# Identification of Nonlinear Dynamic Systems via the Neuro-Fuzzy Computing and Genetic Algorithms

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**Abstract**: In this paper, an effective method for selecting significant input variables in building ANFIS (Adaptive Neuro-Fuzzy Inference System) for nonlinear system modeling is proposed. Dominant inputs in a nonlinear system identification process are extracted by evaluating the performance index and they are applied to ANFIS. The availability of our proposed model is verified with the Box and Jenkins gas furnace data. The comparisons with other methods are also given in this paper to show our proposed method is superior to other models.

Keywords: Adaptive Neuro-Fuzzy Inference System, Genetic Algorithms, Time series data modeling, Nonlinear system.

## **1. INTRODUCTION**

In a modeling process of a real world problem, there usually are a huge number of potential inputs involved. A large number of inputs may increase the complexity in computation and cause other problems related to running time, memory spaces, etc. In the case of modeling process with large input, the number of inputs should be reduced and the priority inputs should be determined by an optimal selection method.

In this paper, an effective method for selecting significant input variables in building ANFIS (Adaptive Neuro-Fuzzy Inference System) [1] model of a nonlinear system is proposed. The important input variables which independently and significantly affect the system output can be extracted by Genetic Algorithms (GAs) [3, 13]. GAs are a powerful search algorithms that use operations found in natural genetics to guide the trek by space searching. Moreover, GAs are theoretically and empirically proven to provide robust search capabilities in complex spaces, offering a valid method to problems requiring efficient and effective searching.

Using GAs, dominant inputs for a nonlinear system identification are extracted through evaluating error function. Regarding to the ANFIS, it combines the power of fuzzy systems with that of neural networks so that the structure of ANFIS is a hybrid system and mainly based on Sugeno-fuzzy model [4].

The way of modeling proposed in this paper is as following. Firstly, we determine the initial conditions for the ANFIS structure. Then, a GA is used as a dominant input selector to enable the system to have reduced input dataset by eliminating irrelevant inputs in problems with a large number of inputs. And then, the selected dominant inputs are applied to the ANFIS to identify the nonlinear system. In order to verify the availability of the performance, Box and Jenkins gas furnace data [2] is applied to our proposed method.

In the simulation results given in section 3, the proposed method combining GA with ANFIS shows a relatively high performance and it indicates that our model is an adequate model for a nonlinear identification.

This paper is composed of four sections. In section 2, the concepts of the Neuro-Fuzzy system and the Genetic Algorithms are introduced. In section 3, the experiment results with a nonlinear dynamic system are shown. This paper is concluded in section 4.

# 2. NEURO-FUZZY AND GENETIC COMPUTING 2.1 Adaptive Neuro-Fuzzy Inference System (ANFIS)

ANFIS [1] is a hybrid system which combines the oral power of a fuzzy system with the numerical power of a neural system. Its structure is mainly based on Sugeno-fuzzy model. For simplicity, we suppose a fuzzy inference system having 2 inputs (x and y) and one output (f). For the first-order Sugeno-fuzzy model, a typical rule containing two fuzzy if-then rules can be expressed as follows.

Rule 1: If x is  $A_1$  and y is  $B_1$ , then  $f_1 = p_1x + q_1y + r_1$ . Rule 2: If x is  $A_2$  and y is  $B_2$ , then  $f_2 = p_2x + q_2y + r_2$ .

Regarding the fuzzy rules shown above, their corresponding fuzzy reasoning and equivalent architecture is depicted in Fig. 1.



(a) Two-input, First-order Sugeno fuzzy model.



(b) ANFIS model corresponding to (a).

Fig. 1. Fuzzy rules and their corresponding ANFIS architecture.

Briefly, we describe the node functions in each layer of the ANFIS as follows. Detailed descriptions can be found in [1]. Note that  $O_i^{j}$  denotes the output of the *i*-th node in layer *j*.

