Natural Frequency Analysis of Spring-Manipulator System for Force Generation Utilizing Mechanical Resonance

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Abstract: This paper describes a natural frequency analysis conducted to find out a suitable working area for a spring-manipulator system generating a large vibrating force with mechanical resonance. Large force generation is one of the functions that we hope for a robot. For example, a weeding robot is required to generate a large force, because some weeds have roots spreading deeply and tightly. The spring-manipulator system has a spring element as an end-effector, so it can be in a state of resonance with the elasticity of the spring element and the inertial characteristics of the manipulator. A force generation method utilizing the mechanical resonance has potential to produce a large force that cannot be realized by a static method. A method for calculating a natural frequency of a spring-manipulator system with the generalized inertia tensor is proposed. Then the suitable working area for the spring-manipulator system is identified based on a natural frequency analysis. If a spring-manipulator system operates in the suitable working area, it can sustain mechanical resonance and generate a large vibrating force. Moreover, it is shown that adding a mass at the tip of the manipulator expands the suitable working area.

Keywords: spring-manipulator system, mechanical resonance, large force generation, natural frequency, suitable working area

1. INTRODUCTION

Large force generation is one of the functions that we hope for a robot. For example, a weeding robot, which is being developed in our laboratory, is required to generate a large force, because some weeds have roots spreading deeply and tightly[1]. The capability can be improved by providing the robot with powerful actuators, but it will cause a bloated robot that consumes energy uselessly because powerful actuators are ordinarily heavy.

In order to achieve efficient large force generation, a robot should exploit its characteristics and capabilities to the full. For this end, Papadopoulos and Gonthier introduced the force workspace, which is a map indicating the locations where a robot can apply a given force, and showed that force capabilities can be improved by employing base mobility and manipulator redundancy[2]. Imamura and Kosuge proposed virtually unactuated joints so that a manipulator could generate a force that is larger than the load capacity[3]. Kobayashi, Kishida, and Ohkawa formulated and solved an optimization problem to find out work postures in which a robot manipulator realizes a force as large as possible[4].

This paper discusses a system that utilizes mechanical resonance. The mechanical resonance is usually treated as an undesirable phenomenon in designing mechanical systems, because it will lead to troubles of devices and become a source of breakdown. Therefore, engineers design systems carefully so that mechanical resonance will never happen. The mechanical resonance is generally considered to be an unfavorable thing. However, our research purpose is to utilize the mechanical resonance for a large force generation.

We propose a manipulator generating a large vibrating force by exploiting mechanical resonance as shown in Fig. 1. The robot manipulator in the concept diagram performs weeding with a vibrating force that is produced by the mechanical resonance. The manipulator with a spring element as an end-effector is called spring-manipulator system. The spring-manipulator system can be in a state of resonance due to the elasticity of the spring and the inertial characteristics of the manipulator. Then the manipulator applies a large vibrating force to an object utilizing the mechanical resonance. The dynamic way exploiting the mechanical resonance will be able to generate a large force that cannot be realized by a static way.

In the case of a simple spring-mass system, when you apply a sinusoidal force to the mass at the natural frequency, a mechanical resonance will appear. If you attempt to excite a resonance in a mechanical system, you need to find out the natural frequency of the system. Since a dynamics of the spring-mass system is simple, linear, and one degree of freedom, you can easily calculate the natural frequency from the mass and the spring constant. But, to the spring-manipulator system, it is not easy to figure out the natural frequency due to multiple degrees of freedom and nonlinearity of the manipulator.

In order to solve the problem, we calculate inertial characteristics of a manipulator with the generalized inertia tensor, which is proposed by Asada[5] to evaluate the manipulator dynamics. Then the natural frequency of the spring-manipulator system is derived from the spring constant and the calculated inertial characteristics.

