Design of a Fu y Model ased Sliding Mode Control for Nonlinear Systems

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Abstract: We proposed the indirect adaptive fuzzy model based sliding mode controller to control a nonaffine nonlinear systems. Takagi-Sugano fuzzy system is used to represent the nonaffine nonlinear system and then inverted to design the controller at each sampling time. Also sliding mode component is employed to eliminate the effects of disturbances, while a fuzzy model component equipped with an adaptation mechanism reduces modeling uncertainties by approximating model uncertainties. The proposed controller and adaptive laws guarantee that the closed-loop system is stable in the sense of Lyapunov and the output tracks a desired trajectory asymptotically.

Keywords: Takagi-Sugano fuzzy system , nonaffine nonlinear system, fuzzy sliding mode control

1. INTRODUCTION

Generally, fuzzy controllers are particularly useful in dealing with complex or ill-defined systems that conventional control methods are hard to handle.[1][2][3] The design of the fuzzy controller is based on the characteristics of the system to be controlled and quite frequently the resulting performance of the control system cannot be tuned efficiently.

However, fuzzy control rules, which are the most important factor in FLC, are generally obtained from intuition and experience of the experts, and such rules represented by linguistic rule sets or fuzzy relation. But there is always something difficult to obtain from such control rules and this makes the design of the controller difficult and the response trajectory of the controlled system is unpredictable. Also,

The sliding mode control approach is one of the robust control methods to handle systems with model uncertainties. Systematic design procedures for sliding mode controller are well known and available in the many literature.[4]-[7] Also the sliding mode control is robust the perturbations of parameters and the external disturbances. Besides, the stability analysis can be made by Lyapunov stability theorem.

Recently, many researchers have managed to two techniques together for better performance of the control systems.[8]-[11] Based on this property, fuzzy sliding mode control is actively researched. The key advantages of using fuzzy sliding mode control are no need of exact plant dynamics and no need of linear in the parameter condition on the unknown nonlinearities.

However, Most of the studies are generally restricted in class of affine nonlinear systems.

In this paper, we proposed the indirect adaptive fuzzy model based sliding mode controller to control a nonaffine nonlinear systems. Takagi-Sugano fuzzy system is used to represent the nonaffine nonlinear system and then inverted to design the controller at each sampling time. Also sliding mode component is employed to eliminate the effects of disturbances, while a fuzzy model component equipped with an adaptation mechanism reduces modeling uncertainties by approximating model uncertainties. The proposed controller and adaptive laws guarantee that the closed-loop system is stable in the sense of Lyapunov and the output tracks a desired trajectory asymptotically.

This paper is organized as follows. Section 2 gives problem statements. Section 3 presents Takagi-Sugeno fuzzy model for nonlinear systems. Then, the proposed fuzzy model based sliding mode control is detailed in section 4. In section 5, The effectiveness for the proposed controller is illustrated with computer simulations. Finally, section 6 presents some conclusions

2. PRO LEM STATEMENT

Consider a single-input single-output(SISO) nonlinear system

$$y^{(n)} = f(y, y^{(1)}, \cdots, y^{(n-1)}, u) \tag{1}$$

where

y $\mathbf{R}$: measured output

u $\mathbf{R}$: control input

$y^{(i)}(i = 1, 2, \cdots, n)$: i-th time derivative of output y

$f(\,) : \mathbf{R}^{n+1}$ $\mathbf{R}$: unknown nonlinear function

Note that, the nonlinearity $f(\,)$ is an implicit function with respective to u. The control objective is to find a control input u such that the output of the system tracks given a desired output trajectory $y_d(t)$, with an acceptable accuracy, under the constraint that all signal involved must be bounded.

Let $\mathbf{x} = [x_1, x_2, \cdots, x_n]^T = [y, y^{(1)}, \cdots, y^{(n-1)}]^T$ $\mathbf{R}^n$ be state vector. Then, we represent system (1) in state model

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(\mathbf{x}, u) \\ y &= x_1 \end{aligned} \tag{2}$$

Let us consider a desired smooth and bounded trajectory vector given by:

$$_d = [y_d, y_d^{(1)}, \cdots, y_d^{(n-1)}] \quad \mathbf{R}^n \tag{3}$$

In this case, the controller synthesis can be investigated using the input-output linearization technique [12] when the system under consideration is so that f can be written in an affine nonlinear system:

$$f(\mathbf{x}, u) = \alpha(\mathbf{x}) + \quad (\mathbf{x})u \tag{4}$$

where $(\mathbf{x})$ 0 for $\mathbf{x}$ $\mathbf{R}^n$.

