# Polyphase Representation of the Relationships Among Fullband, Subband, and Block Adaptive Filters

Chimin Tsai

Department of Electrical Engineering, Chung Hua University, Hsin Chu, Taiwan
(Tel: +886-3-5186037; Fax: +886-3-5186436; E-mail: tsai@chu.edu.tw)

**Abstract:** In hands-free telephone systems, the received speech signal is fed back to the microphone and constitutes the so-called echo. To cancel the effect of this time-varying echo path, it is necessary to device an adaptive filter between the receiving and the transmitting ends. For a typical FIR realization, the length of the fullband adaptive filter results in high computational complexity and low convergence rate. Consequently, subband adaptive filtering schemes have been proposed to improve the performance. In this work, we use deterministic approach to analyze the relationship between fullband and subband adaptive filtering structures. With block adaptive filtering structure as an intermediate stage, the analysis is divided into two parts. First, to avoid aliasing, it is found that the matrix of block adaptive filters is in the form of pseudocirculant, and the elements of this matrix are the polyphase components of the fullband adaptive filter. Second, to transmit the near-end voice signal faithfully, the analysis and the synthesis filter banks in the subband adaptive filtering structure must form a perfect reconstruction pair. Using polyphase representation, the relationship between the block and the subband adaptive filters is derived.

**Keywords:** multirate signal processing, polyphase decomposition, subband adaptive filter, block adaptive filter, pseudocirculant matrix.

## 1. INTRODUCTION

In the context of hands-free telephone systems, the received speech signal $x(n)$ is fed back to the microphone and constitutes the so-called echo, $y(n)$ in Fig. 1. Since the echo path $S(z)$ is changing slowly with time, to cancel this echo, it is necessary to device an adaptive filter $\hat{S}(z)$ between the receiving and the transmitting ends. With an appropriate adaptive algorithm [1], it is possible to make $\hat{y}(n) \approx y(n)$. And consequently, the transmitted signal $e(n)$ is approximately equal to the near-end voice signal $v(n)$.

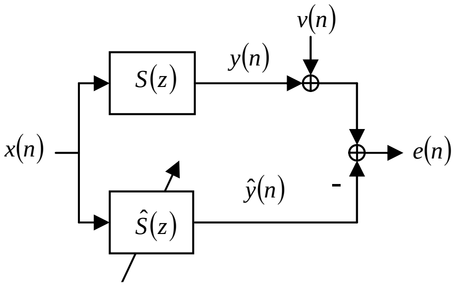


Fig. 1. Echo canceller.

Because the acoustic echo is usually delayed for about several hundred milliseconds, the resulting adaptive filter requires several thousand taps. For a typical FIR realization, the adaptive filter has unacceptable computational complexity and convergence rate. Therefore, a variety of frequency domain and multirate adaptive filtering schemes have been proposed to improve the performance [2,3]. In this presentation, we discuss the structural relationship between the original "fullband" adaptive filtering and a popular multirate scheme, called subband adaptive filtering. The length of each subband adaptive filter is shorter then that of the corresponding fullband adaptive filter, although the total number of filter coefficients is about the same. The performance improvement is thus achieved by using shorter filter length.

In Section 2, we review the researches on the subband adaptive filtering. It is known that the relationship between fullband and subband adaptive filters has not been fully explored. In Section 3, a block adaptive filtering structure is proposed as an intermediate stage for the analysis. Then, the analysis is divided into two parts. First, to avoid aliasing, it is found that the matrix of block adaptive filters must be in the form of pseudocirculant, and the elements of this matrix are the polyphase components of the fullband adaptive filter. In the second step, we consider the transformation from block structure to subband realization. It is known that, to transmit the near-end voice signal faithfully, the analysis and the synthesis filter banks in the subband adaptive filtering structure must form a perfect reconstruction pair. Using polyphase representation, the relationship between block and subband adaptive filters is established. A simple example is given in Section 4 to clarify the above relationships.