You need to carefully decide a work posture of the manipulator, because the inertial characteristics of the

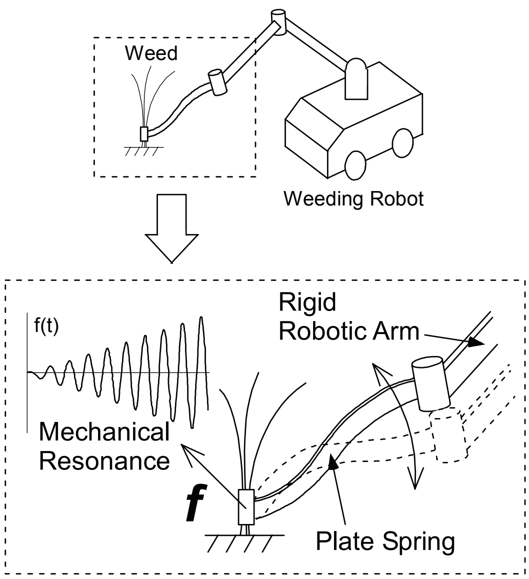

Fig. 1 Concept Diagram

manipulator depend on the posture. It is undesirable that the manipulator poses a configuration where rate of natural frequency change is too high to sustain a mechanical resonance by a sinusoidal force. For this reason, we analyze a relationship between the natural frequency and the posture of a manipulator in order to find out a suitable posture to sustain the mechanical resonance.

This paper is organized as follows. In the section 2, the model of the spring-manipulator system dealt with in this paper is introduced. The generalized inertia tensor of the spring-manipulator system is derived in the section 3. Then the natural frequency of the spring-manipulator system is calculated with the generalized inertia tensor, and the precision of the result is evaluated in the section 4. In the section 5, a suitable working area is figure out based on the calculated natural frequency, and the validity of the area is verified. In the section 6, it is shown that an additional mass at the tip of the manipulator extends the suitable working area. Finally, in the section 7, this paper is concluded.

2. SPRING-MANIPULATOR SYSTEM

The model of the spring-manipulator system used in this paper is shown in Fig. 2. This system consists of a two-link manipulator and a spring as an end-effector. The manipulator applies a force to a point through the spring. The spring-manipulator system is to apply a large vibrating force to the point in mechanical resonance.

We assume in this paper that the end-effector is not a plate spring but an ideal linear spring to simplify the model. If the spring-manipulator system has a plate spring as the concept diagram shown in Fig. 1, the motion of the manipulator is constrained. The model adopted here describes the constraint as a rail that the tip of the manipulator moves on. Depending on a position of a place where the system applies a force, the rail slides in parallel and the length of the spring changes.

Moreover, we suppose that the spring-manipulator system operates horizontal plane so that the gravity effect can be neglected.

The symbols used in this paper are as follows:

q_1, q_2: relative joint angles ($\boldsymbol{q} = [q_1, q_2]^T$)
τ_1, τ_2: joint torques ($\boldsymbol{\tau} = [\tau_1, \tau_2]^T$)
x_e, y_e: tip position of manipulator ($\boldsymbol{p}_e = [x_e, y_e]^T$)
m_1, m_2: mass of links
l_1, l_2: length of links
l_{g1}, l_{g2}: distance from proximal joint to center of mass
I_1, I_2: moment of inertia about center of mass
k_{sp}: spring constant

3. GENERALIZED INERTIA TENSOR

This section introduces an inertia tensor employed for calculating a natural frequency of the spring-manipulator system.

Mechanical resonance is observed in a system when a frequency of a force applied to the system matches its natural frequency. If you know a natural frequency of a system, you can bring about a mechanical resonance intentionally.

Natural frequencies of a mechanical system are calculated from the elasticity and inertia of the system. For example, in the case of a simple spring-mass system, the natural frequency is $\sqrt{k/m}/2\pi$ Hz, where the m is the mass and the k is the spring constant of the spring-mass system. Since the m and k are constant, the natural frequency is also constant. Consequently, you can excite a spring-mass system with a harmonic vibration at the natural frequency. On the other hand, the inertial characteristics of a manipulator changes depending on its posture and a direction of applied force. Therefore natural frequencies of spring-manipulator systems should be calculated in consideration of the characteristic.

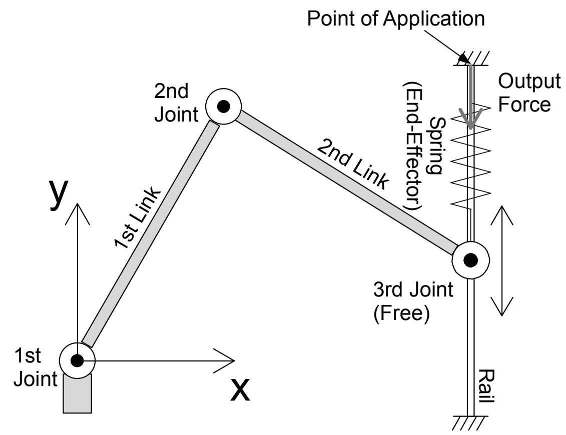

Fig. 2 Model of Spring-Manipulator System

Asada proposed a generalized inertia tenor to represent an inertia property of a manipulator[5]. With this tensor, inertial characteristics of a manipulator can be figured out. In Ref. [6], we calculated the natural frequency of a spring-manipulator system different from the system shown in Fig. 2, and verified the validity of the results. Thus, the natural frequency of the spring-manipulator system dealt with in this paper is also estimated with the calculation method.

