Haptic Friction Display of a Hybrid Active/Passive Force Feedback Interface

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Abstract: This paper addresses both theoretical and experimental studies of the stability of haptic interfaces during the simulation of virtual Coulomb friction. The first objective of this paper is to present an analysis of how friction affects stability in terms of the describing function method and the absolute stability theory. Two different feedback methods are introduced and are used to evaluate the analysis: an active force feedback, using a motor, and a passive force feedback, using controllable brake. The second objective of this paper is to present a comparison of the theoretical and experimental results. The results indicate that the sustained oscillations due to the limit cycle occur when simulating friction with an active force feedback. In contrast, a passive force feedback can simulate virtual friction without the occurrence of instability. In conclusion, a hybrid active/passive force feedback is proposed to simulate a highly realistic friction display.

Keywords: haptic friction display, stability, active force feedback, passive force feedback, hybrid force feedback

1. INTRODUCTION

Haptic displays of virtual Coulomb friction can be challenging. Jex describes four tests that a good haptic interface should pass [1]. One test is the ability to simulate Coulomb friction without sponginess or jitter. Salcudean and Vlaar attempted to emulate stick-slip friction, using magnetically levitated haptic displays [2]. They used a Karnopp model to simulate stick-slip friction. However, their experimental results showed that the operator experienced sustained oscillations when simulating stick-slip friction, due to the zero-crossing limit cycle. Richard extended a Karnopp model by the addition of a virtual coupling, to avoid the limit cycle by a very careful selection of the virtual coupling stiffness and damping [3]. Richard mentions that the discrete time controller for the virtual environment implementation caused the undesirable oscillations. Such oscillations destroyed the realism, and they could be dangerous to the operator. In Richard's research, a conventional single DOF active force feedback interface was used to conduct haptic friction display. The interface consists of a DC motor as the actuator and a digital encoder for position sensing. The present work ascertains, theoretically and experimentally, whether oscillations occur in a virtual friction display with an active force feedback. The present work also proposes an alternative feedback method, termed "hybrid active/passive force feedback" that uses both a motor and a controllable brake to produce a highly stable friction display.

Consider a conventional single DOF haptic display, such as the one pictured in Fig. 1, with a human operator Z_o, a haptic device M, and an environment E [4, 5]. This model uses an ideal active actuator, a position sensor, a sampler, and a zero order hold. Previous studies have been limited to a consideration of the stability of the haptic interface with linear environments mostly dominated by the stiffness and damper. This study concentrates on the stability of the haptic interface with a Coulomb friction nonlinear environment, as depicted in Fig. 2.

In a preliminary study, we experimentally showed that a passive force feedback, using a controllable brake as a passive actuator, could easily display various virtual nonlinear environments, such as Lego block, Styrofoam, and a push button on a door knob [6]; we also suggested that the hybrid active/passive force feedback improved both the stability and the performance on a virtual hard wall display [7]. Now, we

aim to use the hybrid active/passive force feedback interface for other virtual environments, by realization of a haptic friction display.

There are two primary objectives of this paper. The first is to analyze the stability of the haptic interfaces when simulating a virtual friction display. The describing function method and the absolute stability theory are used to derive the conditions for the absence of oscillations and the asymptotical stability, respectively. Two different feedback methods are introduced and used to evaluate the analysis and the experiments: an active force feedback, using a motor, and a passive force feedback, using a controllable brake. The second objective is to show, by experimental results, how the passive force feedback contributes to the stability of the haptic interface.
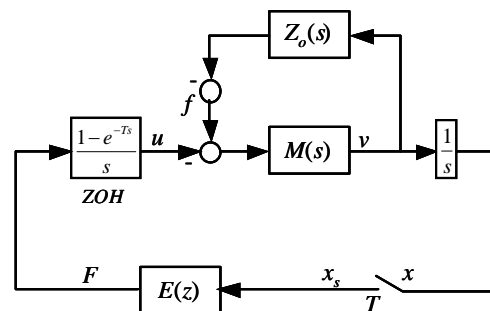

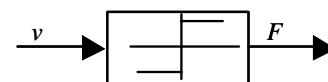Fig. 1 Simplified model of a single DOF haptic display.

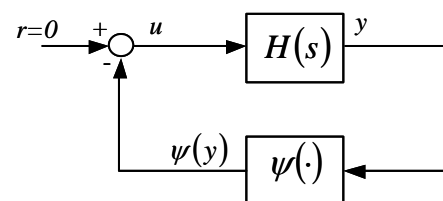Fig. 2 Block diagram representing Coulomb friction environment.

