Gain-scheduling of Acceleration Estimator for Low-velocity Measurement with Encoders

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Abstract: In most of motor-driven motion control systems, an encoder is used to measure a position of the motor and the velocity information is obtained by measuring the position increment over a sampling period. The quantization effect due to limited resolution of the encoder induces some measurement errors, and consequently causes deterioration of the motion performance especially in low velocity. In this paper, we propose a gain-scheduled acceleration estimator which works in wider velocity range than the original acceleration estimator. We investigate and analyze characteristics of the velocity measurement mechanism which takes into account the quantization effect of the encoder. Next, we introduce the acceleration estimator and propose a gain-scheduled acceleration estimator. The bandwidth of the gain-scheduled acceleration estimator is automatically adjusted by the velocity command. Finally, its performance is evaluated by simulation and experiment, and the results are compared with those of a conventional method and the original acceleration estimator.

Keywords: encoder, velocity measurement, acceleration estimator, gain-scheduling

1. INTRODUCTION

In most of motor-driven motion control systems, encoders are used to measure the position, and the velocity information is acquired typically by measuring the position increment over sampling period. But the quantization effect by limited resolution of the encoder and sampling period causes some measurement errors of the velocity, and it may bring out deterioration on the motion performance. The deterioration becomes serious in low velocity range.

For high accuracy of the velocity measurement, a high resolution encoder is required, however it increases the cost of the system. So a number of researchers have proposed several alternatives such as digital filters, model-based observers and some kind of estimators to improve the accuracy of velocity measurement without additional cost. Carpenter *et al.* researched the improvement of the velocity measurement through the use of digital filters [1]. A digital filter based on an adaptive least squares approach was proposed and the performance of various digital filters was compared through experimental results. Yang and Ke developed a closed-loop velocity observer considering a DC motor model [2]. In order to reduce the ripple on the estimated velocity, the measured input position of the velocity observer was compensated with the previous average speed. Kim and Sul suggested a motor speed estimator using Kalman filter [3]. It estimates not only motor speed but also disturbance torque of the motor and has the robust characteristic to parameter variations.

Lee and Song developed an acceleration estimator approach for velocity measurement [4]. It is one of the efficient algorithms to measure the velocity via acceleration estimation. The acceleration estimator can be regarded as a kind of low-pass filter with two gains, and its bandwidth is characterized and limited by these gains. Once the gains of the acceleration estimator are tuned for a specified velocity, its performance may fall off in other velocities.

In this paper, we propose a gain-scheduling method of the acceleration estimator to measure the velocity in wide velocity range. Its bandwidth is automatically adjusted according to the velocity command.

In section 2, we discuss the characteristics of velocity measurement due to the quantization effect of the encoder. In section 3, we introduce the acceleration estimator and suggest to schedule its gains to expand working velocity range. It is also explained that the gains are automatically adjusted by the velocity command of the motion control system. Its performance is evaluated through computer simulation and experiment in section 4. The results are compared with those of a conventional method and the original acceleration estimator.

2. VELOCITY MEASUREMENT IN MOTION CONTROL SYSTEM

In many applications, a fixed-time method which uses backward position difference is typically employed to obtain the velocity from the position of the encoder. The k th velocity v by this method is calculated by Eq. (1).

$$v(kT_s) = \frac{x(kT_s) - x((k-1)T_s)}{T_s} \tag{1}$$

where x is the position of the encoder and T_s is the sampling time.

It is, however, known that the fixed-time method works more accurately in high velocity range than in low velocity range due to the quantization effect by limited encoder resolution and fixed sampling period. This quantization effect triggers the position error maximally of ± 1 encoder pulse per sampling period. Figure 1 shows an example of the measured encoder pulse train. When the actual position increment per sampling period Δx is 1.5, the measured position increment per sampling period $\Delta\hat{x}$ is 1 or 2. It shows that unavoidable errors occur in velocity measurement. The ratio of the error to the actual velocity is relatively large in low velocity range. Therefore it is more difficult to control the motion in low velocity range than in high velocity range.

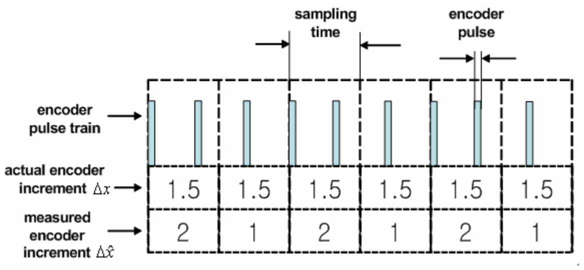

Fig. 1 An example of encoder pulse train

Kavanagh and Murphy researched this quantization effect of the discrete encoder [5]. They presented the spectral characteristics of the differentiation related to quantization error. The power spectral density of the velocity measurement error only by quantization effect is as follows.

$$S_V(f) = 2(1-\cos(2\pi f))\cdot\left(\sum_{k\neq 0}\frac{1}{4\pi^2 k^2}\cdot\delta(f-f_k)\right) \qquad (2)$$

where f_k is $\dfrac{<k\Delta x>}{T_s}$ and the mathematical operation $<x>$ means a fraction part of x.