A procedure to derive the generalized inertia tensor is shown here. First, the kinetic energy of the spring-manipulator system is described as follows:

$$K = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} = \frac{1}{2}\begin{bmatrix}\dot{q}_1 & \dot{q}_2\end{bmatrix}\begin{bmatrix}M_{11} & M_{12}\\ M_{12} & M_{22}\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}, \quad (1)$$

where

$M_{11} = I_1 + I_2 + m_1 l_{g1}^2 + m_2 l_1^2 + m_2 l_{g2}^2 + 2 m_2 l_1 l_{g2}\cos(q_2)$,

$M_{12} = I_2 + m_2 l_{g2}^2 + m_2 l_1 l_{g2}\cos(q_2)$, $M_{22} = I_2 + m_2 l_{g2}^2$.

Second, with the Jacobian matrix $\boldsymbol{J}(\boldsymbol{q})$, the relationship between the joint angular velocity and the tip velocity of the manipulator is given by

$$\dot{\boldsymbol{p}}_e = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}} = \begin{bmatrix}J_{11} & J_{12}\\ J_{21} & J_{22}\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}, \quad (2)$$

where

$J_{11} = -l_1\sin(q_1) - l_2\sin(q_1+q_2)$, $J_{12} = -l_2\sin(q_1+q_2)$,

$J_{21} = l_1\cos(q_1) + l_2\cos(q_1+q_2)$, $J_{22} = l_2\cos(q_1+q_2)$.

Next, substituting Eq. (2) into Eq. (1), the kinetic energy of the spring-manipulator system is expressed in the quadratic form of the tip velocity $\dot{\boldsymbol{p}}_e$ as follows:

$$\begin{aligned} K &= \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &= \frac{1}{2}(\boldsymbol{J}^{-1}(\boldsymbol{q})\dot{\boldsymbol{p}}_e)^T \boldsymbol{M}(\boldsymbol{q})(\boldsymbol{J}^{-1}(\boldsymbol{q})\dot{\boldsymbol{p}}_e) \\ &= \frac{1}{2}\dot{\boldsymbol{p}}_e^T \boldsymbol{J}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}^{-1}(\boldsymbol{q})\dot{\boldsymbol{p}}_e \\ &= \frac{1}{2}\dot{\boldsymbol{p}}_e^T \boldsymbol{G}\dot{\boldsymbol{p}}_e . \end{aligned} \quad (3)$$

This matrix $\boldsymbol{G}$ is called the generalized inertia tensor and defined as

$$\begin{aligned} \boldsymbol{G} &\triangleq \boldsymbol{J}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}^{-1}(\boldsymbol{q}) \\ &= \begin{bmatrix} G_{xx} & G_{xy} \\ G_{xy} & G_{yy} \end{bmatrix}, \end{aligned} \quad (4)$$

where

$G_{xx} = (J_{22}^2 M_{11} - 2J_{21}J_{22}M_{12} + J_{21}^2 M_{22})/g_{den}$,

$G_{xy} = (-J_{12}J_{22}M_{11} + (J_{11}J_{22} + J_{12}J_{21})M_{12} - J_{11}J_{21}M_{22})/g_{den}$,

$G_{yy} = (J_{12}^2 M_{11} - 2J_{11}J_{12}M_{12} + J_{11}^2 M_{22})/g_{den}$,

$g_{den} = l_1^2 l_2^2 sin^2(q_2)$.

The generalized inertia tensor $\boldsymbol{G}$ expresses the inertia property of the spring-manipulator system shown in Fig. 2

4. NATURAL FREQUENCY CALCULATION

In this section, a mathematical equation of the natural frequency for a spring-manipulator system is proposed, and its validity is verified.

