Periodic Sampled-Data Control for Fuzzy Systems: Intelligent Digital Redesign Approach

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Abstract: This paper presents a new linear-matrix-inequality-based intelligent digital redesign (LMI-based IDR) technique to match the states of the analog and the digital T-S fuzzy control systems at the intersampling instants as well as the sampling ones. The main features of the proposed technique are: 1) the affine control scheme is employed to increase the degree of freedom; 2) the fuzzy-model-based periodic control is employed, and the control input is changed n times during one sampling period; 3) The proposed IDR technique is based on the approximately discretized version of the T-S fuzzy system, but its discretization error vanishes as n approaches the infinity. 4) some sufficient conditions involved in the state matching and the stability of the closed-loop discrete-time system can be formulated in the LMIs format.

Keywords: Intelligent digital redesign (IDR), fuzzy-model-based control, digital control, fuzzy system.

1. Introduction

Intelligent Digital redesign (IDR) has gained tremendously increasing attention as yet another efficient design tool of sampled-data fuzzy control [1]-[6]. The IDR problem is the problem of designing a sampled-data state feedback controller such that the sampled-data closed-loop fuzzy system is equivalent to the continuous-time closed-loop fuzzy system in the sense of the state matching.

There have been fruitful researches in the digital control system focusing on IDR method. Historically, Joo et al. first attempted to develop some intelligent digital redesign methodology for complex nonlinear systems [1]. They synergistically merged both the Takagi–Sugeno (T–S) fuzzy-model-based control and the digital redesign technique for a class of nonlinear systems. Chang et al. extended the intelligent digital redesign to uncertain T–S fuzzy systems [2]. These approach [1], [2] to IDR are so called as local approach. The local approach can allows to match the states of the continuous-time and the sampled-data closed-loop fuzzy systems in the analytic way, but it may lead to undesirable and/or inaccurate results. The major reason is that the redesigned digital control gain matrices are obtained by considering only the local state-matching of each sub-closed-loop system [6]. To overcome this weakness, Lee et al. a global state-matching technique based on the convex optimization method, the linear matrix inequalities (LMIs) method, proposed in [6]. Specifically, their method is to globally match the states of the overall closed-loop T–S fuzzy system with the predesigned analog fuzzy-model-based controller and those with the digitally redesigned fuzzy-model-based controller, and further to examine the stabilizability by the redesigned controller in the sense of Lyapunov. However. the IDR problem becomes the overdamped problem according as transferring the local approach to the global one in IDR problem. It may lead to undesirable and/or inaccurate results.

An affine control scheme [19] can be an alternative because the affine control scheme leads to increasing the degree of freedom. At this point, we attempt to IDR for T–S fuzzy system based on an affine control scheme that has not yet been fully tackled under this framework. In addition, the multirate control scheme [13-18] is employed to obtain the some advantages, which allows to consider the intersampling points between sampling points and to decrease the discretization error.

Motivated by the above observations, we studies a periodic control for T-S fuzzy systems by using the LMI-based IDR method. The main features of the proposed method are as follows: First, the affine control scheme is employed to increase the degree of freedom. Second, the fuzzy-model-based periodic control is developed, and the control input is changed n times during one sampling period. Second, the proposed periodic control scheme can improve the state-matching performance in the long sampling limit. Finally, some sufficient conditions involved in the state matching and the stability of the closed-loop discrete-time system can be formulated in the LMIs format.

This paper is organized as follows: Section 2. contains the IDR problem statement of the continuous-time fuzzy system. Section 3. discusses the sampled-data control design for the continuous-time T-S fuzzy systems via the IDR method. This paper is concluded in Section 4.

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2. Problem Statement

Consider a nonlinear system described by

$$\dot{x_c}(t) = f(x_c(t), u_c(t)) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, and $u_c(t) \in \mathbb{R}^m$ is the continuous-time control input, and the subscript "c" means the continuous-time control.

To facilitate the control design, we will develop a simplified model, which can represent the local linear input–output relations of the nonlinear system. This type of models is referred as T–S fuzzy models. The fuzzy dynamical model corresponding to the nonlinear system (1) is described by the following IF–THEN rules [10], [11], [1], [2], [3], [6]:

$$\begin{aligned} R_k : &\text{ IF } z_1(t) \text{ is about } \Gamma_{k1} \text{ and } \cdots \text{ and } z_p(t) \text{ is about } \Gamma_{kp}, \\ &\text{THEN } \dot{x_c}(t) = A_k x_c(t) + B_k u_c(t) \end{aligned} \tag{2}$$