## 2. SUBBAND ADAPTIVE FILTERING

The basic structure of subband adaptive filtering is given in Fig. 2. In general, the number of subbands can be any integer $M$. However, we use $M = 2$ throughout this presentation for simplicity. In this structure, the echo and the near-end voice signals are first filtered by the analysis filter bank

$$H(z) = [H_0(z) \quad H_1(z)]^T,$$

then the subband signals are decimated by a factor of 2 to produce $[y_0(n) \quad y_1(n)]^T$. At the same time, the received speech signal is also filtered by $H(z)$, decimated by 2, and processed by the matrix of subband adaptive filters

$$C(z) = \begin{bmatrix} C_{00}(z) & C_{01}(z) \\ C_{10}(z) & C_{11}(z) \end{bmatrix}.$$

The outputs $[\hat{y}_0(n) \quad \hat{y}_1(n)]^T$ are subtracted by $[y_0(n) \quad y_1(n)]^T$. Finally, the resulting subband errors are interpolated by a factor of 2, then passed through the synthesis filter bank

$$G(z) = [G_0(z) \quad G_1(z)]^T$$

to reconstruct the transmitted signal $e(n)$. Note that the matrix $C(z)$ may be adapted according to the subbands errors instead of the fullband error $e(n)$.

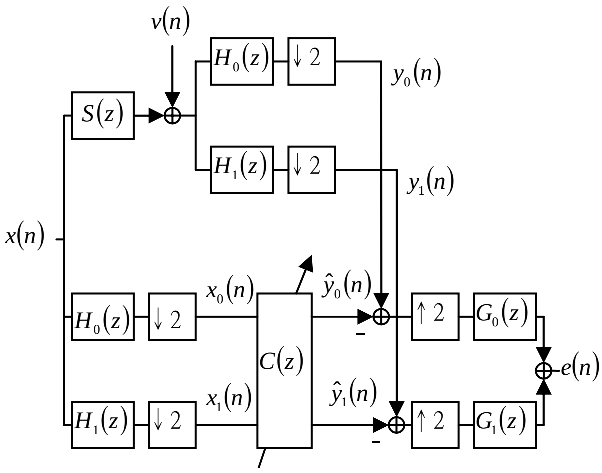


Fig. 2. Subband adaptive filtering.

For the purpose of reducing the computational complexity and improving the rate of convergence, it is desirable to have analysis filter bank with nonoverlapping frequency bins so that the subband adaptive filters can operate independently. Therefore the matrix of the subband adaptive filters in many researches is constrained to be diagonal, i.e.,

$$C(z) = \begin{bmatrix} C_{00}(z) & 0 \\ 0 & C_{11}(z) \end{bmatrix}.$$

Because of the decimation and interpolation, the alias of the input may appear in the output. To avoid this effect, solutions such as "spectral gap" in [4] and "oversampling" in [5] were proposed. However, spectral gaps may impair the subjective quality of the voice signal and oversampling requires higher computational cost. On the other hand, Gilloire and Vetterli showed that the cross-terms in $C(z)$, $C_{01}(z)$ and $C_{10}(z)$, can not be ignored for critical sampling [6]. Although aliasing distortion is reduced by using the cross-terms, the convergence rate is also degraded because the subbands are no longer uncoupled. Besides, the computational complexity is increased due to the cross-terms. Other structures were proposed to further reduce the computational complexity and increase the rate of convergence by avoiding the cross-terms [7,8]. In general, subband adaptive filtering is still an open area of research.

## 3. BLOCK ADAPTIVE FILTERING

To investigate the relationship between the fullband and the subband adaptive filters, i.e., the equation relates $\hat{S}(z)$ and $C(z)$, it is reasonable to use a block adaptive filtering structure as an intermediate stage [9]. It is known that when a scalar filter is implemented in $M$ subbands, the necessary and sufficient conditions for alias cancellation in terms of the polyphase components is that the matrix of block transfer functions must be in the form of pseudocirculant [10]. In this section, we use a different approach to reveal the pseudocirculant nature of block adaptive filtering structure for the case of $M = 2$.

