

Performance Improvement of Low-cost DR/GPS for Land Navigation using Sigma Point Based RHKF Filter

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Abstract: This paper describes a DR construction for land navigation and the sigma point based receding horizon Kalman FIR (SPRHKF) filter for DR/GPS hybrid navigation system. A simple DR construction is adopted to improve the performance both of the pure land DR navigation and the DR/GSP hybrid navigation system. In order to overcome the flaws of the EKF, the SPKF is merged with the receding horizon strategy. This filter has several advantages over the EKF, the SPKF, and the RHKF filter. The advantages include the robustness to the system model uncertainty, the initial estimation error, temporary unknown bias, and etc. The computational burden is reduced. Especially, the proposed filter works well even in the case of exiting the unmodeled random walk of the inertial sensors, which can be occurred in the MEMS inertial sensors by temperature variation. Therefore, the SPRHKF filter can provide the navigation information with good quality in the DR/GPS hybrid navigation system for land navigation seamlessly.

Keywords: DR/GPS, SPKF, receding horizon strategy, SPRHKF filter

1. INTRODUCTION

Commercial navigation technology has been become the core technology in LBS (Location Based) industry. The LBS system has been implemented for a car and the navigation system in LBS system is called CNS (Car Navigation System). CNS is comprised of a GPS receiver and a digital map, generally. And CNS is expanded into DR (Dead Reckoning)/GPS hybrid system to calculate the position information even in the urban area seamlessly. DR system for CNS must be implemented as low-cost to extend a commercial navigation market. Therefore, the DR system may include low-cost sensors instead of an IMU(Inertial Measurement Unit: 6DOP). In this paper, it is assumed that the DR system is implemented using an accelerometer and a gyro [1,2].

In order to calculate a 2D position in DR system, an odometer or a biaxial accelerometer has been utilized. In general, the DR system for a before market uses an odometer. And the DR system for an after market utilizes a biaxial accelerometer. In this paper, it is verified that the performance of the DR system using an accelerometer is better than that of the DR system using a biaxial accelerometer for land navigation.

Currently, the DR/GPS hybrid navigation system has been developed using the extended Kalman filter (EKF). The EKF is the well-known approach in the integration of the nonlinear systems. However, the several flaws of the EKF exist, which may lead to sub-optimal performance and sometimes divergence of the filter. In recent years, various-type filters have been investigated to overcome the flaws. The sigma point kalman filter (SPKF) and the receding horizon Kalman FIR (RHKF) filter are the representative alternative filters [3,4].

If initial estimation error is large in the EKF, this filter may diverge because the Jacobian matrix for implementing the EKF has serious problems. The SPKF, however, does not need to calculate the Jacobian matrix. Therefore, the SPKF is robust to the initial estimation error, unlike the EKF. When system has an unmodeled error or temporary unknown bias, the EKF is under the full influence of the errors. In order to reduce the effect of these kinds of errors, the RHKF filter has been researched. Since the FIR filter utilized finite measurements over the most recent time interval, this filter is known to be

robust against the temporary modeling uncertainties that may cause a divergence phenomenon in the case of the IIR structure filter [5]. However, the SPKF does not have the merits of the RHKF filter and the RHKF filter also does not have the advantages of the SPKF. In this paper, a novel filter, called the SPRHKF (Sigma Point based Receding Horizon Kalman FIR) filter, is presented. The SPRHKF filter is made by fusing the advantages of the two filters.

In this paper, the performance of the EKF, the SPKF, and the SPRHKF filter is analyzed in the various situations of the DR/GPS hybrid navigation system. The results show that the SPKF and the SPRHKF filter work well even in the case of the initial large azimuth error. Moreover, the performance of the SPRHKF filter is better than the other filters in the cases that the inertial sensors have unmodeled random walk errors or have temporary unknown bias. The result is verified by some simulations

2. DR CONSTRUCTION FOR CNS

The differential equation for velocity calculation of INS is as follows:

$$\dot{V}^n = C_b^n f^b - (2\omega_{ie}^n + \omega_{en}^n) \times V^n + g^n \tag{1}$$

