gamma u\\ &- x^T L^T W u + \phi^T(y)\Gamma C_L A_L x + \phi^T(y)\Gamma C_L B_L u\end{aligned}$$

Substituting $u=-\phi(y)$ and using (4c), we obtain

$$\begin{aligned}\dot{V} &= -\frac{1}{2}L^T L x - x^T L^T W u - \frac{1}{2}u^T W^T W u + x^T C_L^T u\\ &= -\frac{1}{2}(Lx+Wu)^T(Lx+Wu) - \phi(y)^T y \le 0\end{aligned}$$

Thus, $\dot{V}$ is semi-negative definite function.

(ii) Assume that $\dot{V}(x(t))=0$ for all $t>T$. It follows from the above expression of $\dot{V}$ that $\phi^T(y(t))y(t)=0$ for all $t>T$. By Assumption 2 this implies $y(t)=0$ for all $t>T$, and hence $u(t)=-\phi(0)=0$ for all $t>T$. Since the input and the output of the system $\{A_L,B_L,C_L\}$ are both identically zero for all $t>T$, its state $x(t)$ is also zero for all $t>T$ by the observability assumption of (C_L,A_L). Thus, by the observability assumption again, no solution can stay identically in S, other than the trivial solution $x(t)\equiv 0$ where $S=\{x\in R^n|\dot{V}(x)=0\}$. ■

Remark 1: Note that we need to extend Popov criterion from strictly positive real condition to positive real condition since the servo compensator for step reference inputs contain an integrator.

4.2. Transformation of the sector bound

In this section, we discuss the transformation of sector restriction. From the equivalent transformation shown in in Fig. 6, sector restriction $\phi(\cdot)\in[K,\infty]$ is transfered to $\tilde{\phi}(\cdot)\in[0,\infty]$. The transfer function of linear part after transformation becomes $\tilde{\ }(s) = \ \ (s)\,[I+K\ \ (s)]^{-1}$. So, we can similarly apply the transfer function $\tilde{\ }$ to Theorem 1.

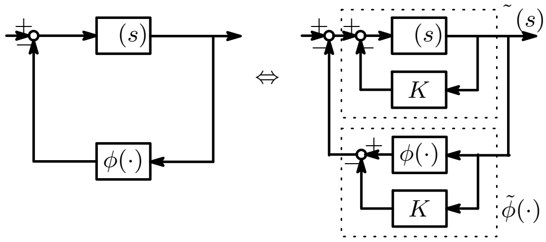

Fig. 6. Equivalent transformation of Fig. 2

5. Design servo system

In this section, we propose a design method of servo systems. Our design method enables to design a specified decoupled transfer function by using the free parameter $Q_B(s)$ of Youla parameterization(See [7]). Since this design method decouples the transfer function seen by the nonlinearity $\phi(y)$ completely, it can be applied easily to multi-input multi-output systems. The transfer function $\ _{yw}(s)$ from w to y in Fig. 4 can be specified by using the free parameter $Q_B(s)$ of Youla parameterization in Fig. 7.

In Fig. 7, $\ _{yw}(s)$ is

$$\begin{aligned}\ _{yw} = -\ \ _{yr}(s)\{K_C\ \ _C(s)\}^{-1}[K_C\frac{I}{s} + K_F(sI-A\\ + LC)^{-1}L + Q_B(s)\{C(sI-A+LC)^{-1}L - I\}]\end{aligned} \tag{5}$$

where $\ _{yr}$ is the transfer function from r to y described by

$$\ _{yr}(s) = C\{sI - A + BK_F + BK_C\frac{I}{s}C\}^{-1}BK_C\frac{I}{s} \tag{6}$$

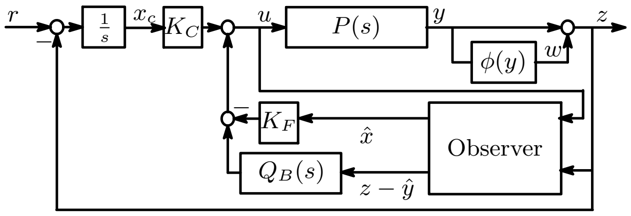

Fig. 7. Observer based servo system

From (5) and (6), we obtain

$$
\begin{aligned}
{}_{yw} =& -C\{sI - A + BK_F + BK_C\frac{I}{s}C\}^{-1}B[K_C\frac{I}{s} \\
&+ K_F(sI - A + LC)^{-1}L \\
&+ Q_B(s)\{C(sI - A + LC)^{-1}L - I\}]. \qquad (7)
\end{aligned}
$$