**Layer 1**: Every node i in this layer is a square node with a node function

$$O_i^1 = \mu_{Ai}(x) \tag{1}$$

 $O_i^{1}$  is the Membership Function (MF) of  $A_i$  and it specifies the degree to which the given x satisfies the quantifier  $A_i$ . In this paper, we used the bell-shaped MFs,  $\mu_{Ai}(x)$ , with maximum equal to 1 and minimum equal to 0, such as

$$O_{i}^{1} = \mu_{Ai}(x),$$

$$\mu_{Ai}(x) = \frac{1}{1 + \left[\left(\frac{x - ci}{ai}\right)^{2}\right]b_{i}} \text{ or } \mu_{Ai}(x) = \exp\left\{1 - \left(\frac{x - ci}{ai}\right)^{2}\right\}$$
(2)

where  $\{a_i, b_i, c_i\}$  is the bell-shaped M.F. parameter set as depicted in Fig. 2.



Fig. 2. Parameter set of the bell M.F.

**Layer 2**: Each node in this layer represents the firing strength of rule and every node is fixed. It multiplies the input signals ( $\mu_{Ai}(x)$ ,  $\mu_{Bi}(y)$ ) as follows

$$O_i^2 = w_i = \mu_{Ai}(x) \times \mu_{Bi}(y), \quad i = 1, 2$$
(3)

**Layer 3**: Every node is also fixed as layer 2. The i-th node in this layer calculates the ratio of the i-th rule's firing strength to the sum of all rule's firing strengths as follows

$$O_i^3 = \overline{w}_i = \frac{w_i}{w_1 + w_2}, \ i = 1, 2$$
 (4)

**Layer 4**: Every node i in this layer is a square node with a node function as follows

$$O_i^4 = \overline{w}_i f_i = \overline{w}_i (p_i x + q_i y + r_i)$$
(5)

where  $w_i$  is a output of Layer 3 and  $(p_i x + q_i y + r_i)$  is the parameter set which is referred to as consequent parameters.

**Layer 5:** The node calculates the overall output as the summation of all input signals as follows

$$O_1^5 = overall \ output, f = \sum_i \overline{w}_i f_i = \frac{\sum_i \overline{w}_i f_i}{\sum_i \overline{w}_i}$$
(6)

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In this paper, ANFIS is used as the identifier with up to ten inputs from Box and Jenkins gas furnace data.

#### 2.2 Genetic algorithm

In conventional identification methods, model structures are selected and the parameters of that model are produced by optimizing an objective function. The methods used for optimization of the objective function are typically based on gradient descent techniques. This process generally requires a large input set due to the complexity and non-linearity of dynamic system. On the other hand, that is not always available as well as it is possible that the obtained parameters are only locally optimal.

Genetic Algorithms (GAs) [3, 13] are proven to be useful in optimization of such problems because of their considerable ability to efficiently use historical information to get the optimal solutions. In addition, Genetic Algorithms are already theoretically and empirically proven to support robust searches in complex space searching. GAs are global search and optimization techniques based on the mechanics of natural selections and natural genetics. GAs persist in such survival evolution principles as the fittest to live, the better to be accepted and the worse to be eliminated throughout natural selections. As shown in Fig. 3, the process of a GA is the combination of artificial crossovers, mutations, and selections. A GA searches for the optimal solution by being operated with population of individuals represented as binary strings [11, 12]. The initial population is randomly generated. The individuals coded as binary strings of finite length represent the potential solution. The performances of individuals are evaluated by the fitness function. The fitness function applied in our experiments is a simplified ANFIS described in next section.



Fig. 3. Operation of a Genetic Algorithm

The first step of the GA process is constructing the initial population. Secondly, each individual in a population is evaluated by the specific fitness function in order to extract dominant strings. In the next step, the dominant strings form a population of new generation by simulating the natural selection processes based on artificial crossover, mutation, and selection. In this procedure, the evolution phase loops until the optimal solution is found or the generation number of GA reaches the maximum. When one of the two conditions is satisfied, the optimal solution is produced.