From Eq. (2), the velocity measurement has a specific characteristic that the power distribution is provided discretely with the Dirac delta function at frequencies, $f_k = \dfrac{<k\Delta x>}{T_s}$. And the phenomenon of the overlap, known as the folding, can occur at f_k above the Nyquist frequency, $f_N = \dfrac{1}{2}f_s = \dfrac{1}{2T_s}$.

Figure 2 shows an example of the velocity obtained by the fixed-time method during the motion control of a motor with $T_s = 1$ msec. Its average value is $\Delta x \cong 5.301$ pulse per sampling period. Figure 3 shows the power spectrum of the experimental data in Figure 2 and the theoretical data with $\Delta x = 5.301$ pulse per sampling period. In the theoretical data, the power distribution appears discretely at $f_1 = 301$ Hz by $\dfrac{<5.301>}{0.001} = 301$, $f_2 = 398$ Hz by $\dfrac{<2\cdot 5.301>}{0.001} = 602$ and the aliasing with $f_N = \dfrac{1}{2T_s} = 500$ Hz, and so on. There is a similar power distribution in dominant frequency range except for low frequency component, which is related to some disturbance of control system.

Consequently, the velocity measurement by the fixed-time method has two representative features in encoder-based motion control systems.

1) The ratio of the error to the actual velocity increases as the velocity slows down.

2) Power spectrum of the velocity error due to quantization effect is distributed discretely with wide frequency range.

These features make it difficult to measure the velocity precisely, and deteriorate the performance of the motion control especially in low velocity. For high accuracy of the velocity measurement, a high resolution encoder is needed; however it increases the cost of the system. In order to improve accuracy of velocity measurement without additional cost, we try to apply a gain-scheduled acceleration estimator to the motion control system in next section.

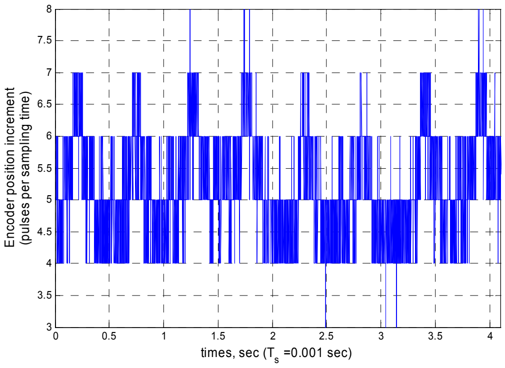

Fig. 2 An experimental velocity data set by a fixed-time method

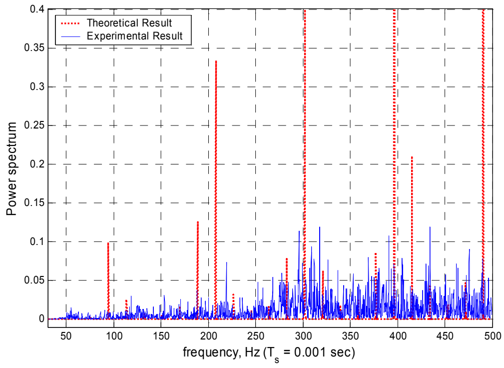

Fig. 3 Power spectrum of velocity for experimental and theoretical data sets

3. ACCELERATION ESTIMATOR WITH GAIN-SCHEDULING

3.1 An acceleration estimator [4]

The acceleration estimator proposed by Lee and Song is useful for low acceleration and velocity range. It is known that the design of the acceleration estimator is based on two facts:

1) The position information based on the encoder signal is quite accurate.

2) Numerical integration is more stable and accurate than numerical differentiation.

As shown in Figure 4, the acceleration estimator is composed of two numerical integrators and a PD controller, of which gains are K_1 and K_2. x is the encoder position, a_e is the estimated acceleration, v_e is the estimated velocity, and x_e is the estimated position. The estimated acceleration is obtained from PD control signal as Eq. (3) instead of a conventional double differentiation of the position. The velocity measurement is acquired by integrating the estimated acceleration.