In the case of 2D positioning for CNS, the gravity term is removed. And the coriolis force term can be ignored when low grade accelerometer is utilized in the CNS. Therefore, the simplified equation for discrete-time systems is as follows:

$$DR_M1: V^n(k) = V^n(k-1) + C_b^n(k) f^b(k) dk \tag{2}$$

where

$$C_b^n = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \tag{3}$$

The DR_M1 must use a biaxial accelerometer. Another construction using just an accelerometer is as follows:

$$DR_M2: V_x^b(k) = V_x^b(k-1) + f_x^b(k) dk \tag{4a}$$

$$V^n(k) = (C_b^n)'(k) V_x^b(k) \tag{4b}$$

where

$$(C_b^n)' = \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix} \tag{5}$$

DR_M1 updates the velocity on the navigation frame by transforming the acceleration on the body frame into that on the navigation frame. If the vehicle turns with constant angular velocity (ω) and constant velocity (V), the accelerometers attached on the vehicle output $f^b = [0 \ V\omega]^T$. The y-axis (lateral axis) term means the centrifugal force. However, in the case that either the velocity or the angular velocity is low, the centrifugal force may not be recognized in the MEMS gyros.

DR_M2 updates the velocity on the body frame using the x-axis accelerometer. Unless a vehicle slips, the vehicle moves only to the direction of the x-axis (longitudinal axis). And the output of the x-axis accelerometer does not contain the centrifugal force on the curve trajectory because the x-axis vector is parallel to the tangential vector of the curve.

The velocity error in DR_M1 is caused by the biaxial accelerometer error and the gyro error. On the other hand, the DR_M2 has the velocity error caused by an accelerometer error and the gyro error. Moreover, the DR_M2 does not have the undetectable error that can be occurred on the curve trajectory in the DR_M1. Therefore, it is expected that the DR_M2 has less error than the DR_M1 as figure 1.

The performance difference between the two methods is also appeared in the DR/GPS integration. In order to analyze the performance of the two methods, the DR/GPS integration is simulated using EKF. Figure 2 shows the gyro bias covariance. The gyro bias of the DR_M2/GPS is observable irrespective of the trajectory. However, the observability of the DR_M1/GPS varies according to the moving trajectory. Therefore, the estimation performance of the DR_M2/GPS is better than that of the DR_M1/GPS.

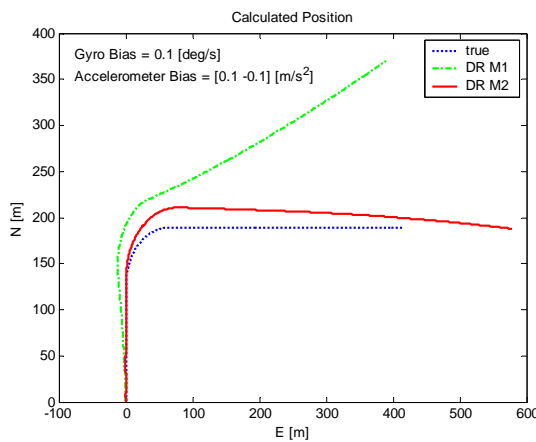


Fig. 1. Comparison between DR M1 and DR M2.

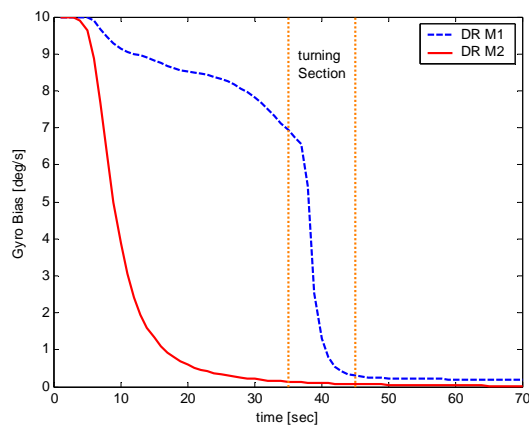
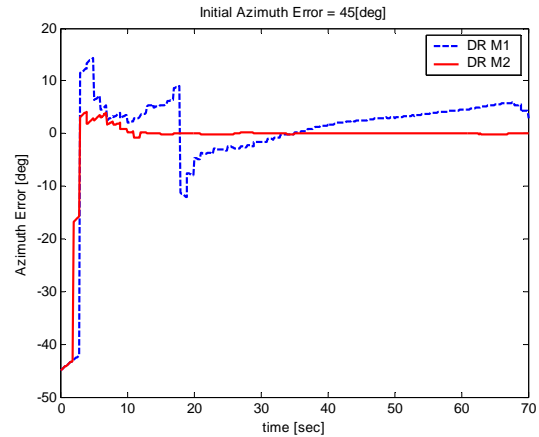
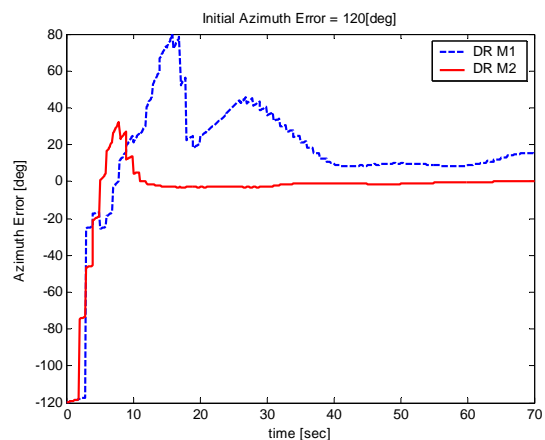