Fig. 3 Feedback system with a nonlinear element.

2. STABILITY VIA DESCRIBING FUNCTION

This section presents the effect of Coulomb friction on the stability of a haptic display. Coulomb friction is highly nonlinear. Unfortunately, it is quite difficult to find strong stability conditions for nonlinear systems. A powerful tool used to overcome this difficulty is the describing function method, which discovers the existence of limit cycles, regardless of whether the nonlinearity involved is "hard" or "soft" [8]. Fortunately, it is quite easy to derive a describing function for Coulomb friction, to examine the existence of oscillations.

First, we should briefly consider the describing function method [8, 9, 10]. Consider a nonlinear system represented by a feedback connection of a linear time-invariant dynamical system and a nonlinear element, as shown in Fig. 3.

The harmonic balance equation, which defines the characteristic equation of a closed-loop system, is derived from investigating the existence of periodic solutions satisfying $y(t+2\pi/\omega)=y(t)$ for all t, where ω is the frequency of oscillation.

$$H(j\omega)\Psi(a)+1=0 \tag{1}$$

The function $\Psi(a)$ defined by (1) is called the describing function of the nonlinearity ψ. It is used for odd, time-invariant, and memoryless nonlinearities, for which $\Psi(a)$ is real, dependent only on a, and given by the following expression:

$$\Psi(a)=\frac{2}{\pi a}\int_0^\pi \psi(a\sin\theta)\sin\theta\,d\theta \tag{2}$$

The power of the describing function method is its graphical solution of (1), which makes the method popular. Equation (1) can now be rewritten as

$$H(j\omega)=-\frac{1}{\Psi(a)} \tag{3}$$

Equation (3) suggests that (1) is solved by plotting the Nyquist plot of $H(j\omega)$ for $\omega > 0$ and the locus of $-1/\Psi(a)$ for $a \geq 0$. Intersections of these loci provide the solutions to (1), which reveal the existence of limit cycles.

The importance of Nyquist plots in classical theory made this graphical implementation of the describing function method a popular tool with control engineers when they faced nonlinearities in control systems.

2.1 Describing Function Analysis of Active Force Feedback

When a conventional haptic interface with an active force feedback, as shown in Fig. 1, simulates a virtual Coulomb friction display, the virtual environment block consists of a differentiator and a Coulomb friction operator, as illustrated in Fig. 4. The virtual environment block is a discrete time block, because of the position differentiation acquired by the digital encoder for the velocity estimation. In this study, the velocity estimation is simply obtained via the backward differentiation of position. The haptic display, as considered here, includes the human operator; it has been discussed in depth elsewhere [5, 11]. One reasonable assumption is that the human operator can be replaced by a set $Z_o(s)$ of linear, time-invariant passive operators.

The effects of an active force feedback in simulating a virtual Coulomb friction display on the closed–loop stability of the haptic interface can be studied via the describing

function analysis. Coulomb friction is constant, except for a sign dependence on the velocity:

$$\psi(v)=c\,\mathrm{sgn}(v) \tag{4}$$

where c is a Coulomb friction constant. The describing function of the Coulomb friction is given as:

$$\Psi(a)=\frac{4c}{\pi a} \tag{5}$$

Since the stability analysis of the haptic interface shown in Fig. 4 is quite complex, it is first transformed to an equivalent system that consists of a linear and a nonlinear block, as shown in Fig. 5. The equivalent system is derived as follows: $G(s)$ is a continuous time block comprising the human operator, the haptic device, the zero-order-hold, and the integrator; $D^*(s)$ is a discrete time block representing the differentiator shown in Fig. 5.

Fig. 4 Block diagram of an active force feedback for displaying virtual Coulomb friction.

Fig. 5 Equivalent block diagram of an active force feedback for displaying virtual Coulomb friction.