Indeed, under this assumption the input-output linearization controller can be implemented using the analytical inversion of (4), that is:

$$u = \frac{\alpha(\mathbf{x})}{(\mathbf{x})} + \frac{v}{(\mathbf{x})} \tag{5}$$

where v is a new input.

However, this method cannot be directly applied to non-affine nonlinear system in the form (1), even when the dynamics of f are exactly known. When the analytic inversion of (1) cannot be determined, one must have recourse to numerical inversion techniques which necessary introduce difficulties inherent in iterative algorithms such as initialization values, convergence conditions and approximation errors.[13]

Moreover, when the dynamics of (1) are unknown, it is even more difficult to design the linearizing controller. As a alternative, when the plant dynamics are represented by a fuzzy system, the linearizing controller can be designed based on a local inversion of the fuzzy model.

3. TAKAGI-SUGENO FU MODEL FOR NONLINEAR S STEMS

Consider an optimal Takaki-Sugeno fuzzy model of the plant (1), whose rule-base consists of fuzzy rule with singleton consequence in the following form:

$$\begin{aligned} &\text{If } x_1 \text{ is } A_1^{i_1} \text{ and } \cdots \text{ and } x_n \text{ is } A_n^{i_n} \text{ and } u \text{ is } B^j \\ &\text{Then } y_f^{(n)} = {}^{i_1,i_2,\cdots,i_n,j} \end{aligned} \tag{6}$$

where $A_k^{i_k}$, $k = 1,2,\cdots,n$, is a linguistic term associated with variable x_k, B^j is a linguistic term associated with variable u and ${}^{i_1,i_2,\cdots,i_n,j}$ $\mathbf{R}$. When considering N_k linguistic terms for describing x_k, i.e. i_k $I_k = \{1,2,\cdots,N_k\}$, $k = 1,\cdots,n$ and M is linguistic terms for describing u, i.e. j $J = \{1,2,\cdots,M\}$, the complete rule-base is composed of $L = \overset{n}{\underset{k=1}{}} N_k M$ rules.

Using the singleton fuzzifier, center average defuzzification and product inference engine, the output of the Takagi-Sugano fuzzy model (6) is given by

$$\begin{aligned} y_f^{(n)} &= \frac{\overset{N_1}{\underset{i_1=1}{}} \cdots \overset{N_n}{\underset{i_n=1}{}} \overset{M}{\underset{j=1}{}} {}^{i_1,i_2,\cdots,i_n,j} \left[\overset{n}{\underset{k=1}{}} {}_{A_k^{i_k}}(x_k) \; {}_{B^j}(u) \right]}{\overset{N_1}{\underset{i_1=1}{}} \cdots \overset{N_n}{\underset{i_n=1}{}} \overset{M}{\underset{j=1}{}} \left[\overset{n}{\underset{k=1}{}} {}_{A_k^{i_k}}(x_k) \; {}_{B^j}(u) \right]} \\ &= \hat{f}(\mathbf{x},u, \;) = {}^T \; (\mathbf{x},u) \end{aligned} \tag{7}$$

where $(\mathbf{x},u)$ is L dimensional vector with its $i_1,i_2,\cdots,i_n,j$ element given by

$${}^{i_1,i_2,\cdots,i_n,j}(\mathbf{x},u) = \frac{\overset{n}{\underset{k=1}{}} {}_{A_k^{i_k}}(x_k) \; {}_{B^j}(u)}{\overset{N_1}{\underset{i_1=1}{}} \cdots \overset{N_n}{\underset{i_n=1}{}} \overset{M}{\underset{j=1}{}} \left[\overset{n}{\underset{k=1}{}} {}_{A_k^{i_k}}(x_k) \; {}_{B^j}(u) \right]} \tag{8}$$

and $= \left[{}^{1,1,\cdots,1,1}, \cdots, {}^{N_1,N_2,\cdots,N_n,M} \right]$ $\mathbf{R}^L$.

The assumption of a strict triangular partitioning of the universe of discourse allows to guarantee that each input variable is described with two linguistic terms at the most[13]. Thus, in the case of a system with $n+1$ inputs, at the most, 2^{n+1} rules are activated for any vector of inputs. The output generated by the fuzzy system is then reduced to that produced by fuzzy subsystem composed of the 2^{n+1} fired rules, i.e. the active subsystem.