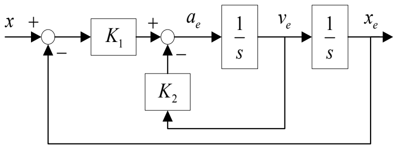

Fig. 4 Block diagram of an acceleration estimator

$$a_e = K_1(x - x_e) - K_2\frac{dx_e}{dt} \tag{3}$$

The transfer function from x to x_e can be derived as Eq. (4).

$$\frac{x_e}{x} = \frac{K_1}{s^2 + K_2 s + K_1} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{4}$$

where ζ, ω_n are the damping ratio and the bandwidth, respectively.

As shown in Eq. (4), the acceleration estimator can be regarded as a kind of second-order low-pass filter expressed by two parameters. The acceleration estimator is characterized by its two gains. If the acceleration estimator is used to measure the velocity in motion control system, ζ and ω_n are important factors which determine its dynamic response. ζ is usually set to 0.707, which is a critical damping, because it provides fast response without overshoot. On the other hand, ω_n is tuned appropriately according to application needs. It has a significant influence on the performance of the motion control.

3.2 Gain-scheduling of the acceleration estimator

In order to measure the velocity effectively, noise filtering and phase delay of the acceleration estimator should be compromised, because narrow bandwidth for noise filtering makes phase delay large; if phase delay is large, motion control may become unstable. As the ratio of the error to the actual velocity increases in low velocity range, narrow bandwidth is needed to use the measured velocity for motion control. On the other side, since the ratio of the error to the actual velocity in high velocity is smaller than that in low velocity range, it is reasonable to set the relatively wide bandwidth which can reduce unnecessary delay of velocity measurement. Once the gains of the acceleration estimator are fixed for a specified velocity, the fixed bandwidth may deteriorate the performance such as noise attenuation and phase delay in other velocity range.

We propose a gain-scheduled acceleration estimator. The purpose of the gain-scheduling is to set the appropriate bandwidth according to the ratio of the error to the actual velocity. The bandwidth of the gain-scheduled acceleration estimator is automatically adjusted by the velocity command. As the velocity command slows down, the gains are tuned to set narrow bandwidth. As shown in Figure 5, its bandwidth ω_n is a function of velocity command v_{ref} of control system. ζ is set to a critical damping with a value of 0.707. The procedure of the gain-scheduling is as follows. First, its minimum bandwidth ω_{n0} which is acceptable in the velocity measurement should be determined. Then, set the desired low velocity $v_{\min}$ and derive ω_n by Eq. (5). An appropriate value of a can be easily found out by investigating acceptable bandwidth at several velocities. Finally, the gains K_1 and K_2 are obtained by Eq. (6).

$$\begin{aligned}&\omega_n = \omega_{n0},\ \text{if}\ v_{ref} < v_{\min}\\&\omega_n = a \times (v_{ref} - v_{\min}) + \omega_{n0},\ \text{if}\ v_{ref} \ge v_{\min}\end{aligned} \tag{5}$$

$$\begin{aligned}K_1 &= \omega_n^2\\K_2 &= 2\zeta\omega_n\end{aligned} \tag{6}$$

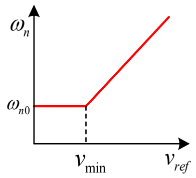

Fig. 5 The relationship between ω_n and v_{ref} for gain-scheduling of an acceleration estimator

4. SIMULATION AND EXPERIMENT

We evaluate the performance of the gain-scheduled acceleration estimator through computer simulation and experiment. Figure 6 shows a block diagram of an experimental setup. The experimental setup is composed of a motor, an encoder, and a PI-controller with the sampling period $T_s = 1$ msec. The motor is a 200W AC servo motor and a load is attached to the motor through a shaft as shown in Figure 7. The resolution of the encoder is 2048 count per revolution and quadrature decoding is used. In the experiment, ζ and ω_n of the acceleration estimator are set to 0.707 and 100Hz, respectively. ζ of the gain-scheduled acceleration estimator is also 0.707 and its ω_n is expressed by Eq. (7). Each component and environment in simulation is modeled identically on the experimental setup.

$$\begin{aligned}&\omega_n = 67.23 \times v_{ref} + 30,\ \text{if}\ v_{ref} \ge 0\\&a = 67.23,\ v_{\min} = 0,\ \omega_{n0} = 30\text{Hz}\end{aligned} \tag{7}$$

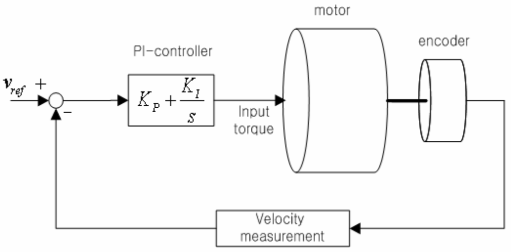

Fig. 6 Block diagram of experimental setup

When a motor velocity is controlled to follow a trapezoidal velocity command, a fixed-time method, an acceleration estimator and the proposed gain-scheduled acceleration estimator are used respectively to measure the velocity of the motor. Maximum velocity command is $\Delta x \cong 1.3653$ pulse per sampling period. It corresponds to the velocity of 1.0472 radian per second.