Inertial characteristics of a manipulator depend on a direction of applied force. Given a unit vector $\boldsymbol{u}$ expressing a direction of applied force, a quadratic form $\boldsymbol{u}^T\boldsymbol{G}\boldsymbol{u}$ gives the inertial characteristics at the tip of the manipulator[5]. Since the tip of the manipulator shown in Fig. 2 moves on a rail along the y axis, the unit vector $\boldsymbol{u}$ is set as $\boldsymbol{u} = [0, 1]^T$, and the required inertial characteristics is obtained as follows:

$$[0\ 1]\begin{bmatrix} G_{xx} & G_{xy} \\ G_{xy} & G_{yy} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = G_{yy} . \quad (5)$$

From analogy to a spring-mass system, the natural frequency of the spring-manipulator system is calculated by the following equation.

$$f_{nGIT} = \frac{1}{2\pi}\sqrt{\frac{k_{sp}}{G_{yy}}} \quad (6)$$

Fig. 3 shows the calculated natural frequency of the spring-manipulator system calculated from Eq. (6). The horizontal axes indicate the tip position of the manipulator, and the vertical axis means the natural frequency. In the calculation, the manipulator's posture, the angles of the 1st and 2nd joints, is calculated from the tip position (x_e, y_e) with the following inverse kinematic equations.

$$q_1 = \cos^{-1}\left(\frac{x_e^2 + y_e^2 + l_1^2 - l_2^2}{2l_1\sqrt{x_e^2 + y_e^2}}\right) + \tan^{-1}\left(\frac{y_e}{x_e}\right) \quad (7)$$

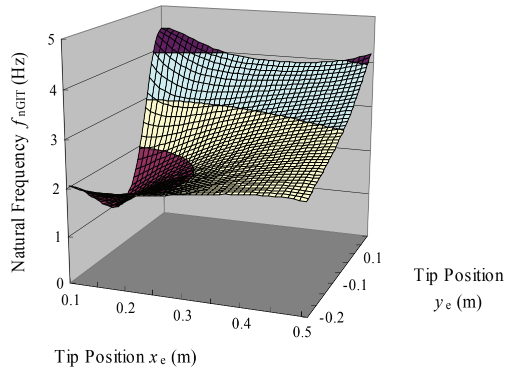

Fig. 3 Natural Frequency f_{nGIT} Calculated from Generalized Inertia Tensor

$$q_2 = -\cos^{-1}\left(\frac{x_e^2 + y_e^2 - l_1^2 - l_2^2}{2l_1 l_2}\right) \quad (8)$$

The physical parameters of the system are m_1=m_2=2.0kg, l_1=l_2=0.30m, l_{g1}=l_{g2}=0.15m, I_1=I_2=0.015kgm^2, k_{sp}=500N/m.

It is confirmed in Fig. 3 that the natural frequency varies depending on the posture of the manipulator. On the conditions established here, the natural frequency changes approximately between 1.0Hz and 4.5Hz.

Next, in order to verify the natural frequency calculated from the generalized inertia tensor, free vibration simulations of the spring-manipulator system are conducted. Then the natural frequencies f_{nGIT} calculated from Eq. (6) are compared with fundamental frequencies of the tip motion of the spring-manipulator system vibrating naturally without any input.

The tip of the manipulator is shifted from the equilibrium position by -1cm at the initial states. The tip motion data obtained from the free vibration simulations are pushed to FFT, and then the resulting fundamental frequency is represented by f_{n1}. Finally, the precision of the natural frequency f_{nGIT} in Eq. (6) is evaluated by the difference from the fundamental frequency f_{n1} as follows:

$$f_{ne} \triangleq |f_{nGIT} - f_{n1}| . \quad (9)$$

Fig. 4 plots the natural frequency error f_{ne}. The f_{nGIT} and f_{n1} were calculated to two places of decimals.

According to the graph in Fig. 4, the maximum error is 0.02Hz; the result demonstrates that Eq. (6) calculates the

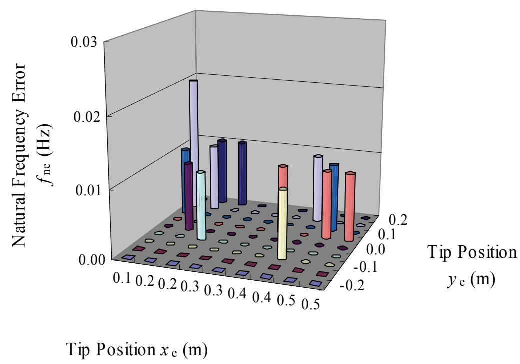

Fig. 4 Natural Frequency Error
$f_{ne} = |f_{nGIT} - f_{n1}|$

natural frequency of the spring-manipulator system with enough precision to excite mechanical resonance. The validity of the natural frequency Eq. (6) derived from the generalized inertia tensor is verified.