Let us consider above the explanation. The membership functions of linguistic variables of u have the form of a triangle and are placed evenly throughout the whole defined space U_u as shown in fig. 1.

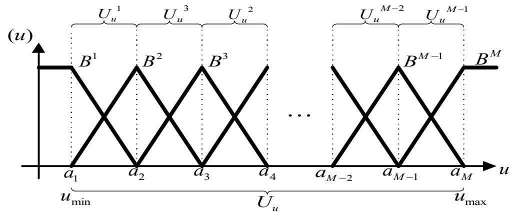

Fig. 1 Membership function of the input linguistic variables.

The space U_u can be decomposed into several subspaces $U_u^\alpha (\alpha = 1,2,\cdots,M-1)$. If u exists in subspace U_u^α, all membership function of linguistic variable of u given by

$${}_{B^m}(u) = \begin{cases} \dfrac{u - a_{m+1}}{a_m - a_{m+1}} & m = \alpha \\ \dfrac{a_{m-1} - u}{a_{m-1} - a_m} & m = \alpha + 1 \\ 0 & otherwise \end{cases} \tag{9}$$

where a_m is a constant satisfying ${}_{B^m}(a_m) = 1$.

Substituting (9) into (7) and considering that u exists in U_u^α, we have

$$\begin{aligned} &\hat{f}^\alpha(\mathbf{x},u, \;) = \\ &\frac{1}{a_\alpha - a_{\alpha+1}} \overset{N_1}{\underset{i_1=1}{}} \cdots \overset{N_n}{\underset{i_n=1}{}} \xi^{i_1,i_2,\cdots,i_n}(\mathbf{x}) \left(a_\alpha \; {}^{i_1,i_2,\cdots,i_n,\alpha+1} - a_{\alpha+1} \; {}^{i_1,i_2,\cdots,i_n,\alpha} \right) \\ &+ \frac{1}{a_\alpha - a_{\alpha+1}} \overset{N_1}{\underset{i_1=1}{}} \cdots \overset{N_n}{\underset{i_n=1}{}} \xi^{i_1,i_2,\cdots,i_n}(\mathbf{x}) \left({}^{i_1,i_2,\cdots,i_n,\alpha} - {}^{i_1,i_2,\cdots,i_n,\alpha+1} \right) u \end{aligned}$$

$$\hat{f}^\alpha(\mathbf{x},u, \;) = {}_1^\alpha(\mathbf{x}, \;) + {}_2^\alpha(\mathbf{x}, \;) u \tag{10}$$

where

$$\xi^{i_1,i_2,\cdots,i_n}(\mathbf{x}) = \frac{\overset{n}{\underset{k=1}{}} {}_{A_k^{i_k}}(x_k)}{\overset{N_1}{\underset{i_1=1}{}} \cdots \overset{N_n}{\underset{i_n=1}{}} \overset{M}{\underset{j=1}{}} \overset{n}{\underset{k=1}{}} {}_{A_k^{i_k}}(x_k)} \tag{11}$$

Therefore, the fuzzy system can be decomposed by $M-1$ fuzzy subsystems which are affine with respect to the control input.

Since $f()$ is unknown, we use Takagi-Sugeno fuzzy system for approximating the unknown function. Suppose that there exists an optimal parameter * which is defined by

$${}^* = \arg\min_{\mathbf{R}^L} \left[\sup_{\mathbf{x} \; {}_x, u \; U_u} \left[\hat{f}(\mathbf{x},u, \;) - f(\mathbf{x},u) \right] \right] \tag{12}$$

where ${}_x$ is a compact set in $\mathbf{R}^L$ and $L = \overset{n}{\underset{k=1}{}} N_k M$. Then the unknown nonlinear function can be expressed as

$$f(\mathbf{x},u) = \hat{f}(\mathbf{x},u, \;^*) + \delta_f \tag{13}$$

where $\hat{f}(\mathbf{x},u, \;^*)$ is the fuzzy approximation of $f(\mathbf{x},u)$, δ_f is approximation error such that $|\delta_f|$ ε^* where ε^* is unknown upper bound.