Fig. 6 Nyquist region for $(mj\omega+b+Z_o(j\omega))^{-1}$.

$$G(s)=\frac{1-e^{-Ts}}{s^2}\frac{1}{ms+b+Z_o(s)} \tag{6}$$

$$D^*(s)=\frac{1-e^{-Ts}}{T} \tag{7}$$

The linear portion of the closed-loop system $H^*(s)$, which defines the harmonic balance equation $1+H^*(j\omega)\Psi(a)=0$ is:

$$H^*(s)=D^*(s)G^*(s) \tag{8}$$

$$G^*(s) = \frac{1}{T} \sum_{k=-\infty}^{\infty} G(s + jk\omega_s) \qquad (9)$$

and $\omega_s = 2\pi/T$. Equation (9) is derived from the z transformation, using the convolution integral in the right half plane and taking the starred Laplace transform [12]; the resulting linear portion of the closed-loop system is drawn.

$$H^*(s) = \frac{1}{T^2} \sum_{k=-\infty}^{\infty} \frac{\left(1 - e^{-Ts}\right)^2}{s^2} \frac{1}{ms + b + Z_o(s)} \Bigg|_{s = s + jk\omega_s} \qquad (10)$$

Consider the $H^*(j\omega)$ to implement the harmonic balance equation for finding the graphical solutions of the limit cycles. The term $(mj\omega + b + Z_o(j\omega))^{-1}$ on the right hand is mapped to a closed disk centered on the real axis at $1/2b$ with a radius of $1/2b$ for every frequency [5, 13]. Let this region be denoted **R**, as represented in Fig. 6.

Because **R** is frequency-independent, it can be removed from the infinite sum when computing the region corresponding to $H^*(j\omega)$, which is defined as $\mathbf{R}_{H^*}$ [5]:

$$\mathbf{R}_{H^*}(\omega) = r(j\omega)\mathbf{R} \qquad (11)$$

where

$$r(j\omega) = \frac{1}{T^2} \sum_{k=-\infty}^{\infty} \frac{\left(1 - e^{-(j\omega + jk\omega_s)T}\right)^2}{(j\omega + jk\omega_s)^2} \qquad (12)$$

$$r(j\omega) = -\frac{1}{2} \sum_{k=-\infty}^{\infty} \frac{\left(1 - e^{-j\omega T}\right)^2}{1 - \cos\omega T} \qquad (13)$$

The multiplier $r(j\omega)$ may be considered a frequency dependent scaling and rotation [14, 15]. Thus, $\mathbf{R}_{H^*}$ is a closed disk at each frequency, as shown in Fig. 7; it has a center and radius as follows:
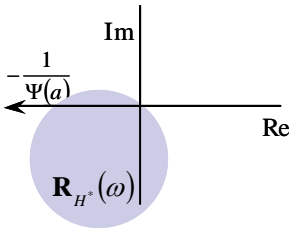
Fig. 7 Nyqiust region of $H^*(j\omega)$ and $-1/\Psi(a)$:

$$Center\{\mathbf{R}_{H^*}(\omega)\} = \frac{1}{4b} \frac{\left(1 - e^{-j\omega T}\right)^2}{\cos\omega T - 1}$$

$$Radius\{\mathbf{R}_{H^*}(\omega)\} = \frac{1}{4b} \left|\frac{\left(1 - e^{-j\omega T}\right)^2}{\cos\omega T - 1}\right| \qquad (14)$$

The describing function analysis can be applied to determine the stability of the haptic interface when simulating a virtual Coulomb friction with an active force feedback. Since the describing function of the Coulomb friction $\Psi(a) = 4c/\pi a$ is positive real, the locus of $-1/\Psi(a)$ in the complex plane will be confined to the negative real axis. From the Nyquist criterion, limit cycles may appear because of the existence of the intersections between $H^*(j\omega)$ and $-1/\Psi(a)$. If small perturbations occur around the intersection, the limit here may be unstable; that is, the human operator will suffer undesirable oscillations in a virtual friction display with an active force feedback.

2.2 Describing Function Analysis of Passive Force Feedback

An and Kwon [7] suggested that a controllable physical brake could be an alternate controllable damper, to improve the stability and the performance of the haptic system when simulating a virtual hard wall display, as shown Fig. 8. A motor and a brake are used to simulating virtual stiffness and damping, respectively. In a physical sense, a brake is a friction device rather than a damper. That is, a brake naturally has a frictional behavior in an opposite direction of motion. Since a controllable brake can control a resistant force against motion, it may successfully simulate a friction with a variable magnitude without any help of a motor. Fig. 9 shows a block diagram of a haptic interface during the simulation of virtual Coulomb friction with a controllable physical brake, which is simply extracted from the Fig. 8. The linear portion of the system, $H(s)$ consists of a human operator and a haptic device. It is straightforwardly derived.

$$H(s) = \frac{1}{ms + b + Z_o(s)} \qquad (15)$$

The Nyquist curve of $H(j\omega)$ is equivalent to that shown in Fig. 6. From the describing function analysis, it is easy to see that there are no limit cycles to cause oscillations because of the absence of intersections between $H(j\omega)$ and the locus of the Coulomb friction, $-1/\Psi(a)$. This shows that the passive force feedback with a controllable brake can successfully display the virtual Coulomb friction, without any oscillations.