where $R_k, k \in \mathcal{I}_q = \{1, 2, \ldots, q\}$, is the kth fuzzy rule, $z_r(t), r \in \mathcal{I}_p = \{1, 2, \ldots, p\}$, is the rth premise variable, and $\Gamma_{kr}, (k, r) \in \mathcal{I}_q \times \mathcal{I}_p$, is the fuzzy set. Then, given a pair $(x_c(t), u_c(t))$, using the center-average defuzzification, product inference, and singleton fuzzifier, the overall dynamics of the IF-THEN rules (2) has the form

$$\dot{x_c}(t) = \sum_{k=1}^{q} \theta_k(z(t))(A_k x_c(t) + B_k u_c(t)) \tag{3}$$

where $\theta_k(z(t)) = \frac{w_k(z(t))}{\sum_{k=1}^{q} w_k(z(t))}$, $w_k(z(t)) = \prod_{r=1}^{p} \Gamma_{kr}(z_r(t))$, and $\Gamma_{kr}(z_r(t))$ is the grade of membership of $z_r(t)$ in Γ_{kr}. The possibly time-varying parameter vector $\theta \in \mathbb{R}^q$ belongs to a convex polytope Θ, where

$$\Theta := \left\{ \sum_{k=1}^{q} \theta_k = 1, \quad 0 \le \theta_k \le 1 \right\}$$

It is clear that as θ varies inside Θ, $\sum_{k=1}^{q} \theta_k(z(t))A_k$ and $\sum_{k=1}^{q} \theta_k(z(t))B_k$ range over a matrix polytope

$$\left[\sum_{k=1}^{q} \theta_k(z(t))A_k, \sum_{k=1}^{q} \theta_k(z(t))B_k \right] \in \mathbf{Co}\{(\mathbf{A_k}, \mathbf{B_k}), \mathbf{k} \in \mathcal{I}_\mathbf{q}\}$$

where **Co** denotes the convex hull. In this note, the stabilization of the polytopic model (3) is equivalent to the simultaneous stabilization of its vertices $(A_k, B_k), k \in \mathcal{I}_q$.

In this paper, a well-constructed continuous-time state feedback controller, which will be employed in redesigning the digital controller, is given. The controller is described by the following IF-THEN rules:

$$\begin{aligned} R_k : &\text{ IF } z_1(t) \text{ is about } \Gamma_{k1} \text{ and } \cdots \text{ and } z_p(t) \text{ is about } \Gamma_{kp}, \\ &\text{THEN } u_c(t) = \widehat{K}_k x_c(t), \end{aligned} \tag{4}$$

and its defuzzified output is

$$u_c(t) = \sum_{k=1}^{q} \theta_k(z(t)) \widehat{K}_k x_c(t) \tag{5}$$

Therefore, main purpose of this paper is to find the digital equivalent of the following continuous-time closed-loop system:

$$\dot{x}_c(t) = \sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(t))\theta_l(z(t))(A_k + B_k \widehat{K}_l) x_c(t) \tag{6}$$

3. Main Results

3.1. Discretization of fuzzy systems

In the following, let h_0 and h be the sampling time and the control update time, respectively. For convenience, we take $h = \frac{h_0}{N}$ for a positive integer N, where N is an input multiplicity. Then, $t = ih_0 + jh$ for $i \in \mathbb{Z}_{\geqslant 0}$ and $j \in \mathbb{Z}_{[0,N-1]}$, where the indexes i and j indicate sampling and control update instants, respectively.

By interfacing an ideal sampler and a zero-order holder between the plant and a controller, the digital fuzzy control system is represented by

$$\dot{x}_d(t) = \sum_{k=1}^{q} \theta_k(z(t))(A_k x_d(t) + B_k u_{dk}(t)). \tag{7}$$

where $u_d(t) = u_d(ih_0 + jh)$ for $t \in [ih_0 + jh, ih_0 + jh + h)$, $i \in \mathbb{Z}_{\geqslant 0}$, $j \in \mathbb{Z}_{[0,N-1]}$ is the periodic control input vector, and the control input is changed N times during one sampling time h_0, the subscript "d" means the sampled-data (digital) control.

Remark 1: This system can be viewed as the affine control system [19].

The periodic control input takes the following form:

$$u_{dk}(ih_0 + jh) = \sum_{l=1}^{q} \theta_l(z(ih_0 + jh)) K_{kl} x_d(ih_0 + jh) \tag{8}$$

where $x_d(ih_0 + jh)$ is not required to obtain $u_d(ih_0 + jh)$ because it will be predicted from $x_d(ih_0)$ after each control update.

