Robust ∞ Fu y Control of Nonlinear Systems with Time-Varying Delay via Static Output Feedback

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Abstract: In this paper, a robust H∞ stabilization problem to a uncertain fuzzy systems with time-varying delay via static output feedback is investigated. The Takagi-Sugeno (T-S) fuzzy model is employed to represent uncertain nonlinear systems with time-varying delayed state, which is a continuous-time or discrete-time system. Using a single Lyapunov function, the globally asymptotic stability and disturbance attenuation of the closed-loop fuzzy control system are discussed. Sufficient conditions for the existence of robust H∞ controllers are given in terms of linear matrix inequalities.

Keywords: Static Output Feedback, Fu y, Time-Varying Delay, Robust Control

1. INTRODUCTION

Most plants used in the real world have a strong nonlinearity and uncertainty. Moreover, when this system is controlled, time-varying delay is generally occurred and disturbance interrupts. Therefore to solve this problem, many efforts have done.

There are many papers that propose the control methodology of the linear system with time-delay. But for the nonlinear system with time-delay, only few papers exist. This arises from the complexity of the nonlinear system. To overcome this difficulty, various schemes have been developed in the last two decades, among which a successful approach is fuzzy control.

Cao et al. first proposed the Takagi-Sugeno (T-S) fuzzy model with time-delay that represents the nonlinear system with time-delay and analyzed the stability of that in [2]. Based on this, Lee et al. [3] proposed a dynamic output feedback robust H∞ control method for a class of uncertain fuzzy systems with time-varying delay. But in this method, to design the robust controller, bilinear matrix inequality (BMI) must be solved. Therefore the design of the controller is very difficult.

Lo et al. proposed the robust static output feedback control method of the nonlinear system without time-delay via fuzzy control approach in [1]. In this method, controller can be easily designed by solving several linear matrix inequalities (LMI).

In this paper, we extend the method that proposed in [1] to the nonlinear system with time-varying delay. We design the H∞ fuzzy controller that robustly control the nonlinear system with time-varying delay subject to external disturbances. To this end, we first represent the nonlinear system with time-delay to fuzzy model with time-delay as did in [2]. Then parallel distributed compensation technique is applied for the design of the static output feedback fuzzy controller. After selecting one Lyapunov function, we derive the sufficient condition for stability of the fuzzy system. But this condition is composed of BMIs. Therefore we convert it LMI by using similarity transform and congruence transform technique. From this, the H∞ fuzzy controller can be easily designed by many current convex optimization algorithm tools.

The remainder of the paper is organized as follows: following the introduction, problem formulation is done in Section 2. In Section 3, the sufficient condition for making the continuous-time T-S fuzzy model with time-varying delay asymptotically stable is derived. In Section 4, the same procedure is done for discrete-time T-S fuzzy system with time-varying delay. Finally, some conclusions are drawn in Section 5.

2. PRO LEM FORMULATION

The T-S fuzzy model is generally known as the universal approximator of nonlinear systems. We consider nonlinear systems represented by the following T-S fuzzy model with time-delay.

Plant Rule i

IF $\theta_1(t)$ is M_{i1} and $\cdots$ and $\theta_n(t)$ is M_{in}

$$\begin{aligned}
\text{THEN } \sigma x(t) &= (A_i + \Delta A(t))x(t) + (A_{di} + \Delta A_d(t))x(t-d(t)) \\
&\quad + (B_{1i} + \Delta \mathrm{B}_1(t))\omega(t) + (B_{2i} + \Delta B_2(t))u(t) \\
\mathrm{z}(t) &= (C_i + \Delta C(t))x(t) + (C_{di} + \Delta C_d(t))x(t-d(t)) \\
&\quad + (D_{1i} + \Delta D_1(t))\omega(t) + (D_{2i} + \Delta D_2(t))u(t) \\
y(t) &= Ex(t), \qquad i = 1,\cdots,r \\
x(t) &= 0, \qquad t \le 0. \qquad (1)
\end{aligned}$$