### 3.1 From fullband to block

To derive the corresponding block filtering structure, we begin with the fullband adaptive filtering structure in Fig.1, insert serial-to-parallel and parallel-to-serial building blocks into the structure, as shown in Fig.3. Note that the serial-to-parallel mechanism is implemented with delay chain followed by decimation and the parallel-to-serial mechanism is realized by interpolation and delay chain. As a consequence, a delay of $M-1$ samples is introduced in both signal paths.

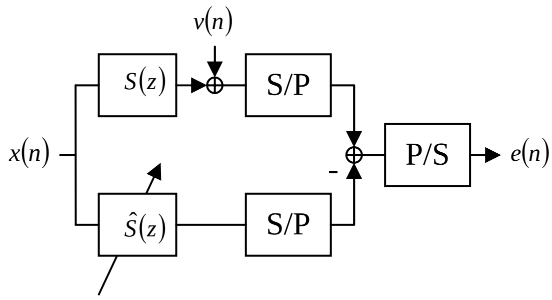


Fig. 3. Conversion of fullband to block processing.

To achieve block processing, the fullband adaptive filter in the lower path of Fig. 3 is moved after the serial-to-parallel mechanism. In result, the scalar transfer function $\hat{S}(z)$ is replaced by a $M$-by-$M$ matrix of transfer functions, $P(z)$ as illustrated in Fig. 4.

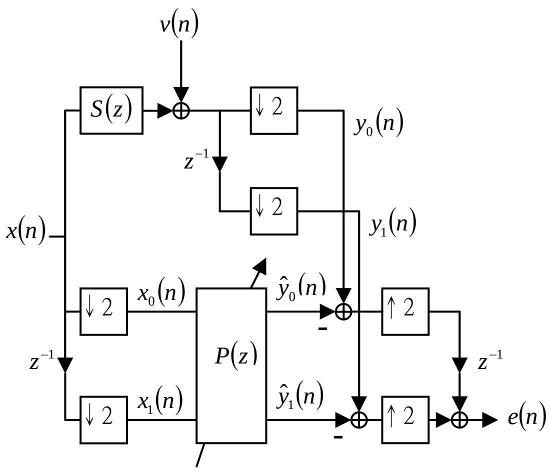


Fig. 4. Block adaptive filtering.

Although the relationship between $\hat{S}(z)$ and $P(z)$ can be found in [10] for the case of alias free. We provide a different approach for this problem. Ignore the upper paths of Fig. 3 and 4, and defined

$$\hat{y}(n) = -e(n).$$

Consequently, the scalar transfer function of the lower path is given in

$$\hat{Y}(z) = z^{-1}\hat{S}(z)X(z). \tag{1}$$

Then we have to derive the following equation

$$\begin{bmatrix} \hat{Y}_0(z) \\ \hat{Y}_1(z) \end{bmatrix} = \begin{bmatrix} P_{00}(z) & P_{01}(z) \\ P_{10}(z) & P_{11}(z) \end{bmatrix} \cdot \begin{bmatrix} X_0(z) \\ X_1(z) \end{bmatrix}, \tag{2}$$

where the matrix $P(z)$ is unknown and has to be determined from $\hat{S}(z)$. For time domain analysis, Eq. (1) is rewritten as

$$\hat{y}(n) = x(n) * \hat{s}(n-1).$$

The outputs of the upper and the lower interpolations in Fig. 4 are

$$\hat{y}(n) = \sum_{k=-\infty}^{\infty} x(k)\hat{s}(n-1-k), \quad n = 1,3,5... \tag{3}$$

and

$$\hat{y}(n) = \sum_{k=-\infty}^{\infty} x(k)\hat{s}(n-1-k), \quad n = 0,2,4... \tag{4}$$

respectively. It is observed from Fig. 4 that the block signals are processed at a lower rate, and the outputs are related to the above scalar signals as

$$\begin{bmatrix} \hat{y}_0(n) \\ \hat{y}_1(n) \end{bmatrix} = \begin{bmatrix} \hat{y}(2n+1) \\ \hat{y}(2n) \end{bmatrix}. \tag{5}$$