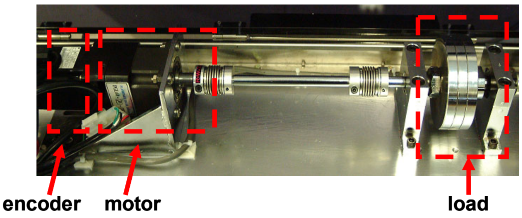

Fig. 7 Experimental hardware

Figure 8 and Figure 9 present the simulation and experimental results, respectively. The experimental result is almost similar to the simulation result except the low frequency ripple observed in experimental result. It is due to uncertainties of the model between the simulation and the experiment. Practically, the cyclic disturbance to the motor takes place in experimental setup because the shaft alignment between the motor and the load is not fitted perfectly. In the fixed-time method, it is shown that the velocity measurement is very inaccurate and the control performance is poor. The input torque of the motor is very noisy, and it can generate the sizzle in the motor. It is dangerous since the noise raises the vibration and shortens the lifetime of a machine. On the other hand, both the acceleration estimator and the proposed gain-scheduling method are more stable. Their velocity is controlled accurately with the smooth torque. The result of the gain-scheduled acceleration estimator differs subtly from that of the acceleration estimator. It is shown that the gain-scheduling method is more effective to reduce the noise near zero velocity than the acceleration estimator. Therefore the gain-scheduling method seems to have an advantage which prevents the motor from undesirable control such as stick-slip or direction change by noisy input torque in extremely low velocity. In addition, it is expected that the gain-scheduling by the velocity command will enable the appropriate bandwidth of the acceleration estimator to be set in high velocity.

5. CONCLUSION

In this paper, we proposed a gain-scheduled acceleration estimator of an appropriate bandwidth for the velocity measurement in wide range of velocity. We discussed characteristics of velocity measurement due to the quantization effect of the discrete encoder. An acceleration estimator was introduced, and its gain-scheduling scheme was proposed. Its performance was evaluated in velocity control system through computer simulation and experiment. The results showed that the gain-scheduling method was stable and more effective to reduce the noise near zero velocity than the acceleration estimator. From the results, the possibility of the gain-scheduling by velocity command was verified to effectively set the bandwidth of the acceleration estimator. In the future, research work to demonstrate the usefulness of the gain-scheduled acceleration estimator for motion control system will be fulfilled. As it's a first step, we are going to evaluate the performance of the gain-scheduled acceleration estimator with respect to the phase delay of the velocity measurement in wide velocity range.

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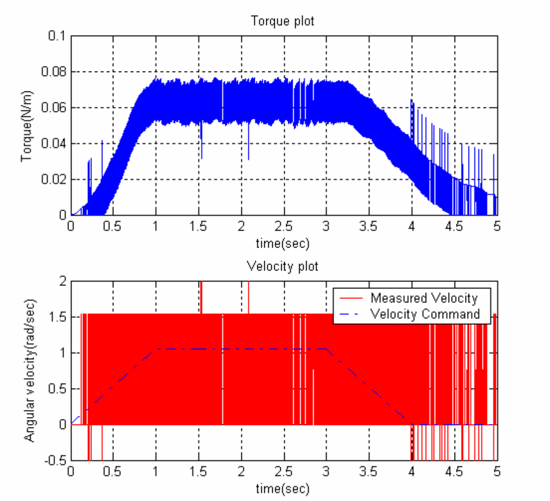
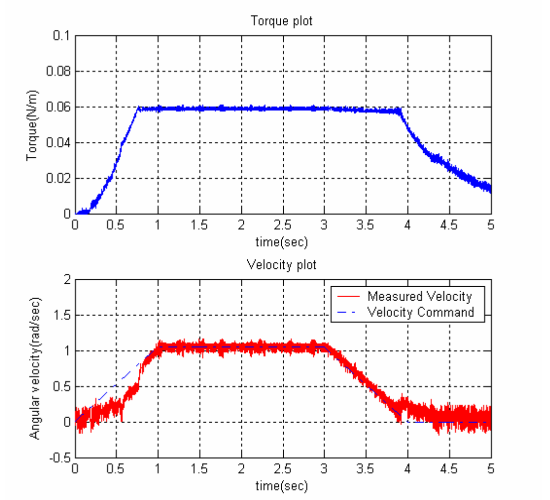
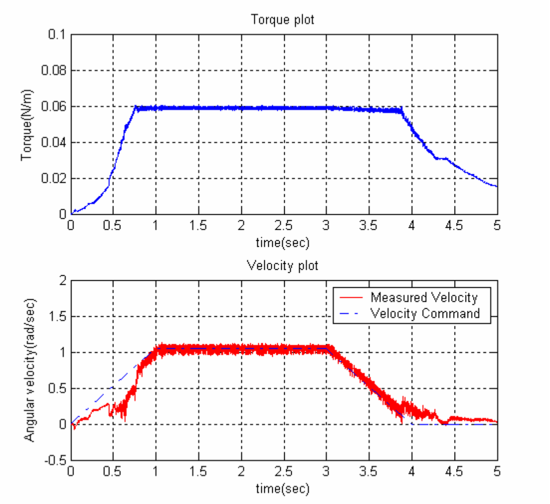