Where M_{ij} is the fuzzy set, $x(t) \in \Re^n$ is the state vector, $\omega(t) \in \Re^q$ is unknown but the energy-bounded disturbance input, $u(t) \in \Re^q$ is the controlled input, $z(t) \in \Re^s$ is the controlled output, $(A_i, A_{di}, B_{1i}, B_{2i}, C_i, C_{di}, D_{1i}, D_{2i})$ are some constant matrices of compatible dimensions, r is the number of IF-THEN rules, and $\theta(t) = \begin{bmatrix}\theta_1(t) & \theta_2(t) & \cdots & \theta_p(t)\end{bmatrix}$ are the premise variables. It is assumed that the premise variables do not depend on the input variables $u(t)$ explicitly. The time-varying delay, $d(t)$ is assumed that

$$0 \le d(t) \le \infty, \quad \dot{d}(t) \le \beta < 1. \qquad (2)$$

The time-varying matrices, $(\Delta \mathrm{A}(t), \Delta A_d(t), \Delta B_1(t), \Delta B_2(t), \Delta C(t), \Delta C_d(t), \Delta D_1(t), \Delta D_2(t))$, is defined as follows:

$$\begin{pmatrix} \Delta \mathrm{A}(t) & \Delta A_d(t) & \Delta B_1(t) & \Delta B_2(t) \\ \Delta C(t) & \Delta C_d(t) & \Delta D_1(t) & \Delta D_2(t) \end{pmatrix} = \begin{pmatrix} M\Delta(t) & (N_1 \;\; N_2 \;\; N_3 \;\; N_4) \\ M_z\Delta_z(t) & (N_{z1} \;\; N_{z2} \;\; N_{z3} \;\; N_{z4}) \end{pmatrix},$$

where $(M, M_z, N_1, N_2, N_3, N_4, N_{z1}, N_{z2}, N_{z3}, N_{z4})$ is known real constant matrices, and (Δ, Δ_z) are unknown matrix functions with Lebesgue-measurable elements and satisfies

$\Delta\ (t)\Delta(t)\le I,\ \Delta_z\ (t)\Delta_z(t)\le I,$ in which I is the identity matrix of appropriate dimension.

Remark 1: The uncertain fuzzy system (1) encompasses the nonlinear system, which it represents.

The defuzzified output of (1) is represented as follows:

$$\begin{aligned}\sigma x(t) &= \sum_{i=1}^{r}\mu_i(\theta(t))[(A_i+\Delta A(t))x(t)+(A_{di}+\Delta A_d(t))x(t-d(t))\\ &\quad +(B_{1i}+\Delta B_1(t))\omega(t)+(B_{2i}+\Delta B_2(t))u(t)]\\ z(t) &= \sum_{i=1}^{r}\mu_i(\theta(t))[(C_i+\Delta C(t))x(t)+(C_{di}+\Delta C_d(t))x(t-d(t))\\ &\quad +(D_{1i}+\Delta D_1(t))\omega(t)+(D_{2i}+\Delta D_2(t))u(t)]\\ y(t) &= Ex(t).\end{aligned}\tag{3}$$

where

$$\mu_i(z(t))=\frac{\omega_i(z(t))}{\sum_{i=1}^{r}\omega_i(z(t))},\quad \omega_i(z(t))=\prod_{k=1}^{n}M_{ik}(z(t)).$$

The controller is a static output feedback fuzzy controller of the following defuzzifed form:

$$u(t)=\sum_{i=1}^{r}\mu_i(x(t))K_i y(t),\tag{4}$$

where K_i are constant control gains to be determined. For simplicity, we represent $\mu_i(\theta(t))$ as μ_i and abbreviate the time index, t, in time-varying matrices.