#### 2.3 Dominant Input Selection Procedure for ANFIS

To eliminate irrelevant inputs and find dominant inputs which influence independently and significantly the system output, the GA is employed with many input candidates. So,

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we can expect to alleviate computational load and required memories with this procedure. The procedure of the dominant input selection is depicted in Fig. 4.



Fig. 4. Scheme of dominant input selection procedure.

Fig. 4. describes how the GA is used for dominant input selection. First of all, the GA produces the initial random population. Each individual in the produced population consists of ten strings which represent the input candidates as shown in Fig. 4. Each string in the individual is either 1 or 0: "O" indicates that the corresponding input should be abandoned, and "1" indicates that the inputs should be selected for the cost function. And then, the selected inputs are applied to the ANFIS in order to evaluate them. After evaluating all individuals of a population, the GA enables the system to have reduced input dataset by eliminating irrelevant inputs within ten candidate inputs based on the evaluation of the cost function. Therefore, the GA can be said to be operated as a dominant input selector. Finally, the selected input set with the smallest error driven by the cost function is applied to the object function as shown Fig. 5.



Fig. 5. Block diagram of overall proposed method

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In Fig. 5, the input selection part shows the determining process of the initial conditions for ANFIS as the cost function and GA as the dominant input selector described in previous section. They are: desired error, maximum iteration number, for ANFIS and population size, generation number, etc for GA. Next, the best input set extracted by the GA is applied to the object function, the complete version of ANFIS. Here the object function, ANFIS identifies Box and Jenkins gas furnace modeling problem with the selected input set. Finally, in order to compare our result with other methods, mean square error (MSE) is calculated, defined as,

$$MSE = \frac{1}{m} \sum_{i=1}^{m} (y_i - \hat{y}_i)^2$$
(7)

where *m* is the number of data sets,  $y_i$  is the actual output, and  $\hat{y}_i$  is the estimated output.

## **3. EXPERIMENT RESULTS**

In order to verify the availability of the proposed method, Box and Jenkins gas furnace data [2] is applied and tested. Box and Jenkins gas furnace data is a time series data which have many inputs such as y(t-1), y(t-2), y(t-3), and u(t-1), u(t-2), u(t-3), etc, where, u(t) is the gas flow rate and y(t) is the  $CO_2$  concentration. Box and Jenkins data is frequently used as a benchmark example for testing nonlinear identification problems.

To predict outputs y(t) using ten input candidates: u(t), u(t-1), u(t-2), u(t-3), u(t-4), y(t-1), y(t-2), y(t-3), y(t-4), and y(t-5) are employed initially. Consequently, 291 data sets are extracted and the first 146 data sets are used as training dataset and the remaining 145 data sets are used as test dataset as shown in Fig. 5.





Fig. 6. Box -Jenkins gas furnace data.

As mentioned in section 2.3, we reduced the input dimension among ten input candidates by dominant input selection using GAs with parameters of maximum epoch 160, desired error 0.01, population size 10, and maximum generation 30. After the dominant input selection, the best input set which has the smallest training RMSE was found as the input set [u(t-3), u(t), y(t-4), y(t-1)]. Then this set is applied to the object function, ANFIS to identify the problem.



Fig. 7. MSE curve at each generation



Fig. 8. Simulation results of training phase



Fig. 9. Simulation results of testing phase.

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Consequently, Fig. 7 shows the minimum MSE at each generation. ANFIS training result is shown in Fig. 8(a) and its error (Fig. 8(b)) is given for 160 epochs. Fig. 9 shows the result (Fig. 9(a)) and its error (Fig. 9(b)) of ANFIS test for 160 epochs. In Fig. 8-9, a) the solid and dotted line mean actual data and estimated data, respectively. From the results in Fig. 8, and Fig. 9, b), the MSE values of our model for training and testing are 0.0195 and 0.2595, respectively. In Table 1, our proposed method is compared with other methods identifying the same data. It can be seen from Table 1. that the performance of our model is superior to those of other models including Sugeno's model in [10].