4. FU MODEL ASED SLIDING MODE CONTROL

We define the tracking error vector $\mathbf{e}$ as

$$\mathbf{e} = \left[e, e^{(1)}, \cdots, e^{(n-1)} \right]^T \quad \mathbf{R}^n \tag{14}$$

where $e = y - y_d$.

The VSC design consists in achieving the following steps:

1) Design a switching manifold S in the state space to represent a desired system dynamics, which is of lower order than the dimension of the given plant; S is defined by

$$S = \left\{ \mathbf{e} \quad \mathbf{R}^n \left| s(\mathbf{e}) \quad = \quad 0 \right. \right\} \tag{15}$$

2) Design a variable structure control

$$u(t) = \begin{cases} u^+ & when \;\; s \quad 0 \\ u^- & when \;\; s \quad 0 \end{cases} \tag{16}$$

such that any state x out side the switching surface is driven to reach this surface in finite time, that is the condition $s = 0$ is satisfied in finite time.

To specify the reaching condition a Lyapunov function is used. Let the Lyapunov function candidate be defined as:

$$V(t) = \frac{1}{2} s^2 \tag{17}$$

Then the reaching condition for existence of the sliding mode motion of the system under consideration is given as follows:

$$\dot{V}(t) = s\dot{s} \quad 0 \quad for \;\; \mathbf{x} \quad R^n - S \tag{18}$$

The control of the dynamics of a VSC system in the reaching mode may be made possible by specifying the dynamics of the switching function s. More specifically, the dynamics of the switching function s are described by a differential equation of the form:

$$\dot{s} = -K \operatorname{sgn}(s) \quad K \quad 0 \tag{19}$$

Note that we no longer to verify the reaching condition because it is inherent in the differential equation of the function s. By specifying the dynamics of the function s, we can predetermine the speed with which the system state approaches the switching manifold.

Define the switching surface

$$s = e^{(n-1)} + k_{n-1} e^{(n-2)} + \cdots + k_1 e + k_0 \int e dt = 0 \tag{20}$$

When the coefficients $k_i, i = 0, \cdots, n-1$ are chosen such that the following polynomial is Hurwitz

$$s^{(n)} + k_{n-1} s^{n-1} + \cdots + k_1 s + k_0 = 0 \tag{21}$$

then the tracking error $e(t) \quad 0$, as $t \quad \infty$.

Differentiating of s with respect to t, yields

$$\begin{aligned} \dot{s} &= e^{(n)} + k_{n-1} e^{(n-1)} + \cdots + k_1 \dot{e} + k_0 e \\ &= x^{(n)} - x_d^{(n)} + \sum_{i=0}^{n-1} k_i e^{(i)} \\ &= f(\mathbf{x}, u) - x_d^{(n)} + \sum_{i=0}^{n-1} k_i e^{(i)} \end{aligned} \tag{22}$$

Since $f(\,)$ is unknown, we substitute (13) into (22) and have as following equations

$$\dot{s} = \hat{f}(\mathbf{x}, u, \;\; ^*) + \delta_f - x_d^{(n)} + \sum_{i=0}^{n-1} k_i e^{(i)} \tag{23}$$

where $\hat{f}(\mathbf{x}, u, \;\; ^*) = \;\; ^{*T} \;\; (\mathbf{x}, u)$.

With easy (23) can be expressed as

$$\dot{s} = \hat{f}(\mathbf{x}, u, \quad) + \delta_f - x_d^{(n)} + \sum_{i=0}^{n-1} k_i e^{(i)} - \;\tilde{}\;^T \;\; (\mathbf{x}, u) \tag{24}$$

where $\tilde{} \;=\; - \;^*$.

The main objective in this section is to design an adaptation law for , so that fuzzy system can approximate unknown nonlinear function and to design sliding mode control law such that the tracking error converges to zero.

Consider the Lyapunov function candidate

$$V = \frac{1}{2} s^2 + \frac{1}{2\gamma} \;\tilde{}\;^T \tilde{} + \frac{1}{2\gamma_\varepsilon} \tilde{\varepsilon}^2 \tag{25}$$

where γ and γ_ε are positive constants, $\tilde{\varepsilon} = \varepsilon - \varepsilon^*$.