Substituting Eq. (4) into Eq. (3), the closed-loop system is obtained as follows:

$$\begin{aligned}\sigma x(t) &= \left[A_\mu + B_{2\mu}K_\mu E + M\Delta\left(N_1+N_4K_\mu E\right)\right]x(t)\\ &\quad +\left(A_{d\mu}+M\Delta N_2\right)x\left(t-d(t)\right)+\left(B_{1\mu}+M\Delta N_3\right)\omega(t)\\ z(t) &= \left[C_\mu + D_{2\mu}K_\mu E + M_z\Delta_z\left(N_{z1}+N_{z4}K_\mu E\right)\right]x(t)\\ &\quad +\left(C_{d\mu}+M_z\Delta_z N_{z2}\right)x\left(t-d(t)\right)+\left(D_{1\mu}+M_z\Delta_z N_{z3}\right)\omega(t)\\ y(t) &= Ex(t).\end{aligned}\tag{5}$$

where

$$_\mu=\sum_{i=1}^{r}\mu_i\ _i,\quad _i\in\left\{A_i,A_{di},B_{1i},B_{2i},C_i,C_{di},D_{1i},D_{2i}\right\}.$$

The performance considered here is an H∞ criterion such that the following is satisfied:

In the continuous-time case

$$\int_0^\infty z\ (\tau)z(\tau)d\tau<\gamma^2\int_0^\infty\omega\ (\tau)\omega(\tau)d\tau.\tag{6}$$

In the discrete-time case

$$\sum_{\tau=0}^{\infty}z\ (\tau)z(\tau)<\gamma^2\sum_{\tau=0}^{\infty}\omega\ (\tau)\omega(\tau).\tag{7}$$

Definition 1: H∞ fuzzy controller

1) The controller makes the system (1), (3) robustly stable in the presence of $\omega(t)$.

2) Given γ, the closed-loop system (5) must satisfy the criterion (6) in continuous-time case or (7) in discrete-time case, in which the initial condition is zero.

3. CONTINUOUS-TIME S STEMS

In this section, a continuous-time static output feedback fuzzy controller satisfying Definition 1 will be addressed. It should be noted that the result presented in Theorem 1, arising from static output feedback stabilization problems, is only existential and can not be solved by present convex algorithm. Therefore further work is required for easy application.

Given a constant $\gamma>0$, the system (1) is robustly stabilizable by the controller (4) if there exists the positive symmetric matrices P, S and control gains, K_j satisfied the following matrix inequalities. In other words, (4) is the H∞ fuzzy controller.

$$\begin{cases} M_{ii}<0, & i=1,\cdots,r\\ \dfrac{1}{r-1}M_{ii}+\dfrac{1}{2}\left(M_{ij}+M_{ji}\right)<0, & 1\le i\ne j\le r\end{cases}\tag{8}$$

where the second equation at the bottom of the next page holds, and

$$\Phi_{ij}=A_jP+E\ K_jB_{2i}P+PA_i+PB_{2i}K_jE+\frac{1}{1-\beta}S,$$

$$\Gamma_{ij}=C_i+D_{2i}K_jE,\qquad \Psi_j=N_1+N_4K_jE,$$

$$\Psi_{zj}=N_{z1}+N_{z4}K_jE.$$

Proof:

Consider the following Lyapunov function:

$$V\left(x(t)\right)=x\ (t)Px+\frac{1}{1-\beta}\int_{t-d(t)}^{t}x\ (\tau)Sx(\tau)d\tau$$

where P and S is positive symmetric matrices, and β is the constant defined by (2). Clearly, $V\left(x(t)\right)$ is positive definite and radially unbounded. The time derivative of $V\left(x(t)\right)$ is

$$\begin{aligned}\dot V &= \dot x\ (t)Px(t)+x\ (t)P\dot x(t)+\frac{1}{1-\beta}x\ (t)Sx(t)\\ &\quad -\frac{1-\dot d(t)}{1-\beta}x\ \left(t-d(t)\right)Sx\left(t-d(t)\right).\end{aligned}$$