The following rational transfer function is defined based on the equation (7).

$$
\begin{aligned}
T_1(s) :=& \bar{\ }_{yw}(s) + C\{sI - A + BK_F + BK_C\frac{I}{x}C\}^{-1}B \\
&\times [K_C\frac{I}{s} + K_F(sI - A + LC)^{-1}L] \qquad (8a)
\end{aligned}
$$

$$
T_2(s) := C\{sI - A + BK_F + BK_C\frac{I}{s}C\}^{-1}B \qquad (8b)
$$

$$
T_3(s) := -C(sI - A + LC)^{-1}L + I \qquad (8c)
$$

Here, we assume the specified transfer function $\bar{\ }_{yw}$ of the decoupled form

$$
\bar{\ }_{yw}(s) = \underset{1\le i\le m}{\text{diag}}\left\{\frac{N_i(s)}{M_i(s)}\right\} \qquad (9)
$$

Here, we define the achievable class $\mathcal{G}_{yw}$ of $\bar{\ }_{yw}(s)$ in the sense that $\bar{\ }_{yw} - \ _{yw} \equiv T_1(s) - T_2(s)Q_B(s)T_3(s) = 0$, which is characterized by the following theorem.

Theorem 2: The transfer function $\bar{\ }_{yw}(s)$ is achievable if and only if the following two conditions hold.

(1) $M_i(s) + N_i(s)$ has s in the factor.

(2) $\deg M_i(s) - \deg N_i(s) \ge \tilde{d}_i$.

where $\tilde{d}_i$ is defined as the maximal value of difference between the degree of the numerator and that of denominator among all elements in the i-th row.

$$
(s)^{-1} = \begin{bmatrix} \frac{v_{11}(s)}{w_{11}(s)} & \dots & \frac{v_{1m}(s)}{w_{1m}(s)} \\ & \vdots & \\ \frac{v_{m1}(s)}{w_{m1}(s)} & \dots & \frac{v_{mm}(s)}{w_{mm}(s)} \end{bmatrix}
$$

($w_{ij}(s)$ and $v_{ij}(s)$ is irreducible)

[Design Procedure]

(1) Obtain an L such that $A - LC$ is stable.

(2) Obtain the achievable class $\mathcal{G}_{yw}$ and specify a desired transfer function $\bar{\ }_{yw}$.

(3) Obtain $Q_B(s)$ by $Q_B(s) = T_2(s)^{-1}T_1(s)T_3(s)^{-1}$

6. Stability of servo system

In this section, we discuss the stability of the servo system shown in Fig. 8 which consists of a plant described as a nonlinear Wiener model and a servo controller such that an output signal tracks a step reference asymptotically. For this purpose, we first show that there exist a unique equilibrium point in the servo system, and then the output follows the step reference without offset. A state space representation of the servo system can be described by

$$
\begin{aligned}
\dot{x} &= Ax + Bu && (10a)\\
\dot{x}_C &= r - z && (10b)\\
\dot{\hat{x}} &= (A - LC)\hat{x} + Bu + Lz && (10c)\\
\dot{x}_q &= A_q x_q + B_q(z - \hat{y}) && (10d)\\
u &= -K_f\hat{x} + K_C x_C + y_q && (10e)\\
y &= Cx && (10f)\\
z &= \psi(y) && (10g)\\
\hat{y} &= C\hat{x} && (10h)\\
y_q &= C_q x_q + D_q(z - \hat{y}) && (10i)
\end{aligned}
$$

where $Q_B(s) = C_q(sI - A_q)^{-1}B_q + D_q$, x_q is the state and y_q is the output. We assume that the origin of the system (10) is absolutely stable for the step reference $r = 0$ and the sector restriction of $\psi(y) \in [0, \infty]$.