5. SUITABLE WORKING AREA

In this paper, a posture in which the rate of natural frequency change becomes low is called a suitable posture. If the rate of natural frequency change is low enough, the spring-manipulator system can keep being a state of resonance by a sinusoidal input, and increasing an effort force. Moreover, a collection of tip positions when a manipulator takes this suitable posture is defined as a suitable working area.

As mentioned above, a natural frequency of a spring-manipulator system varies with changes in the posture of the manipulator, because the inertial characteristics of the manipulator depend on its posture. The fact means that the natural frequency constantly fluctuates during operations. It is desirable that the fluctuation of the natural frequency is low, because you will be able to excite the spring-manipulator system in mechanical resonance by a sinusoidal input on that condition. In this section, the suitable working area for a spring-manipulator system is investigated based on the rate of the natural frequency change.

Since the tip of the manipulator shown in Fig. 2 is allowed to move on the straight rail, the natural frequency is differentiated with respect to the y_e-coordinate of the tip position, and the absolute value of the partial differential coefficient $|\partial f_{nGIT}/\partial y_e|$ is used to obtain a suitable working area for the spring-manipulator system.

Fig. 5 represents the rate of natural frequency change $|\partial f_{nGIT}/\partial y_e|$ in contour. The x and y axes of the graph express the tip position's coordinates.

The area where the rate of natural frequency change is less than 5Hz/m spreads on the bottom half of the graph shown in Fig. 5. On the other hand, on the neighborhood of the point (0.1m, 0.1m), the rate is higher, over 30Hz/m. From the graph shown in Fig. 5, the spring-manipulator system should take a posture where the y_e-coordinate of the tip position is negative when you drive the spring-manipulator system at its natural frequency. If the tip of the manipulator is near (0.1m, 0.1m), it will be difficult to keep the mechanical resonance by a sinusoidal input, because the rate of natural frequency change is high.

In order to verify the result about the suitable working area, excitation simulations of the spring-manipulator system are carried out.

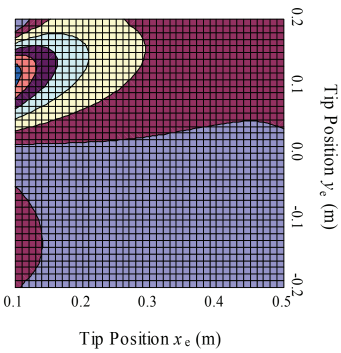

Fig. 5 Rate of Natural Frequency Change

The following input joint torque is applied for producing mechanical resonance in the spring-manipulator system.

$$\boldsymbol{\tau} = \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{f}_{ref} \tag{10}$$

The vector f_{ref} is a reference force vector, and it is chosen as follows:

$$\boldsymbol{f}_{ref} = \left[0,\ \alpha\{cos(2\pi f_{nGIT}t) - 1\}\right]^T . \tag{11}$$

This reference force vector is in direction of y axis, and it is sinusoidal at a calculated natural frequency f_{nGIT}. The amplitude is adjusted with the parameter α , which is 1.0 in the following excitation simulations

First, an excitation simulation is conducted on condition that the tip of the manipulator is put at (0.3m, -0.1m) at initial state. In this case, the natural frequency is 2.13Hz from Eq. (6), and the rate of its change is less than 5Hz/m.

Fig. 6 is the time history of generated force through the spring in the simulation. From this figure, it is confirmed that a mechanical resonance appears and the generated force increases with time; the system produces approximately 60N within 5s. The torques applied to the joints are shown in Fig. 7. If the amount of each joint torque is limited to 1.2Nm and the reference force vector is slope, not vibrating, the system can generate only about 4N. With this fact, the dynamic way utilizing mechanical resonance succeeds in producing a large force that cannot be realized by the static way.

Next, another excitation simulation is carried out on the condition that the rate of natural frequency change is high. The initial position of the tip is (0.1m, 0.1m). Then the natural frequency is 2.52Hz.