Let $n \to 2n+1$ and $n \to 2n$ for Eqs. (3) and (4) respectively, then substitute into Eq. (5), we have

$$\begin{bmatrix} \hat{y}_0(n) \\ \hat{y}_1(n) \end{bmatrix} = \begin{bmatrix} \sum_{k=-\infty}^{\infty} x(k)\hat{s}(2n-k) \\ \sum_{k=-\infty}^{\infty} x(k)\hat{s}(2n-1-k) \end{bmatrix}.$$

The even and the odd terms are separated as

$$\begin{bmatrix} \hat{y}_0(n) \\ \hat{y}_1(n) \end{bmatrix} = \begin{bmatrix} \sum_{k\ even} x(k)\hat{s}(2n-k) + \sum_{k\ odd} x(k)\hat{s}(2n-k) \\ \sum_{k\ even} x(k)\hat{s}(2n-1-k) + \sum_{k\ odd} x(k)\hat{s}(2n-1-k) \end{bmatrix}.$$

The dummy variables in the above equation are changed as $k \to 2k$ and $k \to 2k-1$ for the even and the odd terms respectively. In result,

$$\begin{bmatrix} \hat{y}_0(n) \\ \hat{y}_1(n) \end{bmatrix} = \begin{bmatrix} \sum_{k=-\infty}^{\infty} x(2k)\hat{s}(2(n-k)) + \sum_{k=-\infty}^{\infty} x(2k-1)\hat{s}(2(n-k)+1) \\ \sum_{k=-\infty}^{\infty} x(2k)\hat{s}(2(n-k)-1) + \sum_{k=-\infty}^{\infty} x(2k-1)\hat{s}(2(n-k)) \end{bmatrix}.$$

Rewrite the above equation in the notation of convolution, we get

$$\begin{bmatrix} \hat{y}_0(n) \\ \hat{y}_1(n) \end{bmatrix} = \begin{bmatrix} x(2n) * \hat{s}(2n) + x(2n-1) * \hat{s}(2n+1) \\ x(2n) * \hat{s}(2n-1) + x(2n-1) * \hat{s}(2n) \end{bmatrix}. \tag{6}$$

From Fig. 4, is it also observed that the block inputs of $P(z)$ are related to the scalar input by

$$\begin{bmatrix} x_0(n) \\ x_1(n) \end{bmatrix} = \begin{bmatrix} x(2n) \\ x(2n-1) \end{bmatrix}. \tag{7}$$

On the other hand, the polyphase representation of $\hat{S}(z)$ is

$$\hat{S}(z) = \sum_{k=0}^{M-1} z^{-k} \hat{S}_k(z^M),$$

in which $\hat{S}_k(z) = Z\{\hat{s}_k(n)\}$ and

$$\begin{bmatrix} \hat{s}_0(n) \\ \hat{s}_1(n) \end{bmatrix} = \begin{bmatrix} \hat{s}(2n) \\ \hat{s}(2n+1) \end{bmatrix}. \tag{8}$$

Substitute Eqs. (7) and (8) into Eq. (6), we obtain

$$\begin{bmatrix} \hat{y}_0(n) \\ \hat{y}_1(n) \end{bmatrix} = \begin{bmatrix} x_0(n) * \hat{s}_0(n) + x_1(n) * \hat{s}_1(n) \\ x_0(n) * \hat{s}_1(n-1) + x_1(n) * \hat{s}_0(n) \end{bmatrix}.$$