Table 1 The	comparisons	with	other	methods
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Models	Inputs	MSE		
11204010	Input	Training	Testing	
Tong [5]	u(t-4), y(t-1)	0.469		
Xu [6]	u(t-4), y(t-1)	0.328		
Wang [7]	u(t-4), y(t-1)	0.158		
Pedrycz [8]	u(t-4), y(t-1)	0.320		
Box [2]	u(t-3), u(t-4), (t-5), y(t-1), (t-2)	0.202		
Lin [9]	u(t-3), u(t-5), (t-6), y(t-1), y(t-2)	0.071	0.261	
Sugeno [10]	u(t-1), u(t-2), (t-3), y(t-1), y(t-2), (t-3)	0.068		
Proposed method	u(t-3), u(t), y(t-4), y(t-1)	0.0195	0.2595	

#### **4. CONCLUSION**

In this paper, a system identification method using ANFIS and Genetic algorithms is proposed. The proposed model is applied to Box and Jenkins gas furnace problem. As shown in the performance comparison of the proposed method with other model, our model possesses higher availability to other models. In addition, it is expected that our proposed method might be successfully used to resolve the computation and memory space problems as well as it is a powerful solution to the identification of nonlinear system problems. The applications of the proposed method to the variety of nonlinear system modeling problems will be considered as future works.

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### REFERENCES

- J.-S. R. Jang, "ANFIS: Adaptive-network-based fuzzy inference systems,"IEEE Trans. on System, Man and Cybernetics, vol. 23, pp. 665-685, May 1993.
- [2] G. Box and G. Jenkins. Time Series Analysis: Forecasting

and Control, pages 532-533. Holden Day, San Francisco, 1970.

- [3] D. E. Goldberg, "Genetic Algorithms in Search, Optimization, and Machine Learning," Addison-Wesley, Reading, MA, 1989.
- [4] J. S. R. Jang and C. T. Sun, "Neuro-fuzzy modeling and control," Proc. IEEE, Vol.83, Issue.3, pp. 378-406, 1995.
- [5] R. M. Tong, 'The construction and evaluation of fuzzy models." In Advances in Fuzzy Set Theory and Applications, M. M. Gupt, R. K. Ragade, and R. R. Yager, Eds. Amsterdam: North-Holland, 1979, 559-576.
- [6] C. W. Xu and Y. Z. Lu, "Fuzzy model identification and self-learning for dynamic systems," IEEE Trans. Syst., Man, Cybern, vol. 17, pp. 683-689, 1987.
- [7] L. Wang and R. Langari, "Complex systems modeling via fuzzy logic," IEEE Trans. Syst., Man, Cybern., 1995, accepted for publication.
- [8] W. Pedrycz, "An identification algorithm in fuzzy relational system," Fuzzy Sets Syst., vol. 13, pp. 153-167, 1984.
- [9] Y. Lin and G. A. Cunningham . A New Approach to Fuzzy-Neural System Modeling, IEEE Transactions on Fuzzy Systems, 3 (2): 190-198, May 1995.
- [10] M. Sugeno and K. Tanaka. Successive identification of a fuzzy model and its application to prediction of a complex system, Fuzzy Sets and Systems, 42: 315-334, 1991.
- [11] D.-W. Kim, G.-T. Park, "Optimization of Polynomial Neural Networks: An Evolutionary Approach," The Korean Institute of Electrical Engineers, vol. 52, no. 7, pp. 421-433, July 2003
- [12] D.-W. Kim, G.-T. Park, "A novel design of self-organizing approximator technique: an evolutionary approach," Systems, Man and Cybernetics, 2003. IEEE International Conference on vol. 5, pp. 4643 – 4648, Oct. 2003.
- [13] J. Holland, "Adaptation in Natural and Artificial Systems," The University of Michigan Press, Ann Arbor, M.I., 1975.