Then, we show (8) is the sufficient condition to satisfy the following inequality:

$$\dot V(t)+z\ (t)z(t)-\gamma^2\omega\ (t)\omega(t)<0\tag{9}$$

From (5), we have

$$\dot V(t)+z\ (t)z(t)-\gamma^2\omega\ (t)\omega(t)\le$$

$$\begin{aligned}&\begin{bmatrix}x(t)\\ x\left(t-d(t)\right)\\ w(t)\end{bmatrix}\left(\begin{bmatrix}A_\mu P+E\ K_\mu B_{2\mu}P+*+\dfrac{1}{1-\beta}S & * & *\\ A_{d\mu}P & -S & *\\ B_{1\mu}P & 0 & -\gamma^2 I\end{bmatrix}\right.\\ &+\begin{bmatrix}C_\mu+E\ K_\mu D_{2\mu}+\left(N_{z1}+N_{z4}K_\mu E\right)\ \Delta_z M_z\\ C_{d\mu}+N_{z2}\Delta_z M_z\\ D_{1\mu}+N_{z3}\Delta_z M_z\end{bmatrix}*\\ &\left.+\begin{bmatrix}PM\\ 0\\ 0\end{bmatrix}\Delta\left[N_1+N_4K_\mu E\quad N_2\quad N_3\right]+*\right)\begin{bmatrix}x(t)\\ x\left(t-d(t)\right)\\ w(t)\end{bmatrix}\end{aligned}$$

After applying a matrix fact

$$M\Delta N + N\Delta M \le \varepsilon MM + \varepsilon^{-1} N N$$

on the third and the fourth terms within ( ), a direct Schur complement to the resulting upper bound 0 yields

$$\begin{bmatrix} A_\mu P + E K_\mu B_{2\mu} P + * + \frac{1}{1-\beta} S & * & * & * & * & * \\ A_{d\mu} P & -S & * & * & * & * \\ B_{1\mu} P & 0 & -\gamma^2 I & * & * & * \\ C_\mu + D_{2\mu} K_\mu E & C_{d\mu} & D_{1\mu} & -I & * & * \\ \varepsilon_1 M P & 0 & 0 & 0 & -\varepsilon_1 I & * \\ N_1 + N_4 K_\mu E & N_2 & N_3 & 0 & 0 & -\varepsilon_1 I \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ 0 \\ 0 \\ M_z \\ 0 \\ 0 \end{bmatrix} \Delta_z \begin{bmatrix} N_{z1} + N_{z4} K_\mu E & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + *$$

$$+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ M_z \\ 0 \end{bmatrix} \Delta_z \begin{bmatrix} N_{z2} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + *$$

$$+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ M_z \end{bmatrix} \Delta_z \begin{bmatrix} N_{z3} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + * < 0$$

The matrix fact and a second Schur complement again, the inequality is converted to (10) where

$$\Phi_\mu = A_\mu P + E K_\mu B_{2\mu} P + P A_\mu + P B_{2\mu} K_\mu E + \frac{1}{1-\beta} S,$$

$$\Gamma_\mu = C_\mu + D_{2\mu} K_\mu E, \qquad \Psi_\mu = N_1 + N_4 K_\mu E,$$

$$\Psi_{z\mu} = N_{z1} + N_{z4} K_\mu E.$$

Inequality (10) can be expressed as follows:

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j M_{ij} < 0$$

Based on parameterized linear matrix inequality (PLMI) technique [5], (10) is converted to less conservative and computationally efficient condition (8).

In the zero initial condition, integrating (9) from $t = 0$ to ∞ yields

$$J_\infty = \int_0^\infty \left[z(\tau) z(\tau) - \gamma^2 \omega(\tau) \omega(\tau) \right] d\tau$$
$$< V(0) - V(\infty) \le x(0) P x(0) = 0.$$

$$\begin{bmatrix} \Phi_\mu & * & * & * & * & * & * & * & * & * & * & * \\ A_{d\mu} P & -S & * & * & * & * & * & * & * & * & * & * \\ B_{1\mu} P & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * & * \\ \Gamma_\mu & C_{d\mu} & D_{1\mu} & -I & * & * & * & * & * & * & * & * \\ \varepsilon_1 M P & 0 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * & * \\ \Psi_\mu & N_2 & N_3 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * \\ 0 & 0 & 0 & \varepsilon_2 M_z & 0 & 0 & -\varepsilon_2 I & * & * & * & * & * \\ \Psi_{z\mu} & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * & * & * & * \\ 0 & 0 & 0 & 0 & \varepsilon_3 M_z & 0 & 0 & 0 & -\varepsilon_3 I & * & * & * \\ N_{z2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_4 M_z & 0 & 0 & 0 & -\varepsilon_4 I & * \\ N_{z3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix} < 0 \qquad (10)$$