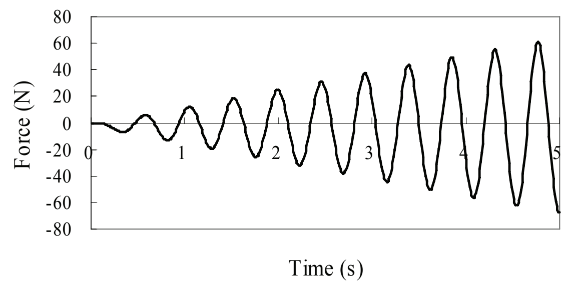

Fig. 6 Generated Force by Excitation inside Suitable Working Area: (x_e, y_e) = (0.3m, -0.1m)

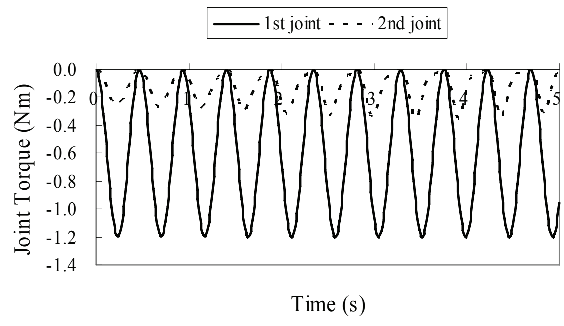

Fig. 7 Input Joint Torques (x_e, y_e) = (0.3m, -0.1m)

Fig. 8 shows the result of the excitation simulation. The torques applied to the joints are shown in Fig. 9. It is revealed from the result that the mechanical resonance cannot be persisted in, and the spring-manipulator system can only generate about 20N.

The results shown in Fig. 6 and Fig. 8 prove that operating in the suitable working area is important for the spring-manipulator utilizing mechanical resonance.

6. EXPANSION OF SUITABLE WORKING AREA BY ADDITIONAL MASS

In this section, in order to show that adding a mass at the tip of the manipulator is effective in expansion of the suitable working area, the sensitivity of the natural frequency f_{nGIT} to the inertial characteristics G_{yy} is analyzed.

Differentiating the natural frequency Eq. (6) with respect to the inertial characteristics G_{yy}, the sensitivity is obtained by

$$\left|\frac{\partial f_{nGIT}}{\partial G_{yy}}\right| = \frac{1}{4\pi}\sqrt{\frac{k_{sp}}{G_{yy}^3}}\ . \tag{12}$$

Fig. 10 graphs the sensitivity $\left|\partial f_{nGIT}/\partial G_{yy}\right|$. According to Fig. 10, the change of the spring constant k_{sp} hardly influences the sensitivity, but it sharply becomes large as the inertial characteristics G_{yy} is less than about 3kg.

Fig. 10 reveals that increasing inertial characteristics G_{yy} causes reducing the sensitivity $\left|\partial f_{nGIT}/\partial G_{yy}\right|$, and then it results in expansion of a suitable working area. Therefore, we propose adding a mass at the tip of manipulator in order to enlarge the suitable working area.

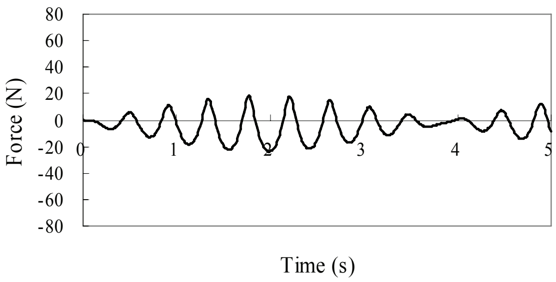

Fig. 8 Generated Force by Excitation outside Suitable Working Area: $(x_e, y_e) = (0.1\text{m}, 0.1\text{m})$