To match the states of the continuous-time and the sampled-data closed-loop systems, we first have to know that the pointwise dynamical behavior, the discretized version of them at every sampling and control update instants. Because of the highly complex nonlinearities among the linear subsystems, it is typically impossible to obtain an exact discretized version of fuzzy system. So, the previous approach [6] is to approximate $\theta_k(z(t))$ as $\theta_k(z(ih_0 + jh))$ for $t \in [ih_0 + jh, ih_0 + jh + h)$ so that the nonlinear matrices $\sum_{k=1}^{q} \theta_k(z(t))A_k$ and $\sum_{k=1}^{q} \theta_k(z(t))B_k$ can be handled as the constant matrices $\sum_{k=1}^{q} \theta_k(z(ih_0 + jh))A_k$ and $\sum_{k=1}^{q} \theta_k(z(ih_0 + jh))B_k$.

Theorem 1 ([6]) If $\theta_k(z(t)) = \theta_k(z(ih_0 + jh))$ for $t \in [ih_0 + jh, ih_0 + jh + h)$, $i \in \mathbb{Z}_{\geqslant 0}$, $j \in \mathbb{Z}_{[0,N-1]}$, and $\mathrm{e}^{\sum_{k=1}^{q} \theta_k(z(ih_0+jh))A_k h} = \sum_{k=1}^{q} \theta_k(z(ih_0+jh))\, \mathrm{e}^{A_k h}$, then the discretized system of the sampled-data fuzzy control system (7) with sampling time h is as follows:

$$\begin{aligned} &x_d(ih_0 + jh + h) \\ &= \sum_{k=1}^{q} \theta_k(z(ih_0 + jh))(G_k x_d(ih_0 + jh) + H_k u_{dk}(ih_0 + jh)) \end{aligned} \tag{9}$$

where $G_k = \mathrm{e}^{A_k h}$ and $H_{kl} = (G_k - I)A_k^{-1}B_l$.

In order to predict $x_d(ih_0+jh)$ in (8), we will develop a general form of solutions to (9) controlled by (8) for $x_d(ih_0+jh)$ with the arbitrary initial state $x_d(ih_0)$.

Corollary 1: The solution to (9) closed by (8) for $x_d(ih_0 + jh)$ with the arbitrary initial state $x_d(ih_0)$ is given by

$$\begin{aligned} &x_d(ih_0 + jh) \\ &= \prod_{v=1}^{j} (\sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(ih_0 + jh - vh))\theta_l(z(ih_0 + jh - vh)) \\ &\quad \times (G_k + H_k K_{kl}))x_d(ih_0) \end{aligned} \tag{10}$$

for $i \in \mathbb{Z}_{\geqslant \nvdash}$ and $j \in \mathbb{Z}_{[\nVdash, \mathbb{N} - \nVdash]}$.

Proof: The closed-loop system (9) with (8) is described by

$$\begin{aligned} x_d(ih_0 + jh + h) = &\sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(ih_0 + jh))\theta_l(z(ih_0 + jh)) \\ &\times (G_k + H_k K_{kl})x_d(ih_0 + jh) \end{aligned} \tag{11}$$

Replacing j in (11) to $j - 1$ leads

$$\begin{aligned} x_d(ih_0 + jh) = &\sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(ih_0 + jh - h))\theta_l(z(ih_0 + jh - h)) \\ &\times (G_k + H_k K_{kl})x_d(ih_0 + jh - h) \end{aligned}$$

We compute

$$\begin{aligned} x_d(ih_0 + h) &= \sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(ih_0))\theta_l(z(ih_0))(G_k + H_k K_{kl})x_d(ih_0) \\ x_d(ih_0 + 2h) &= \sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(ih_0 + h))\theta_l(z(ih_0 + h)) \\ &\quad \times (G_k + H_k K_{kl})x_d(ih_0 + h) \\ &= \sum_{k_0=1}^{q} \sum_{l_0=1}^{q} \sum_{k_1=1}^{q} \sum_{l_1=1}^{q} \theta_{k_0}(z(ih_0 + h))\theta_{l_0}(z(ih_0 + h)) \\ &\quad \times \theta_{k_1}(z(ih_0))\theta_{l_1}(z(ih_0))(G_{k_0} + H_{k_0}K_{k_0 l_0}) \\ &\quad \times (G_{k_1} + H_{k_1}K_{k_1 l_1})x_d(ih_0) \end{aligned}$$

for $(k_0, j_0, k_1, j_1) \in \underbrace{\mathcal{I}_{\mathrm{II}} \times \cdots \times \mathcal{I}_{\mathrm{II}}}_{4}$. Proceeding forward, we can readily obtain (10) for $j > 0$. ■

Substituting (10) to $x_d(ih_0+jh)$ in (9) controlled by (8), we can obtain the following discretized version of the closed-loop digital fuzzy system with (7) and (8):

$$\begin{aligned} &x_d(ih_0 + jh + h) \\ &= \prod_{v=0}^{j} (\sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(ih_0 + jh - vh))\theta_l(z(ih_0 + jh - vh)) \\ &\quad \times (G_k + H_k K_{kl}))x_d(ih_0) \end{aligned} \tag{12}$$

for $i \in \mathbb{Z}_{\geqslant 0}$ and $j \in \mathbb{Z}_{[0,N-1]}$.