Fig. 2. Gyro bias covariance.



(a) $\delta\psi_0 = 45^\circ$, $\dot{\psi} = 20^\circ / s$ [15 ~ 25sec]



(b) $\delta\psi_0 = 120^\circ$, $\dot{\psi} = 20^\circ / s$ [15 ~ 53sec]

Fig. 3. Comparison between DR_M1 and DR_M2.

Figure 3 shows the azimuth error according to the initial azimuth estimation error. It can be seen that the azimuth error in the DR_M2/GPS converges into 0 even in the case that the initial azimuth estimation error is large. However, the convergence of the azimuth error in the DR_M1/GPS is dependent on the moving trajectory and the initial estimation error size. Therefore, it is confirmed that the DR_M2/GPS has better performance than the DR_M1/GPS.

3. SIGMA POINT BASED RHKF FILTER

The EKF has various drawbacks in the estimation problem. One of the main drawbacks is that the state distribution is approximated by a Gaussian random variable, which is then propagated through the first-order linearization of the nonlinear system. When the initial estimation error is large, the propagated mean and covariance may have large errors, which may lead to sub-optimal performance and sometimes divergence of the filter. Another weak point is that EKF may have large error when there is model uncertainty, unknown time varying bias, etc. because of IIR structure. In this chapter the alternative filters are introduced.

3.1 Sigma point Kalman filter

The main idea of the SPKF: *with a fixed number of parameters it should be easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function* [3]. The fixed number in the SPKF is the minimal set

of weighted sample points chosen deterministically, called sigma points. Generally, the number of sigma points is $2L+1$ (state dimension L). The SPKF is constructed as follows [4]:

0) A discrete time nonlinear system

$$x_{k+1} = f(x_k) + Gw_k, w_k \sim N(0, Q) \quad (6a)$$

$$y_k = h(x_k) + v_k, v_k \sim N(0, R) \quad (6b)$$

1-1) Initialization: augmented states and covariance

$$\hat{x}_0^a = E[x_0^T \quad v_0^T]^T = [\hat{x}_0^T \quad 0^T]^T \quad (7)$$

$$P_0^a = E[(x_0 - \hat{x}_0^a)(x_0 - \hat{x}_0^a)^T] = \begin{bmatrix} P_0 & 0 \\ 0 & Q \end{bmatrix} \quad (8)$$

1-2) Initialization: weights

$$W_0^{(m)} = \lambda / (L + \lambda) \quad (9a)$$

$$W_0^{(c)} = \lambda / (L + \lambda) + (1 - \alpha^2 + \beta) \quad (9b)$$

$$W_i^{(m)} = W_i^{(c)} = 1 / 2(L + \lambda), i = 1, \dots, 2L \quad (9c)$$

where $\lambda = (\alpha^2 - 1)L$ is a scaling parameter. α means the spread of the sigma points around \hat{x}_0 (set to $1 \leq \alpha \leq 1e^{-3}$) and β is used to incorporate prior knowledge of the distribution of x (2 for Gaussian distribution).