Differentiating (25), we have

$$\begin{aligned} \dot{V} &= s\dot{s} + \frac{1}{\gamma} \;\tilde{}\;^T \;\dot{}\; + \frac{1}{\gamma_\varepsilon} \tilde{\varepsilon}\dot{\varepsilon} \\ &= s(\hat{f}(\mathbf{x}, u, \quad) - x_d^{(n)} + \sum_{i=0}^{n-1} k_i e^{(i)}) + s\delta_f \\ &\quad + \frac{1}{\gamma} \;\tilde{}\;^T \left[\;\dot{}\; - \gamma \;\; s \;\; (\mathbf{x}, u) \right] + \frac{1}{\gamma_\varepsilon} \tilde{\varepsilon}\dot{\varepsilon} \end{aligned} \tag{26}$$

We consider the following sliding mode control input in order to eliminate the part $(\hat{f}(\mathbf{x}, u, \quad) - x_d^{(n)} + \sum_{i=0}^{n-1} k_i e^{(i)})$ of (26) using (10).

$$u = \frac{1}{\;_2^\alpha(\mathbf{x}, \quad)} \quad - \;_1^\alpha(\mathbf{x}, \quad) + x_d^n - \sum_{i=0}^{n-1} k_i e^{(i)} + u_s \quad U_u^\alpha \tag{27}$$

where u_s is switching input.

And we design the adaptive law for updating parameter vector as follows

$$\dot{} = \gamma \;\; s \;\; (\mathbf{x}, u) \tag{28}$$

Substituting (27) and (28) into (26), we obtain

$$\begin{aligned} \dot{V} &= s\delta_f + s u_s + \frac{1}{\gamma_\varepsilon} \tilde{\varepsilon}\dot{\varepsilon} \\ &\quad |s| |\delta_f| + s u_s + \frac{1}{\gamma_\varepsilon} \tilde{\varepsilon}\dot{\varepsilon} \quad |s| \varepsilon^* + s u_s + \frac{1}{\gamma_\varepsilon} \tilde{\varepsilon}\dot{\varepsilon} \end{aligned} \tag{29}$$

If the switching input u_s is taken as

$$u_s = -\varepsilon \operatorname{sgn}(s) \tag{30}$$

then (29) is given by

$$\dot{V} \quad \frac{1}{\gamma_\varepsilon} \tilde{\varepsilon} (\dot{\varepsilon} - \gamma_\varepsilon |s|) \tag{31}$$

If we choose the adaptation law for the upper bound of approximation error as

$$\dot{\varepsilon} = \gamma_\varepsilon |s| \tag{32}$$

then $\dot{V} \quad 0$ which ensures the closed-loop stability and the convergence of the tracking error to zero.

5. COMPUTER SIMULATIONS

We apply the proposed fuzzy model based sliding mode control algorithm given in the previous section to the following nonlinear equation

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1^2 + 0.15u^3 + 0.1(1 + x_2^2)u + \sin(0.1u) \\ y &= x_1 \end{aligned} \tag{33}$$

It appears clearly that the above nonlinear system is non-affine. In this case, a feedback linearization controller based on an analytic inversion cannot be directly implemented.

As an alternative method, a fuzzy model based sliding mode controller is designed with a local inversion of a fuzzy model.

In this simulation example, the control objective is to determine u so that the output $y(t)$ of the closed-loop systems follows the desired reference output given by

$$y_d(t) = \sin(t) + \cos(0.5t) \tag{34}$$

The initial state is $\mathbf{x}(0) = [0.6, 0.5]^T$. The initial parameters for fuzzy model are randomly selected and the switching surface is chosen $s = \dot{e} + 4e + 4\int e dt = 0$, $\gamma_f = 9$, $\gamma_\varepsilon = 0.1$. The universe of discourse of inputs x_1, x_2 and u are respectively [-2,2],[-2,2] and [-4,4]. In order to determine the control input u so that the fuzzy output follows the given desired trajectory, the inversion principle is applied, at each sampling time, to the 5 subsystems that can be activated.

The simulation results of the application of the proposed control design approach are depicted in from Fig. 2 to Fig. 5. Fig.2 shows that the output y tracks the reference input y_d effectively and Fig. 3 illustrates the history of the control input u. In Fig. 4 and Fig. 5, we display respectively response of switching surface and the estimated approximation error. The results of the simulation show the proposed controller have satisfactory transient performance and small tracking error.