Corollary 2: In continuous-time closed-loop system(6),

- the approximate discrete-time model can be also obtained as

$$\begin{aligned} x_c(ih_0 + jh + h) = &\sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(ih_0 + jh))\theta_l(z(ih_0 + jh)) \\ &\times \Xi_{kl} x_c(ih_0 + jh) \end{aligned} \tag{13}$$

where $\Xi_{kl} = \mathrm{e}^{(A_k + B_k \widehat{K}_l)h}$.

- the solution to (13) for $x_c(ih_0 + jh)$ with the arbitrary initial state $x_c(ih_0)$ is given by

$$\begin{aligned} &x_c(ih_0 + jh) \\ &= \prod_{v=1}^{j} (\sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(ih_0 + jh - vh))\theta_l(z(ih_0 + jh - vh))\Xi_{kl}) \\ &\quad \times x_c(ih_0) \end{aligned} \tag{14}$$

for $i \in \mathbb{Z}_{\geqslant 0}$ and $j \in \mathbb{Z}_{[1,N-1]}$.

Therefore, from (13) and (14), we directly obtain the following discrete-time representation of (6):

$$\begin{aligned} &x_c(ih_0 + jh + h) \\ &= \prod_{v=0}^{j} (\sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(ih_0 + jh - vh))\theta_l(z(ih_0 + jh - vh))\Xi_{kl}) \\ &\quad \times x_c(ih_0) \end{aligned} \tag{15}$$

for $i \in \mathbb{Z}_{\geqslant 0}$ and $j \in \mathbb{Z}_{[0,N-1]}$.

Proof: It can be straightforwardly proven by Lemma 1 and Corollary 1. ■

3.2. Design of the Periodic Control using IDR method

The IDR problem for the system (7) is the problem to design a periodic control law (8) such that i) the origin $x = 0$ is a globally asymptotically stable equilibrium point of the closed-loop system

$$\begin{aligned} \dot{x_d}(t) = &\sum_{k=1}^{q} \sum_{l=1}^{q} \theta_k(z(t))\theta_l(z(ih_0 + jh)) \\ \times &\quad (A_k x_d(t) + B_k K_{kl} x_d(ih_0 + jh)), \end{aligned} \tag{16}$$

and ii) by comparing (12) and (15), to realize $x_c(ih_0+jh) = x_d(ih_0 + jh)$ under the assumption that $x_c(ih_0) = x_d(ih_0)$, K_{kl} was numerically synthesized for to be a minimizer in the induced 2-norm sense.

Theorem 2: If there exist $Q = Q^T \succ 0$ and constant matrices F_{kl} such that the following generalized eigenvalue problem (GEVP) has solutions:

$$\underset{Q,F_{kl}}{Minimize} \quad \gamma \quad \text{subjectto}$$

$$\begin{bmatrix} -\gamma Q & (\bullet)^T \\ \Xi_{kl}Q - G_k Q - H_k F_{kl} & -\gamma I \end{bmatrix} \prec 0, \quad k,l \in \mathcal{I}_q \tag{17}$$

$$\begin{bmatrix} -Q & (\bullet)^T \\ G_k Q + H_k F_{kk} & -Q \end{bmatrix} \prec 0, \quad k \in \mathcal{I}_q \tag{18}$$

$$\begin{bmatrix} -Q & (\bullet)^T \\ \frac{G_k Q + H_k F_{kl} + G_l Q + H_l F_{lk}}{2} & -Q \end{bmatrix} \prec 0, \quad k,l \in \mathcal{I}_q \tag{19}$$

then the state $x_d(ih_0+jh)$ of the discrete-time representation (12) closely matches the discrete-time representation (15), and (12) is globally asymptotically stable in the sense of Lyapunov, where $(\bullet)^T$ denotes the transposed element in symmetric positions.

Proof: It can be straightforwardly proven by Theorem 2 in [6] ■

4. Conclusions

This paper proposed the periodic control design using the LMI approach for the fuzzy system. Some sufficient conditions were derived for stabilization and state matching of the discretized model by the fast discretization. The proposed periodic control scheme can improve the state-matching performance in the long sampling limit.

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