2) Sigma points Calculation

$$\chi_{k-1} = [\hat{x}_{k-1} \quad \hat{x}_{k-1} \pm \sqrt{(L + \lambda)P_{k-1}}] \quad (10)$$

3) Time propagation

$$\chi_{k|k-1}^x = f(\chi_{k-1}^x) + G\chi_{k-1}^v \quad (11)$$

$$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} \chi_{i,k|k-1}^x \quad (12)$$

$$P_k^- = \sum_{i=0}^{2L} W_i^{(c)} [\chi_{i,k|k-1}^x - \hat{x}_k^-][\chi_{i,k|k-1}^x - \hat{x}_k^-]^T \quad (13)$$

$$\hat{y}_k^- = \sum_{i=0}^{2L} W_i^{(m)} h(\chi_{i,k|k-1}^x) \quad (14)$$

4) Measurement update

$$P_{y_k y_k} = \sum_{i=0}^{2L} W_i^{(c)} [h(\chi_{i,k|k-1}^x) - \hat{y}_k^-][h(\chi_{i,k|k-1}^x) - \hat{y}_k^-]^T \quad (15)$$

$$P_{x_k y_k} = \sum_{i=0}^{2L} W_i^{(c)} [\chi_{i,k|k-1}^x - \hat{x}_k^-][h(\chi_{i,k|k-1}^x) - \hat{y}_k^-]^T \quad (16)$$

$$K_k = P_{x_k y_k} P_{y_k y_k}^{-1} \quad (17)$$

$$\hat{x}_k = \hat{x}_k^- + K_k (y_k - \hat{y}_k^-) \quad (18)$$

$$P_k = P_k^- - K_k P_{y_k y_k} K_k^T \quad (19)$$

The equations (7)~(9) are preprocessed before processing the main SPKF. Then the SPKF is processed using the equations (10)~(19), recursively.

It is well-known fact that the SPKF can overcome the flaws of the EKF such as inaccurate Jacobian matrices caused by the linear approximations of nonlinear functions with large initial estimation error. Therefore, it can be expected that the SPKF can drive the DR/GPS hybrid navigation system no matter what the estimated initial heading error is large.

3.2 Receding horizon Kalman FIR filter

If a filter has a model uncertainty or an unknown temporary time-varying bias, the estimation performance depends on the filter property. Unfortunately, the EKF cannot estimate the state variables exactly because the EKF has an IIR structure.

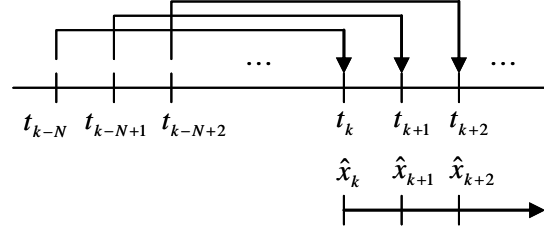


Fig. 4. The concept of the RHKF filter.

In order to enhance the filter performance in the system that has a model uncertainty or a time-varying bias, this paper introduces the RHKF filter.

Figure 4 shows the concept of the RHKF filter. As can be seen in this figure, the current state, x_k , is estimated only using the current measurements on the horizon $[k-N, k]$ (N is a horizon size). The RHKF filter has a fast estimation property and is influenced restrictively by the errors such as model uncertainty, temporary time-varying bias, etc. due to the FIR construction. And it can be also utilized irrespective of singularity problems caused by unknown information about the horizon initial state in the linear systems.

However, the research on the RHKF filter for nonlinear systems is insufficient by this time. The linear filters for nonlinear systems need the linearization of the nonlinear functions, which problem has decelerated the studies of the RHKF filter for nonlinear systems [5]. In order to apply the merits of the RHKF filter into the DR/GPS hybrid navigation system, this paper utilizes the concept of the sigma point. And an advanced RHKF filter for nonlinear systems is presented in the next chapter.

3.3 Sigma point based RHKF filter

The RHKF filter is designed using the inverse covariance form of the Kalman filter because it is assumed that the initial information of the states is unknown in the linear system. So, the initial value of the inverse covariance matrix is set by 0. However, the initial information must be obtained with small error in the nonlinear system because of the linear approximations of nonlinear functions. So, the RHKF filter has a restriction in the linearization process. In this paper, the SPRHKF filter is designed to weaken the restriction of the RHKF filter. As mentioned previously, the SPKF works well even in case of large initial estimation errors. The SPRHKF filter merges the merits of the RHKF filter and the SPKF to guarantee the robustness in the state estimation.