3. STABILITY VIA ABSOLUTE STABILITY CRITERIA

The describing function analysis is, strictly speaking, a criterion for the absence of oscillations rather than the stability condition of a whole system. Absolute stability criteria, on the other hand, can determine the asymptotic stability of a nonlinear system represented as a feedback connection of a linear dynamical system and a nonlinear element satisfying the so-called sector condition [16]. Absolute stability is graphically interpreted in the Nyquist plane in a similar manner as the describing function analysis. There are many well-known absolute stability criteria. Among these, the circle criterion is an appropriate criterion to apply to continuous time nonlinear systems, and the Tsypkin's criterion is effective for the stability analysis of discrete time systems.

The Tsypkin's criterion [17] provides the sufficient condition for the stability of a haptic interface during a simulation of virtual Coulomb friction with an active force feedback, as shown in Fig. 5. The Coulomb friction falls into sector boundaries for the sampled velocity v^* as follows:

$$0 \le \frac{\psi(v^*)}{v^*} \le \infty \qquad (16)$$

The Tsypkin's criterion for an asymptotically stable system is that the Nyquist plot of the linear portion $H^*(j\omega)$ must lie wholly within the closed RHP.

$$Re[H^*(j\omega)] > 0 \qquad \forall \ 0 \le \omega \le \frac{\omega_s}{2} \qquad (17)$$

The Nyquist region of $H^*(j\omega)$, represented in Fig. 7, however, does not satisfy the above stability condition. Thus, the active force feedback becomes unstable when simulating virtual friction. This instability may be caused by the sample-and-hold effect in the feedback loop.

The circle criterion [8, 9, 16] is the most widely used to investigate the stability of continuous time nonlinear systems. Accordingly, a sufficient condition can be also derived for the stability of a haptic interface during the display of virtual Coulomb friction with the passive force feedback, as illustrated in Fig. 9. The sector condition of the Coulomb friction is similar to that of the case using discrete time.
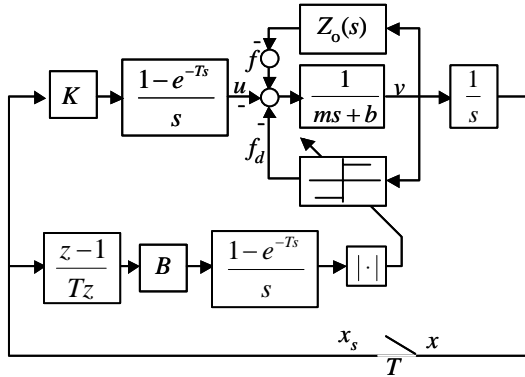
Fig. 8 A hybrid active/passive force feedback when displaying a virtual hard wall using a motor for "stiffness (K)" and a brake for "damping (B)"
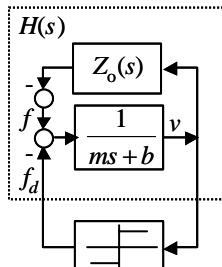
Fig. 9 Block diagram of a passive force feedback for displaying virtual Coulomb friction.

$$0 \le \frac{\psi(v)}{v} \le \infty \qquad (18)$$

The sufficient condition for asymptotic stability given by the circle criterion is that the Nyquist region of $H(j\omega)$ must lie outside the LHP. An examination of Fig. 6 shows that the haptic interface with a passive force feedback using a controllable brake is asymptotically stable during the virtual Coulomb friction display.