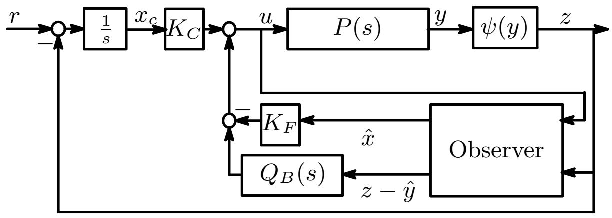

Fig. 8. Observer based servo system

6.1. Equilibrium point

In this section, We prove that there is an equilibrium point of the system (10). For this purpose, we expresses the system (10) in a compact form as follows

$$
\frac{d}{dt}\begin{bmatrix} x \\ x_c \\ \hat{x} \\ x_q \end{bmatrix} =
$$

$$
\underbrace{\begin{bmatrix} A + BD_qC & BK_c & -F & BC_q \\ -C & 0 & 0 & 0 \\ LC + BD_qC & BK_c & A - LC - F & BC_q \\ B_qC & 0 & -B_qC & A_q \end{bmatrix}}_{\mathcal{A}} \begin{bmatrix} x \\ x_c \\ \hat{x} \\ x_q \end{bmatrix}
$$

$$
+ \underbrace{\begin{bmatrix} BD_q \\ -I \\ L + BD_q \\ B_q \end{bmatrix}}_{\mathcal{B}_w} w + \underbrace{\begin{bmatrix} 0 \\ I \\ 0 \\ 0 \end{bmatrix}}_{\mathcal{B}_r} r \qquad (11)
$$

$$
y = \begin{bmatrix} C & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_c \\ \hat{x} \\ x_q \end{bmatrix}
$$

where $\phi(y) = \psi(y) - y$, $w = \phi(y)$ and $F = BK_F + BD_qC$. First, we prove the nonsingularity of the matrix $\mathcal{A}$. Adding the first row of $\mathcal{A}$ to the third row of $\mathcal{A}$, and subtracting the first column of $\mathcal{A}$ from the third column of $\mathcal{A}$ yields the following matrix.

$$\begin{bmatrix} A-BK_F & BK_c & -F & BC_q \\ -C & 0 & 0 & 0 \\ 0 & 0 & A-LC & 0 \\ 0 & 0 & -B_qC & A_q \end{bmatrix}$$

This matrix is nonsingular by the stability of $A_e - B_eK_e = \begin{bmatrix} A-BK_F & BK_c \\ -C & 0 \end{bmatrix}$, $A-LC$ and A_q. From this fact, $\mathcal{A}$ is nonsingular. Now, we prove that the equilibrium point of the servo system is uniquely obtained. Here, we define $\bar{\mathcal{X}}$ as the equilibrium point of $\mathcal{X}$. From $\dot{x}_c = r - z = r - (y + \phi(y))$, it follows that $\bar{y} + \bar{w} = \bar{y} + \phi(\bar{y}) = r$. Using the equation (11) and nonsingularity of $\mathcal{A}$, we obtain the following equation.

$$\bar{\zeta} = -\mathcal{A}^{-1}\left(\mathcal{B}_w\bar{w} + \mathcal{B}_r r\right), \quad (\zeta = [x, x_C, \hat{x}, x_q]^T).$$

This implies the existence and uniqueness of equilibrium states $\bar{\zeta}$, from which the equilibrium output and input $\bar{y}, \bar{\hat{y}}, \bar{y}_q, \bar{u}, \bar{z}$ can be obtained uniquely by using the equation (10).

6.2. Steady state error of the servo system

In this section, we prove that there is no steady state error in the servo system. From the equation (10), the following equations hold in steady state.

$$0 = A\bar{x} + B\bar{u} \tag{12a}$$
$$0 = r - \bar{z} \tag{12b}$$
$$0 = (A - LC)\bar{\hat{x}} + B\bar{u} + L\bar{z} \tag{12c}$$
$$0 = A_q\bar{x}_q + B_q(\bar{z} - \bar{\hat{y}}) \tag{12d}$$
$$\bar{u} = -K_F\bar{\hat{x}} + K_C\bar{x}_C + \bar{y}_q \tag{12e}$$
$$\bar{y} = C\bar{x} \tag{12f}$$
$$\bar{z} = \psi(\bar{y}) \tag{12g}$$
$$\bar{\hat{y}} = C\bar{\hat{x}} \tag{12h}$$
$$\bar{y}_q = C_q\bar{x}_q + D_q(\bar{z} - \bar{\hat{y}}) \tag{12i}$$