The concept of the SPRHKF filter is shown in figure 5. In this figure, k_N means the receding interval. The SPKF driven from time t_k provides the estimated solution in the interval $[t_{k+k_N}, t_{k+2k_N}]$. Simultaneously, the SPKF for the posterior horizon is processed from time t_{k+k_N} . And the estimated solution is provided by the SKPF for the posterior horizon in the interval $[t_{k+2k_N}, t_{k+3k_N}]$.

The SPRHKF filter has three advantages over the EKF, the RHKF filter, and the SPKF. First, the SPRHKF filter has a robust estimation property by the FIR characteristics of the RHKF filter. Secondly, the SPRHKF filter also has robustness to the horizon initial condition due to the strong point of the SPKF. Finally, the SPRHKF filter solved the heavy computational burden of the RHKF filter by extending the receding interval from 1 to k_N .

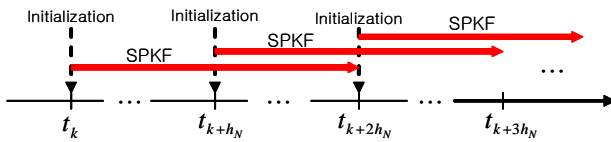


Fig. 5. The concept of the SPRHKF filter.

4. DR/GPS USING THE SPRHKF FILTER

In this chapter, the loosely coupled DR/GPS hybrid navigation system is designed using the SPRHKF filter. Figure 6 shows the block diagram of the DR/GPS hybrid navigation system using the SPRKKF filter.

It is assumed that the DR system is constructed by an accelerometer and a gyro. The accelerometer measures the forward acceleration of the vehicle and the gyro measures the z-axis (vertical axis) angular velocity.

The states to be estimated are set by 2D position on the navigation frame, velocity on the body frame, azimuth, accelerometer bias (∇), and gyro bias (ε).

First, the sigma points are generated using (10). Then the time propagation is processed as follows:

$$\begin{aligned}
 \chi_{k|k-1}^x(1, j) &= \chi_{k-1}^x(1, j) + \chi_{k-1}^x(3, j) \cos(\chi_{k-1}^x(4, j))dt + \chi_{k-1}^v(1, j) \\
 \chi_{k|k-1}^x(2, j) &= \chi_{k-1}^x(2, j) + \chi_{k-1}^x(3, j) \sin(\chi_{k-1}^x(4, j))dt + \chi_{k-1}^v(2, j) \\
 \chi_{k|k-1}^x(3, j) &= \chi_{k-1}^x(3, j) + (f_x^b - \chi_{k-1}^x(5, j))dt + \chi_{k-1}^v(3, j) \\
 \chi_{k|k-1}^x(4, j) &= \chi_{k-1}^x(4, j) + (\omega_z - \chi_{k-1}^x(6, j))dt + \chi_{k-1}^v(4, j) \\
 \chi_{k|k-1}^x(5, j) &= \chi_{k-1}^x(5, j) + \chi_{k-1}^v(5, j) \\
 \chi_{k|k-1}^x(6, j) &= \chi_{k-1}^x(6, j) + \chi_{k-1}^v(6, j)
 \end{aligned}
 \tag{20}$$

where $j = 1, 2, \dots, 2L + 1$. f_x^b denotes the accelerometer output and ω_z means the gyro output.

In the EKF, the relations between states are denoted clearly in the Jacobian matrix. In the RHSPKF filter, the relations are shown in the time propagation of the sigma points as equations (20).

After time propagation, the measurement update is carried out by equations (15)~(19).

As can be seen in this chapter, the SPRHKF filter does not have any complex Jacobian matrixes. Moreover, this filter does not have complex equations required in the RHKF filter. Therefore, the proposed filter can be easily utilized to implement the DR/GPS hybrid navigation system.

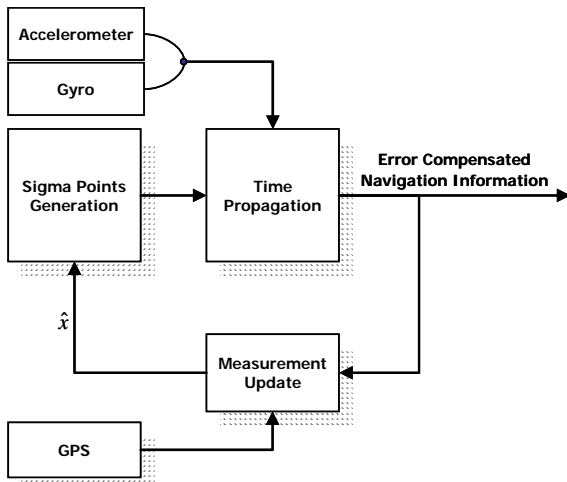


Fig. 6. The block diagram of the DR/GPS.