$$M_{ij} = \begin{bmatrix} \Phi_{ij} & * & * & * & * & * & * & * & * & * & * & * \\ A_{di} P & -S & * & * & * & * & * & * & * & * & * & * \\ B_{1i} P & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * & * \\ \Gamma_{ij} & C_{di} & D_{1i} & -I & * & * & * & * & * & * & * & * \\ \varepsilon_1 M P & 0 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * & * \\ \Psi_j & N_2 & N_3 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * \\ 0 & 0 & 0 & \varepsilon_2 M_z & 0 & 0 & -\varepsilon_2 I & * & * & * & * & * \\ \Psi_{zj} & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * & * & * & * \\ 0 & 0 & 0 & 0 & \varepsilon_3 M_z & 0 & 0 & 0 & -\varepsilon_3 I & * & * & * \\ N_{z2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_4 M_z & 0 & 0 & 0 & -\varepsilon_4 I & * \\ N_{z3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix}$$

Therefore, K_j satisfying (8) satisfies the Definition 1.

.E.D.

The inequality (8) is BMIs which is not solvable by the convex programming technique. Therefore further manipulations are required. Here we use the method that proposed in [1]. First we define new state variables

$$x = T\tilde{x}.$$

Then (5) is converted followings:

$$\begin{aligned}\sigma\tilde{x}(t) &= \left[\tilde{A}_\mu + \tilde{B}_{2\mu}K_\mu\tilde{E} + \tilde{M}\Delta\left(\tilde{N}_1 + N_4K_\mu\tilde{E}\right)\right]\tilde{x}(t) \\ &\quad + \left(\tilde{A}_{d\mu} + \tilde{M}\Delta\tilde{N}_2\right)\tilde{x}\left(t-d(t)\right) + \left(\tilde{B}_{1\mu} + \tilde{M}\Delta N_3\right)\omega(t) \\ z(t) &= \left[\tilde{C}_\mu + D_{2\mu}K_\mu\tilde{E} + M_z\Delta_z\left(\tilde{N}_{z1} + N_{z4}K_\mu\tilde{E}\right)\right]\tilde{x}(t) \\ &\quad + \left(\tilde{C}_{d\mu} + M_z\Delta_z\tilde{N}_{z2}\right)\tilde{x}\left(t-d(t)\right) + \left(D_{1\mu} + M_z\Delta_z N_{z3}\right)\omega(t). \quad (11)\end{aligned}$$

where

$$\tilde{A}_\mu = T^{-1}A_\mu T,\ \tilde{B}_{2\mu} = T^{-1}B_{2\mu},\ \tilde{A}_{d\mu} = T^{-1}A_{d\mu}T,$$
$$\tilde{B}_{1\mu} = T^{-1}B_{1\mu},\ \tilde{C}_\mu = C_\mu T,\ \tilde{C}_{d\mu} = C_{d\mu}T,\ \tilde{E} = ET,$$
$$\tilde{M} = T^{-1}M,\ \tilde{N}_1 = N_1T,\ \tilde{N}_2 = N_2T,\ \tilde{N}_{z1} = N_{z1}T,\ \tilde{N}_{z2} = N_{z2}T,$$

Let $Q = P^{-1}$.

$$Q_{n\times n} = \begin{bmatrix} Q_{1p\times p} & 0 \\ 0 & Q_2 \end{bmatrix}$$

The transformation matrix, T, is selected in order to satisfy the following condition:

$$\tilde{E} = ET = \begin{bmatrix} I_p & 0 \end{bmatrix}.$$

That is

$$T = \left[E\left(EE\right)^{-1}\Big|_{n-p} \quad \mathrm{ortc}\left(E\right)\right].$$

where $ortc\left(E\right)$ denotes orthogonal complement of E. Applying Theorem 1 to (11), the sufficient condition to stabilize (11) is the following:

$$\begin{cases} \tilde{M}_{ii} < 0, & i = 1,\cdots,r \\ \dfrac{1}{r-1}\tilde{M}_{ii} + \dfrac{1}{2}\left(\tilde{M}_{ij} + \tilde{M}_{ji}\right) < 0, & 1 \le i \ne j \le r \end{cases} \quad (12)$$