(a) fixed-time method

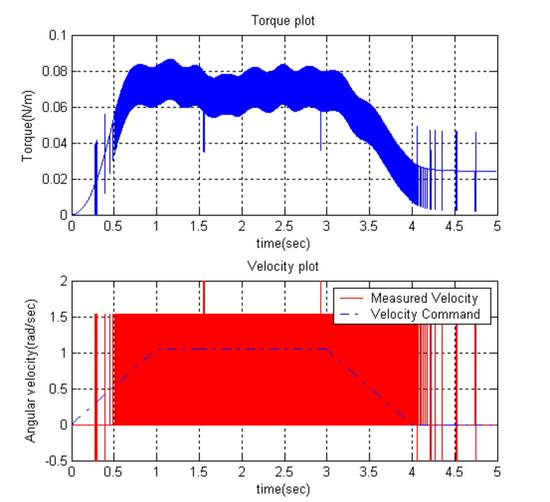


(b) acceleration estimator
with $\zeta = 0.707$ and $\omega_n = 100$

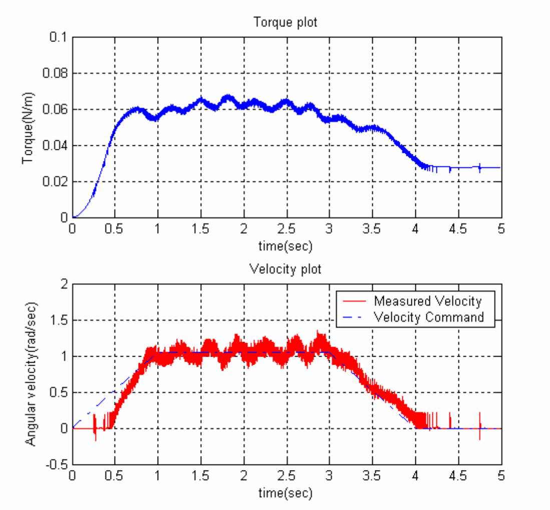


(c) gain-scheduled acceleration estimator
with $\zeta = 0.707$, $\omega_{n0} = 30$ and $a = 67.23$

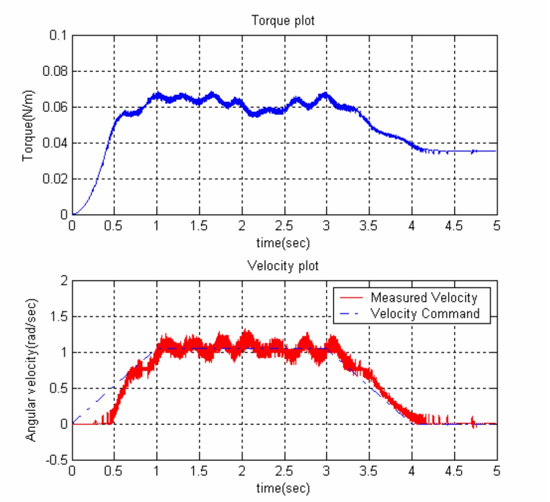
Fig. 8 Computer simulation results



(a) fixed-time method



(b) acceleration estimator
with $\zeta = 0.707$ and $\omega_n = 100$



(c) gain-scheduled acceleration estimator
with $\zeta = 0.707$, $\omega_{n0} = 30$ and $a = 67.23$

Fig. 9 Experimental results