5. SIMULATION AND RESULT

In order to verify the performance of the proposed filter, some simulations are carried. The five situations are made and the EKF, the SPKF, and the SPRHKF filter are driven in these situations. Then the performance of these filters is analyzed. In the SPRHKF filter, the size of the receding horizon is set by 15sec. The simulation results are summarized in table 1.

5.1 Situation I

Usually, the biases of low-cost inertial sensors show non-zero mean and non-stationary behavior, the errors are modeled as random walk.

$$\nabla_k = \nabla_{k-1} + w_{\nabla,k}, w_{\nabla} \sim N(0, Q_{\nabla})
 \tag{21a}$$

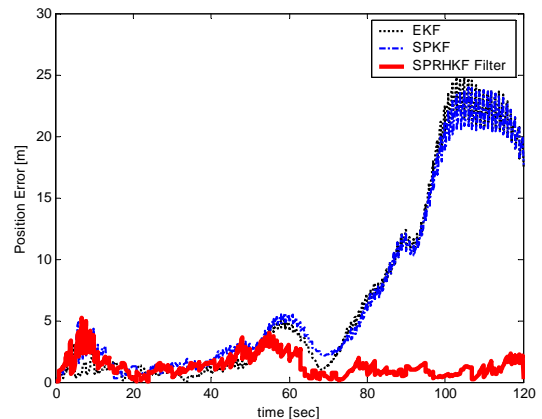
$$\varepsilon_k = \varepsilon_{k-1} + w_{\varepsilon,k}, w_{\varepsilon} \sim N(0, Q_{\varepsilon})
 \tag{21b}$$

where the process noise must be set by Q_{∇} and Q_{ε} .

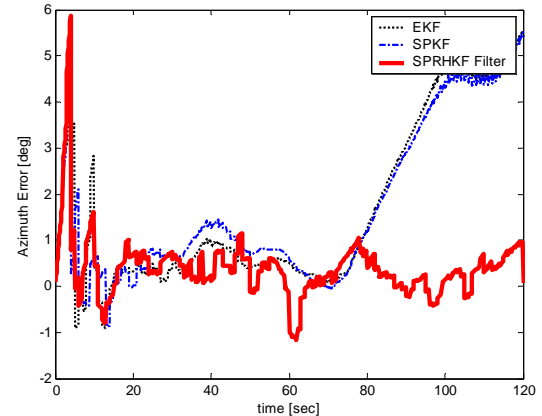
If the filters consider the sensor errors as random walk, the performance of the filters is similar to one another as can be seen in table 1.

5.2 Situation II

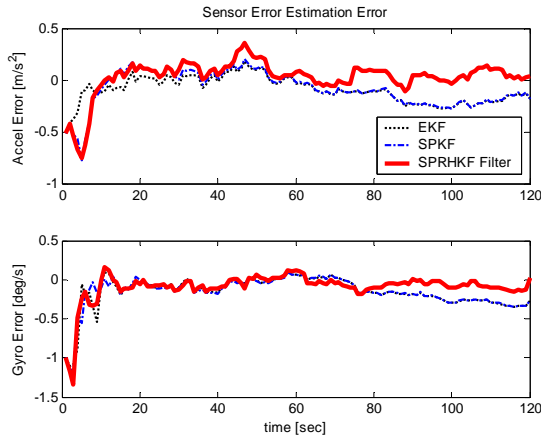
In general, the errors of inertial sensors can be modeled as random constant. However, the errors of low-cost inertial sensors may have random walk process. In this situation, the sensor errors are modeled as random walk. But the filters are considered the sensor errors as random constant.



(a) Position error



(b) Azimuth error



(c) Sensor error estimation error
Fig. 7. Results of the situation II.