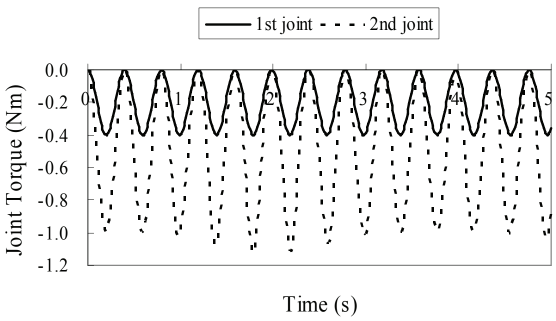

Fig. 9 Input Joint Torques $(x_e, y_e) = (0.1\text{m}, 0.1\text{m})$

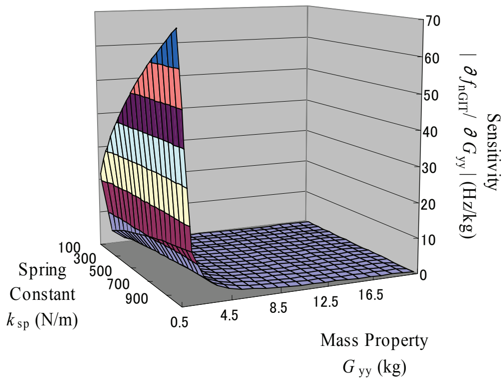

Fig. 10 Sensitivity of Natural Frequency to Inertial Characteristics $\left|\partial f_{nGIT}/\partial G_{yy}\right|$ Hz/kg

Next, the effectiveness of the additional mass is verified. If a manipulator has a mass at the tip, which is m_w kg, the kinetic energy of the manipulator is describes as follows:

$$\begin{aligned} K &= \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}m_w \dot{\boldsymbol{p}}_e^T \dot{\boldsymbol{p}}_e \\ &= \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}m_w \dot{\boldsymbol{q}}^T \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &= \frac{1}{2}\dot{\boldsymbol{q}}^T \left\{\boldsymbol{M}(\boldsymbol{q}) + m_w \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q})\right\}\dot{\boldsymbol{q}} \\ &= \frac{1}{2}\dot{\boldsymbol{p}}_e^T \left\{\boldsymbol{J}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}^{-1}(\boldsymbol{q}) + m_w \boldsymbol{I}\right\}\dot{\boldsymbol{p}}_e\ . \end{aligned} \tag{13}$$

Thus, the generalized inertia tensor $\boldsymbol{G}_w$ of the manipulator with an additional mass is given by

$$\begin{aligned} \boldsymbol{G}_w &\triangleq \boldsymbol{J}^{-T}(\boldsymbol{q})\boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}^{-1}(\boldsymbol{q}) + m_w \boldsymbol{I} \\ &= \begin{bmatrix} G_{wxx} & G_{wxy} \\ G_{wxy} & G_{wyy} \end{bmatrix}, \end{aligned} \tag{14}$$

where

$G_{wxx} = G_{xx} + m_w$, $G_{wxy} = G_{xy}$, $G_{wyy} = G_{yy} + m_w$.

Then the natural frequency is calculated by the following equation.

$$f_{nGIT} = \frac{1}{2\pi}\sqrt{\frac{k_{sp}}{G_{wyy}}} \tag{15}$$

Fig. 11 is the rate of natural frequency change recalculated with Eq. (15) on the condition that a mass of 0.5kg is added at the tip of the manipulator. Compared with Fig. 5, it is clear that adding a mass at the tip extends the suitable working area.

Since the necessary joint torques increase due to adding a mass, you should design a spring-manipulator system considering a trade-off between the amount of suitable working area and the joint torques.

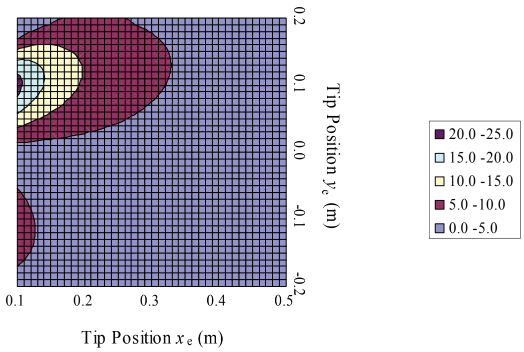

Fig. 11 Rate of Natural Frequency Change: A mass of 0.5kg is added at the tip of the manipulator.

7. CONCLUSION

In this paper, the calculation method for the natural frequency of the spring-manipulator system with the generalized inertia tensor was proposed, and the precision of the method was investigated. Then the suitable working area for the spring-manipulator system utilizing the mechanical resonance was identified based on the rate of natural frequency change. From the result, it was shown that the system could sustain the mechanical resonance and could generate a large vibrating force if the spring-manipulator system operates in the suitable working area. Moreover, the effectiveness of an additional mass at the tip of the manipulator for suitable working area expansion was demonstrated.

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