Where the equation at the bottom of the page holds, and

$$\tilde{\Phi}_{ij} = \tilde{A}_jP + \tilde{E}\,K_j\tilde{B}_{2i}P + P\tilde{A}_i + P\tilde{B}_{2i}K_j\tilde{E} + \frac{1}{1-\beta}S,$$
$$\tilde{\Gamma}_{ij} = \tilde{C}_i + D_{2i}K_j\tilde{E}, \qquad \tilde{\Psi}_j = \tilde{N}_1 + N_4K_j\tilde{E},$$
$$\tilde{\Psi}_{zj} = \tilde{N}_{z1} + N_{z4}K_j\tilde{E}.$$

Let

$$\Theta = diag\begin{bmatrix} Q & Q & I & I & I & I & I & I & I & I & I & I \end{bmatrix}.$$

Pre- and post-multiplying (12) by Θ, the inequality expounded is displayed as

$$\begin{cases} \bar{M}_{ii} < 0, & i = 1,\cdots,r \\ \dfrac{1}{r-1}\bar{M}_{ii} + \dfrac{1}{2}\left(\bar{M}_{ij} + \bar{M}_{ji}\right) < 0, & 1 \le i \ne j \le r \end{cases} \quad (13)$$

where the first equation at the bottom of the next page holds, and

$$\bar{\Phi}_{ij} = Q\tilde{A}_j + \begin{bmatrix} {}_j \\ 0 \end{bmatrix}\tilde{B}_{2i} + \tilde{A}_iQ + P\tilde{B}_{2i}\begin{bmatrix} {}_j & 0 \end{bmatrix} + \frac{1}{1-\beta} \quad ,$$
$$\bar{\Gamma}_{ij} = \tilde{C}_iQ + D_{2i}\begin{bmatrix} {}_j & 0 \end{bmatrix}, \qquad \bar{\Psi}_j = \tilde{N}_1Q + N_4\begin{bmatrix} {}_j & 0 \end{bmatrix},$$
$$\tilde{\Psi}_{zj} = \tilde{N}_{z1}Q + N_{z4}\begin{bmatrix} {}_j & 0 \end{bmatrix}, \qquad = QSQ > 0, \qquad {}_j = K_jQ_1.$$

Remark 2: The matrices, and Q, are independent.

Remark 3: By defining the new variable, ${}_j = K_jQ_1$, the inequality (13) is linear matrix inequality that has following 7 variables: $\left(Q, \quad , \ {}_j, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\right)$.

4. DISCRETE-TIME S STEMS

As we did for the continuous-time case, the same procedure is applicable to the discrete-time systems. The result is stated below.

Given a constant $\gamma > 0$, the system (1) is robustly stabilizable by the controller (4) if there exists the positive symmetric matrices P, S and control gains, K_j satisfied the following matrix inequalities. In other words, (4) is the H∞ fuzzy controller.

$$\begin{cases} M_{ii} < 0, & i = 1,\cdots,r \\ \dfrac{1}{r-1}M_{ii} + \dfrac{1}{2}\left(M_{ij} + M_{ji}\right) < 0, & 1 \le i \ne j \le r \end{cases} \quad (14)$$

where the second equation at the bottom of the next page holds

$$\Gamma_{ij} = A_i + B_{2i}K_jE, \quad \Omega_j = N_1 + N_4K_jE, \quad \Psi_{ij} = C_i + D_{2i}K_jE,$$