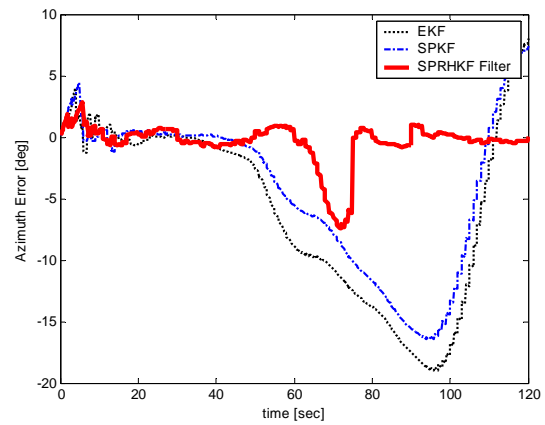
As can be seen in figure 7, the estimation errors of the EKF and the SPKF diverge gradually. First, the sensor error estimation error increases with time. Second, the azimuth error is expanded under the influence of the gyro error estimation error. Finally, the position data diverges. On the other hand, the SPRHKF filter has bounded errors. Therefore, the SPRHKF filter is robust against the model uncertainty.

5.3 Situation III

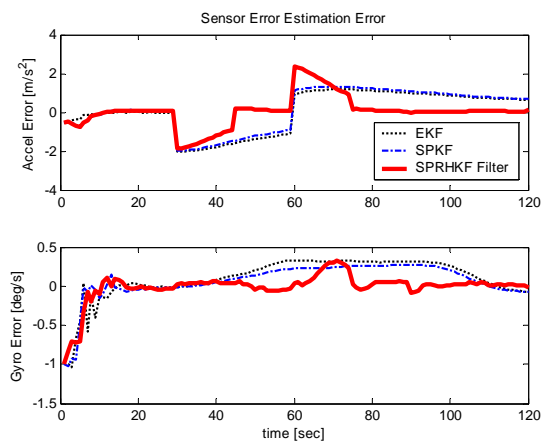
In this situation, the sensor error is modeled as random walk and the filters consider the sensor error as random walk. And a temporary unknown accelerometer bias is occurred in the interval [30, 60] with size of $1[m/s^2]$. As can be seen in table 1, the results of the three filters are similar to one another. This phenomenon is caused by that the temporary unknown bias is treated as a random walk bias.

5.4 Situation IV

In this situation, the sensor error is modeled as random constant and the filters consider the sensor error as random constant. And the temporary accelerometer unknown bias is occurred as equation (22). The result is shown in figure 8. It can be seen that the errors of the SPRHKF filter are less than that of the EKF and the SPKF. The reason is that the SPRHKF filter can estimate the sensor error as the situation II and the filter is influenced restrictively by the unknown bias due to the FIR construction.



(b) Azimuth error

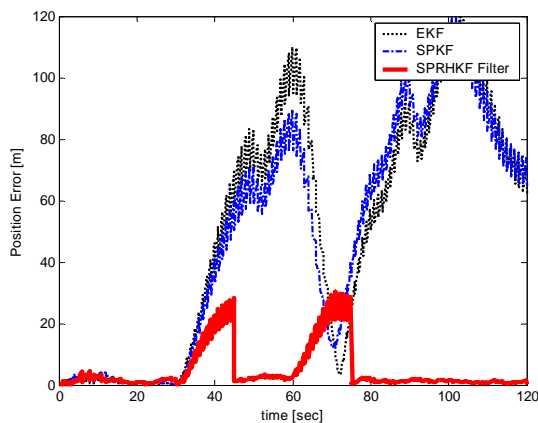


(c) Sensor error estimation error
Fig. 8. Results of the situation IV.

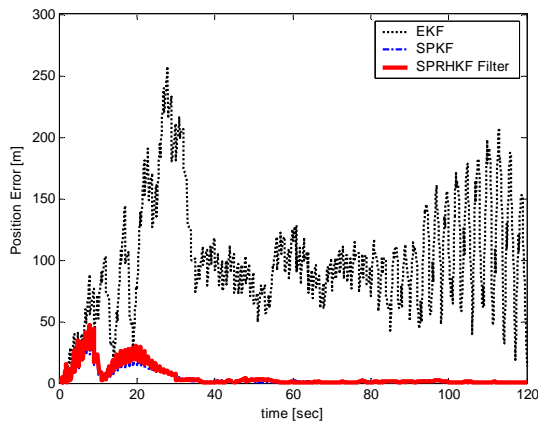
On the other hand, the EKF and the SPKF can not estimate the temporary unknown sensor error because of the IIR construction. It can be guessed that the temporary unknown sensor error can be estimated accurately if the sensor error is modeled as random walk in the filters.