. CONCLUSIONS

In this paper, we proposed the indirect adaptive fuzzy model based sliding mode controller to control nonaffine nonlinear systems. Takagi-Sugano fuzzy systems is used to represent the nonaffine nonlinear system and then inverted to design the controller at each sampling time. Also sliding mode component is employed to eliminate the effects of disturbances, while a fuzzy model component equipped with an adaptation mechanism reduces modeling uncertainties by approximating model uncertainties. The proposed controller and adaptive laws guarantee that the closed-loop system is stable in the sense of Lyapunov and the output tracks a desired trajectory asymptotically.

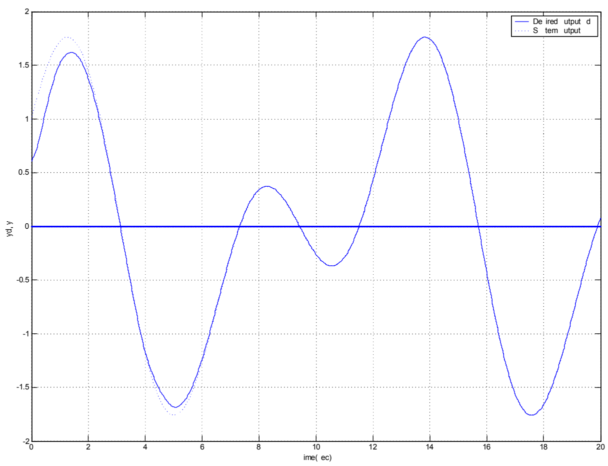

Fig. 2 system output and desired output

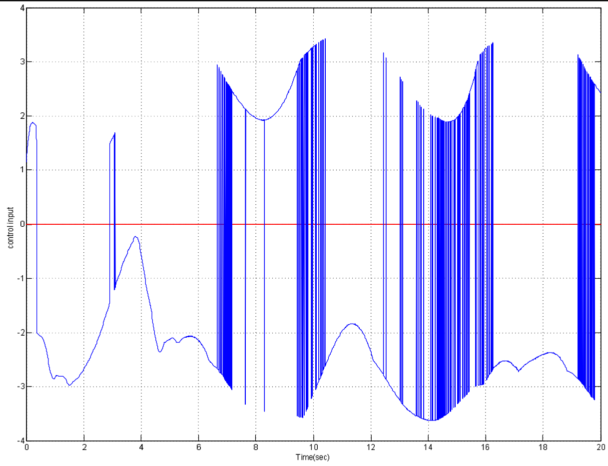

Fig. 3 Control input

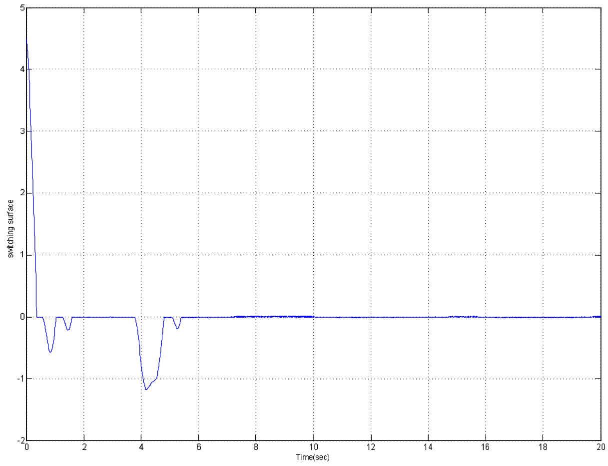

Fig. 4 Switching surface

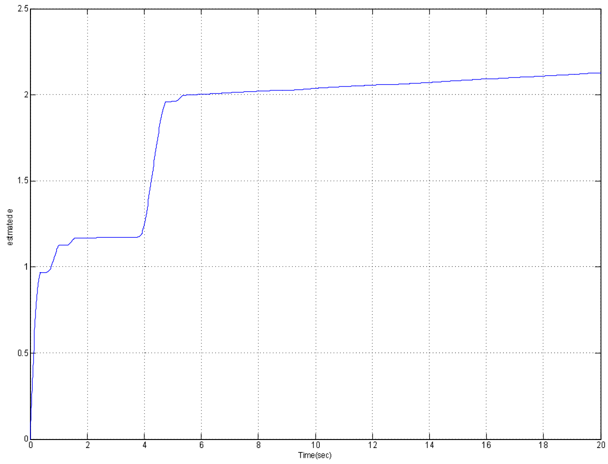

Fig. 5 Estimated approximation error

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