From the equation (12b), it can be seen that there exist no steady state error for the step reference signal. Next, we prove that the equilibrium point is absolutely stable. Define $\tilde{\mathcal{X}} = \mathcal{X} - \bar{\mathcal{X}}$, and subtracting the system (10) from the system (12) yields

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u} \tag{13a}$$
$$\dot{\tilde{x}}_C = \bar{z} - z = -\tilde{z} \tag{13b}$$
$$\dot{\tilde{\hat{x}}} = (A - LC)\tilde{\hat{x}} + B\tilde{u} + L\tilde{z} \tag{13c}$$
$$\dot{\tilde{x}}_q = A_q\tilde{x}_q + B_q(\tilde{z} - \tilde{\hat{y}}) \tag{13d}$$
$$\tilde{u} = -K_f\tilde{\hat{x}} + K_C\tilde{x}_C + \tilde{y}_q \tag{13e}$$
$$\tilde{y} = C\tilde{x} \tag{13f}$$
$$\tilde{z} = \psi(y) - \psi(\bar{y}) = \psi(\tilde{y} + \bar{y}) - \psi(\bar{y}) := \varphi(\tilde{y}) \tag{13g}$$
$$\tilde{\hat{y}} = C\tilde{\hat{x}} \tag{13h}$$
$$\tilde{y}_q = C_q\tilde{x}_q + D_q(\tilde{z} - \tilde{\hat{y}}). \tag{13i}$$

From this operation, we can see that the equilibrium point of the system (10) for the step reference signal is equal to the origin of the system (13). To construct the servo system, we assume the existence and uniqueness of y such that $\psi(y) = r$. From this assumption and Assumption 1, the transformed nonlinearity φ has also sector restriction $\varphi \in [0, \infty]$. So, the servo system has no steady state error for step references.

7. Numerical Example

In this section, we present a numerical example that illustrates the main feature of our result. In the system illustrated in Fig. 8, the the linear part $P(s)$ is

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{ccc|cc} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -6 & -5 & -3 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array}\right]$$

and the nonlinear part $\psi(y)$ is

$$\psi = \mathrm{diag}[2\tan^{-1}(y_1), y_2^3] \in [0, \infty]$$

The observer poles are $[-11, -12, -10]$ and the poles of the servo system are $[-2.8, -2.6, -3.0, -2.5, -2.7]$. A specified decoupled transfer function $\bar{\ }_{yw}$ is given by $\bar{\ }_{yw} = \mathrm{diag}[\frac{1}{(1+s)^2}, \frac{1}{1+s}]$. The simulation result of z for the step reference signal $r = 1$ is shown in Fig. 9.

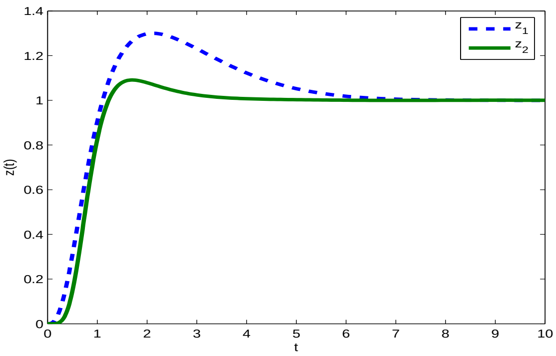

Fig. 9. The simulation result for the step reference signal $r = 1$

From this figure, we can see that the output of the servo system tracks the reference signal with no steady state error.

8. Conclusions

In this paper, we have presented a servo system design using a free parameter of Youla parameterization for MIMO Wiener systems with nonlinear uncertainty. By using the KYP lemma for stabilizable systems proposed by [4], we ensure that Theorem 1 holds in the case where the closed loop system (3) with the same assumption as in [4] is stabilizable and detectable(the proof of this fact was omitted). Our future work is to obtain the stability conditions when the $P(s)$ in (2) has unstable poles.

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