5.5 Situation V

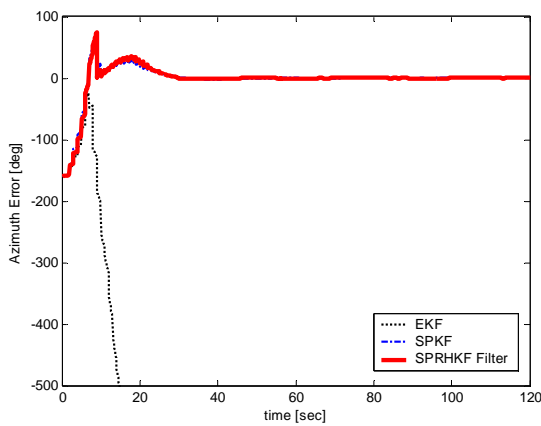
The initial azimuth information cannot be obtained unless a magnetic compass or a high-grade gyro module is utilized. Therefore, the initial azimuth error exists unavoidably. In this situation, the initial azimuth error is set by 160degrees. Figure 9 shows the simulation results. As can be seen in figure 9, the EKF errors diverge with time. However, the SPKF and the SPRHKF filter have good performance. This phenomenon is owing to the Jacobian matrix error. The SPKF and the SPRHKF filter need not to calculate the Jacobian matrix. Therefore, the proposed filter is robust to the initial large estimation error, also.



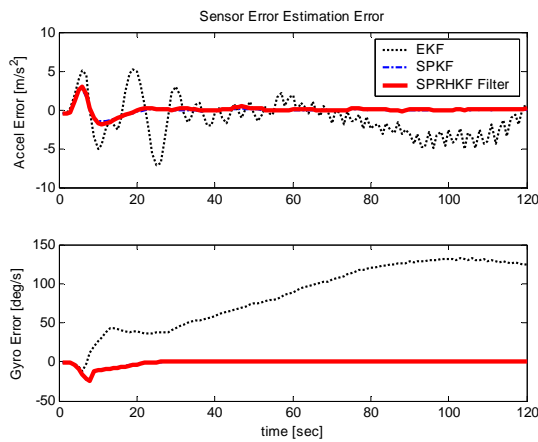
(a) Position error



(a) Position error



(b) Azimuth error



(c) Sensor error estimation error
Fig. 9. Results of the situation V.

6. CONCLUSION

The SPRHKF filter for DR/GPS hybrid navigation system is developed and simulated in the various situations. The proposed filter has a robust estimation property by the FIR strategy. This filter also has robustness to the initial large estimation error due to the merit of the SPKF. And the flaw of the RHKF filter, heavy computational burden, is overcome in this filter. It can be expected that the RHSPKF filter can be utilized in DR/GPS hybrid navigation system with robust properties.

Table 1. Results of the situations.

(Mean value of the estimation error)

		Position [m]	Azimuth [deg]	Acc Error [m/s ²]	Gyro Error [deg/sec]
I	(1)	1.1182	0.3141	-0.0324	-0.0934
	(2)	1.2885	0.2304	-0.0373	-0.0949
	(3)	1.3704	0.4455	0.0128	-0.1059
II	(1)	6.9248	1.6791	-0.0959	-0.1538
	(2)	7.1681	1.6970	-0.0895	-0.1528
	(3)	1.3201	0.3820	0.0330	-0.0928
III	(1)	2.5022	0.1633	-0.0354	-0.0888
	(2)	2.7712	0.1723	-0.0373	-0.0802
	(3)	2.6391	0.1707	0.0097	-0.0749
IV	(1)	50.6778	-5.7703	0.0638	0.1037
	(2)	49.2438	-4.1851	0.1409	0.0694
	(3)	4.9754	-0.4085	0.0482	-0.0079
V	(1)	0.1005e3	-4.5025e3	-0.0012e3	0.0838e3
	(2)	4.3238	-8.3788	-0.0596	-1.4181
	(3)	4.9269	-2.4147	-0.0053	-1.5011

(1) EKF (2) SPKF (3) SPRHKF filter

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