An important aim of this work is to prove the asymptotic stability of the haptic interfaces during simulation of virtual Coulomb friction with two different feedback methods--active or passive force feedback--using absolute stability criteria, such as Tsypkin's criterion and the circle criterion, as opposed

4. NUMERICAL SIMULATIONS

A simulation study was performed to verify the validity of the stability analysis that was described in the foregoing sections. As previously mentioned, the human operator is considered a force-generating component with the impedance

$Z_o(s)$. Thus, the dynamics of the operator can be modeled as follows [18, 19]:

$$f_h - f = Z_o(s)v \qquad (19)$$

where f_h and f represent the force generated by the operator's muscles and the force applied by the operator to the haptic interface, respectively. For the purposes of the simulation, the force produced by the operator's muscles f_h is assumed to be a predefined force trajectory $f_h=40\sin(\pi^2/4\cdot t)+\sin(\pi/8\cdot t)+2[1-\cos(\pi/2\cdot t)(N)]$. The impedance of a human operator can be represented approximately as a simple mass-damper-spring model. Tsuji et al. estimated the human hand impedance using an impedance ellipse as a graphical representation tool [20]. In this work, it is assumed that $Z_o(s)=1.287s+29.33+178/s$ (Ns/m) is calculated from the root-mean-square of the sizes of inertia, stiffness, and viscosity ellipses, respectively. Assuming a haptic device with mass $m = 0.1$ (kg), damping $b = 0.5$ (Ns/m), and a sampling frequency 1kHz, an attempt was made to investigate whether two types of feedbacks--active or passive--cause the haptic interface to be unstable when simulating the Coulomb friction display. The magnitude of the static friction is given 10 (N). Fig. 10 shows the responses of the velocity and forces; it also shows that active force feedback becomes unstable with zero-crossing limit cycle. Passive force feedback, on the other hand, simulates the friction without oscillations.
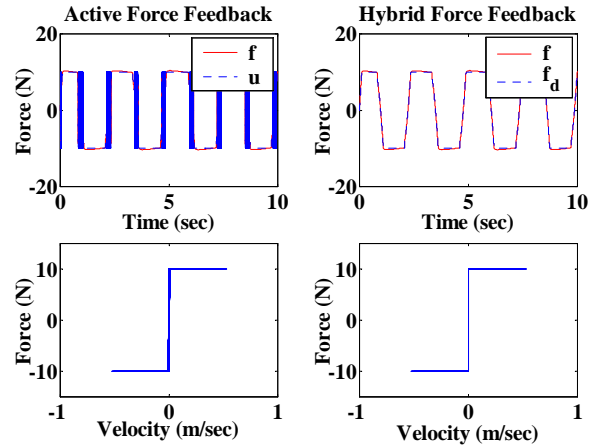

Fig. 10   Simulation results of virtual Coulomb friction display

5. EXPERIMENTAL RESULTS

A single DOF haptic device, as shown in Fig. 11, is used to verify experimentally the theoretical predictions for the stability of the haptic interface during virtual Coulomb friction display. The device consists of a DC motor for the active force feedback, an MR controllable brake for the passive force feedback, a cable transmission, and a handle. A rotary-type torque transducer, placed between the handle and the brake shaft, measures the interaction force between the device and the operator. The angular position of the motor shaft is measured by a digital encoder, which has an 8192 pulse/revolution. The position of the brake shaft can be obtained by dividing the transmission ratio into the position of the motor shaft. A 166 MHz PC, running QNX as a real-time operating system, is used for data acquisition and control. Torque commands are sent to each motor and brake driver via a D/A converter.

The Coulomb friction model is not causal because of

multi-valued friction at zero velocity. Karnopp proposed another approach, switching the input-output causalities to represent stick-slip friction, as shown in Fig. 12 [21]. Haessig and Friedland suggested that the Karnopp model is highly appropriate for friction simulation, with its advantages of computational efficiency and ease of implementation [22]. This model circumvents the discontinuity at zero velocity by defining a small stick region of speed, in which the velocity is considered zero. It is difficult to calculate a precisely zero velocity using digital computer simulations because of the encoder's quantization and discrete differentiation. Accordingly, in this study, the model of choice for friction display is the Karnopp model.

$$F_f = \begin{cases} F_d \, \text{sgn}(v) & |v| > \Delta v \\ \max(F_s, F_a) & |v| \le \Delta v \end{cases} \quad (20)$$

where F_d, F_s, F_a, and Δv denote, respectively, the dynamic friction, the static friction, the force applied to the system, and the velocity band, below which the velocity is considered to be zero.

The experimental results should be thoroughly examined after the friction compensation of the haptic device has been performed. Fig. 13 shows the result of friction compensation, using a well-known state feedback method, which is described in [23]. The solid lines represent the predicted friction for each case using a least square regression. Uncompensated friction is predicted 0.14 (Nm). In contrast, the compensated friction is estimated 0.004 (Nm).