$$\tilde{M}_{ij} = \begin{bmatrix}
\tilde{\Phi}_{ij} & * & * & * & * & * & * & * & * & * & * & * \\
\tilde{A}_{di}P & -S & * & * & * & * & * & * & * & * & * & * \\
\tilde{B}_{1i}P & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * & * \\
\tilde{\Gamma}_{ij} & \tilde{C}_{di} & D_{1i} & -I & * & * & * & * & * & * & * & * \\
\varepsilon_1\tilde{M}\ P & 0 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * & * \\
\tilde{\Psi}_j & \tilde{N}_2 & N_3 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * \\
0 & 0 & 0 & \varepsilon_2 M_z & 0 & 0 & -\varepsilon_2 I & * & * & * & * & * \\
\tilde{\Psi}_{zj} & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * & * & * & * \\
0 & 0 & 0 & 0 & \varepsilon_3 M_z & 0 & 0 & 0 & -\varepsilon_3 I & * & * & * \\
\tilde{N}_{z2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_4 M_z & 0 & 0 & 0 & -\varepsilon_4 I & * \\
N_{z3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I
\end{bmatrix}$$

$\Lambda_j = N_{z1} + N_{z4} K_j E.$

Proof:
Considet the following Lyapunov function:

$$V\left(x(k)\right) = x(k)^T Px(k) + \sum_{\sigma=k-d(k)}^{k-1} x^T(\sigma) Sx(\sigma).$$

The procedure of the proof is almost identical to that of the continuous-time case.

.E.D.

Let $x = T\tilde{x}$. Then (5) is converted (11). Applying Theorem 2 to (11), the sufficient condition to stabilize (11) is the following:

$$\begin{cases} \tilde{M}_{ii} < 0, & i = 1, \cdots, r \\ \dfrac{1}{r-1}\tilde{M}_{ii} + \dfrac{1}{2}\left(\tilde{M}_{ij} + \tilde{M}_{ji}\right) < 0, & 1 \le i \ne j \le r \end{cases} \tag{15}$$

where the first equation at the bottom of the next page holds, and

$\tilde{\Gamma}_{ij} = \tilde{A}_i + \tilde{B}_{2i} K_j \tilde{E}$, $\tilde{\Omega}_j = \tilde{N}_1 + N_4 K_j \tilde{E}$, $\tilde{\Psi}_{ij} = \tilde{C}_i + D_{2i} K_j \tilde{E}$
$\tilde{\Lambda}_j = \tilde{N}_{z1} + N_{z4} K_j \tilde{E}$ ·

Let $Q = P^{-1}$ and

$$\Theta = diag\left[Q \quad Q \quad I \quad I \quad I \quad I \quad I \quad I \quad I \quad I \quad I\right].$$

Pre- and post-multiplying (15) by Θ , the inequality expounded is displayed as

$$\begin{cases} \bar{M}_{ii} < 0, & i = 1, \cdots, r \\ \dfrac{1}{r-1}\bar{M}_{ii} + \dfrac{1}{2}\left(\bar{M}_{ij} + \bar{M}_{ji}\right) < 0, & 1 \le i \ne j \le r \end{cases} \tag{16}$$

where the second equation at the bottom of the next page holds, and

$$\bar{\Gamma}_{ij} = \tilde{A}_i Q + \tilde{B}_{2i}\left[\ _j \quad 0\right], \qquad \bar{\Omega}_j = \tilde{N}_1 Q + N_4\left[\ _j \quad 0\right],$$
$$\bar{\Psi}_{ij} = \tilde{C}_i Q + D_{2i}\left[\ _j \quad 0\right], \qquad \bar{\Lambda}_j = \tilde{N}_{z1} Q + N_{z4}\left[\ _j \quad 0\right],$$
$$= QSQ > 0, \qquad _j = K_j Q_1.$$

5. CONCLUSION

In this paper, the fuzz control approach is proposed to robustly control the nonlinear system with time-varying delay. Using Lyapunov theory, the sufficient condition is derived. Through the manipulation, bilinear matrix inequality is converted to linear matrix inequality. Therefore, we can easily design the controller via current convex algorithm.