Since the friction is ultimately a function of velocity, a virtual spring is used to convert the input motion to an input force. That is, a small motion of the system away from its stuck position will generate a force proportion corresponding to the motion, as follows: $F_a = -K_p(x - x_{stuck})$ [2, 3].

Fig. 14 compares the result of the active force feedback with that of the hybrid (active/passive) force feedback during a simulation of virtual friction display. The word "hybrid" means that a motor is used to reduce the inherent friction of the system (active force feedback), and a controllable brake is simultaneously used to display the virtual friction (passive force feedback). The velocity band $\Delta v = 0.16$ (rad/s) = 2 x (encoder resolution)/sampling time is used all through experiments. The other parameters used in the active force feedback are $K_p = 100$ (Nm-sec/rad), $F_s = 1.2$ (Nm), and $F_d = 1.0$ (Nm). The parameters in the hybrid force feedback are also characterized by $K_p = 10000$ (Nm-sec/rad), $F_s = 3.5$ (Nm), and $F_d = 3.0$ (Nm). A remarkable difference between the active force feedback and the proposed hybrid force feedback is shown in Fig. 14. In the hybrid force feedback, the virtual friction is completely displayed. With the active force feedback, the system exhibits instability in the form of severe vibrations. Consequently, the operator can feel a more realistic "stick-slip friction" with the proposed hybrid force feedback.

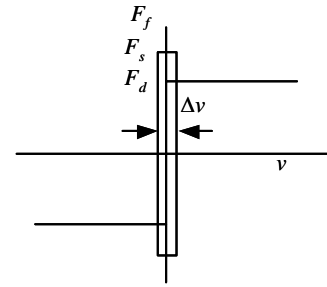Fig. 11   Single DOF hybrid active/passive haptic device
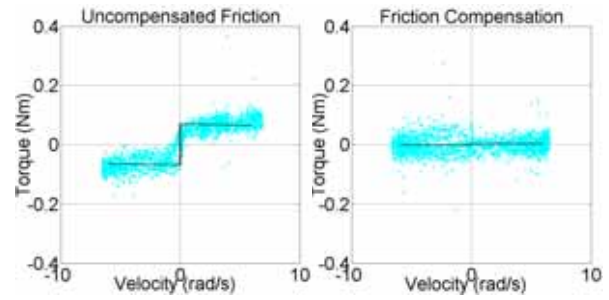

Fig. 12   The Karnopp model
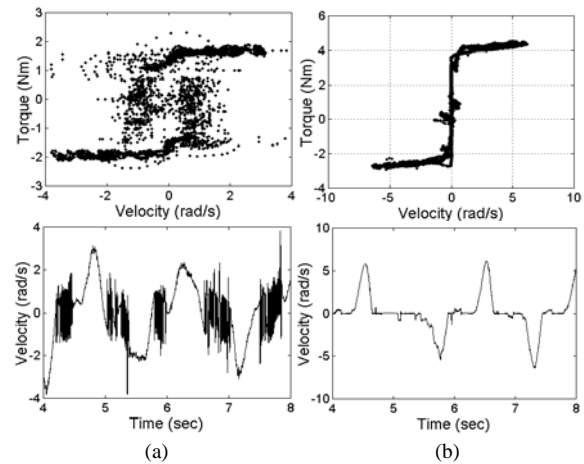

Fig. 13   Friction compensation result

(a)   (b)
Fig. 14   Friction display results: (a) active force feedback and (b) hybrid force feedback

6. CONCLUSIONS

A single DOF haptic device, as shown in Fig. 11, is used to verify experimentally the theoretical predictions for the stability of the haptic interface during virtual Coulomb friction display. This paper discusses the stability of haptic systems during virtual friction display, which is theoretically based on describing function and absolute stability criteria. The describing function is used to investigate the absence of oscillations; the sufficient conditions for the stability of haptic systems coupled to a virtual friction are presented by means of the circle criterion and Tsypkin's criterion. Two sets of analyses and experiments--involving the active force feedback and the passive force feedback--were performed and compared. The results show that the active force feedback becomes unstable during a virtual friction display due to limit cycle. In contrast, the passive force feedback can successfully simulate a virtual friction display without any instability. It is also demonstrated that the presented hybrid force feedback device

can display a more realistic stick-slip friction.

An important topic for future work is the development of methodologies that can handle restrictions on simulating the multi DOF constraints with the passive force feedback. For instance, if a haptic device has a coupled Jacobian matrix in a kinematic configuration, then it cannot simulate multi DOF virtual environments, including friction [6].

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