$$\bar{M}_{ij} = \begin{bmatrix} \bar{\Phi}_{ij} & * & * & * & * & * & * & * & * & * & * & * \\ Q\tilde{A}_{di} & - & * & * & * & * & * & * & * & * & * & * \\ \tilde{B}_{1i} & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * & * \\ \bar{\Gamma}_{ij} & \tilde{C}_{di}Q & D_{1i} & -I & * & * & * & * & * & * & * & * \\ \varepsilon_1 \tilde{M} & 0 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * & * \\ \bar{\Psi}_j & \tilde{N}_2 Q & N_3 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * \\ 0 & 0 & 0 & \varepsilon_2 M_z & 0 & 0 & -\varepsilon_2 I & * & * & * & * & * \\ \bar{\Psi}_{zj} & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * & * & * & * \\ 0 & 0 & 0 & 0 & \varepsilon_3 M_z & 0 & 0 & 0 & -\varepsilon_3 I & * & * & * \\ \tilde{N}_{z2} Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_4 M_z & 0 & 0 & 0 & -\varepsilon_4 I & * \\ N_{z3} Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix}$$

$$M_{ij} = \begin{bmatrix} S-P & * & * & * & * & * & * & * & * & * & * & * & * & * \\ 0 & -S & * & * & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * & * & * & * \\ \Gamma_{ij} & A_{di} & B_{1i} & -P^{-1} & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -P^{-1} & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & \varepsilon_1 M & -\varepsilon_1 I & * & * & * & * & * & * & * & * \\ \Omega_j & N_2 & N_3 & 0 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * & * \\ \Psi_{ij} & C_{di} & D_{1i} & 0 & 0 & 0 & 0 & -I & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_2 M_z & -\varepsilon_2 I & * & * & * & * & * \\ \Lambda_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_3 M_z & 0 & 0 & -\varepsilon_3 I & * & * & * \\ 0 & N_{z2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_4 M_z & 0 & 0 & 0 & 0 & -\varepsilon_4 I & * \\ 0 & 0 & N_{z3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix}$$

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$$\tilde{M}_{ij}=\begin{bmatrix}
S-P & * & * & * & * & * & * & * & * & * & * & * & * & * \\
0 & -S & * & * & * & * & * & * & * & * & * & * & * & * \\
0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * & * & * & * \\
\tilde{\Gamma}_{ij} & \tilde{A}_{di} & \tilde{B}_{1i} & -P^{-1} & * & * & * & * & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & -P^{-1} & * & * & * & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & \varepsilon_1 M & -\varepsilon_1 I & * & * & * & * & * & * & * & * \\
\tilde{\Omega}_j & \tilde{N}_2 & N_3 & 0 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * & * \\
\tilde{\Psi}_{ij} & \tilde{C}_{di} & D_{1i} & 0 & 0 & 0 & 0 & -I & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_2 M_z & -\varepsilon_2 I & * & * & * & * & * \\
\tilde{\Lambda}_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_3 M_z & 0 & 0 & -\varepsilon_3 I & * & * & * \\
0 & \tilde{N}_{z2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_4 M_z & 0 & 0 & 0 & 0 & -\varepsilon_4 I & * \\
0 & 0 & N_{z3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I
\end{bmatrix}$$

$$\bar{M}_{ij}=\begin{bmatrix}
-Q & * & * & * & * & * & * & * & * & * & * & * & * & * \\
0 & - & * & * & * & * & * & * & * & * & * & * & * & * \\
0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * & * & * & * \\
\bar{\Gamma}_{ij} & \tilde{A}_{di}Q & \tilde{B}_{1i} & -Q & * & * & * & * & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & -Q & * & * & * & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & \varepsilon_1 M & -\varepsilon_1 I & * & * & * & * & * & * & * & * \\
\bar{\Omega}_j & \tilde{N}_2 Q & N_3 & 0 & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * & * & * \\
\bar{\Psi}_{ij} & \tilde{C}_{di} Q & D_{1i} & 0 & 0 & 0 & 0 & -I & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_2 M_z & -\varepsilon_2 I & * & * & * & * & * \\
\bar{\Lambda}_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_3 M_z & 0 & 0 & -\varepsilon_3 I & * & * & * \\
0 & \tilde{N}_{z2} Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_4 M_z & 0 & 0 & 0 & 0 & -\varepsilon_4 I & * \\
0 & 0 & N_{z3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I
\end{bmatrix}$$