Using z-transform, we obtain

$$\begin{bmatrix} \hat{Y}_0(z) \\ \hat{Y}_1(z) \end{bmatrix} = \begin{bmatrix} \hat{S}_0(z) & \hat{S}_1(z) \\ z^{-1}\hat{S}_1(z) & \hat{S}_0(z) \end{bmatrix} \cdot \begin{bmatrix} X_0(z) \\ X_1(z) \end{bmatrix}. \tag{9}$$

By comparing Eqs. (2) and (9), it is clear that for a given $\hat{S}(z)$, the block adaptive filters must be in the form of pseudocirculant matrix [10], i.e.,

$$P(z) = \begin{bmatrix} \hat{S}_0(z) & \hat{S}_1(z) \\ z^{-1}\hat{S}_1(z) & \hat{S}_0(z) \end{bmatrix}. \tag{10}$$

On the other hand, $\hat{S}(z)$ can be expressed in terms of $P(z)$ as

$$\hat{S}(z) = \sum_{k=0}^{M-1} z^{-k} P_{0k}(z^M).$$

In other words, $P_{jk}(z)$ given in Eq. (2) constitute the polyphase components of $\hat{S}(z)$.

### 3.2 Revisit subband structure

In this subsection, we show that the above is consistent with the result obtained in [6]. Using the standard procedure for quadrature mirror filter bank to analyze the multirate system in Fig. 2, the output of the upper path is

$$\begin{bmatrix} Y_0(z) \\ Y_1(z) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} H_0(z^{1/2}) & H_0(-z^{1/2}) \\ H_1(z^{1/2}) & H_1(-z^{1/2}) \end{bmatrix} \cdot \begin{bmatrix} S(z^{1/2}) & 0 \\ 0 & S(-z^{1/2}) \end{bmatrix} \cdot \begin{bmatrix} X(z^{1/2}) \\ X(-z^{1/2}) \end{bmatrix}$$

and the output of the lower path is

$$\begin{bmatrix} \hat{Y}_0(z) \\ \hat{Y}_1(z) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} C_{00}(z) & C_{01}(z) \\ C_{10}(z) & C_{11}(z) \end{bmatrix} \cdot \begin{bmatrix} H_0(z^{1/2}) & H_0(-z^{1/2}) \\ H_1(z^{1/2}) & H_1(-z^{1/2}) \end{bmatrix} \cdot \begin{bmatrix} X(z^{1/2}) \\ X(-z^{1/2}) \end{bmatrix}.$$

The subband errors are zero if

$$\begin{bmatrix} \hat{Y}_0(z) \\ \hat{Y}_1(z) \end{bmatrix} = \begin{bmatrix} Y_0(z) \\ Y_1(z) \end{bmatrix},$$

or equivalently

$$\begin{bmatrix} C_{00}(z^2) & C_{01}(z^2) \\ C_{10}(z^2) & C_{11}(z^2) \end{bmatrix} \cdot \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix} = \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix} \cdot \begin{bmatrix} S(z) & 0 \\ 0 & S(-z) \end{bmatrix}.$$

Compare Fig. 2 and Fig. 4, it is known that the block filtering structure is a special case of the subband filtering. As a consequence, these two structures are the same if $H(z)$ and $G(z)$ in Fig. 2 are replaced by $\begin{bmatrix} 1 & z^{-1} \end{bmatrix}^T$ and $\begin{bmatrix} z^{-1} & 1 \end{bmatrix}^T$, respectively. In this case, we have

$$\begin{aligned} &\begin{bmatrix} C_{00}(z^2) & C_{01}(z^2) \\ C_{10}(z^2) & C_{11}(z^2) \end{bmatrix}. \\ &= \begin{bmatrix} 1 & 1 \\ z^{-1} & -z^{-1} \end{bmatrix} \cdot \begin{bmatrix} S(z) & 0 \\ 0 & S(-z) \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ z^{-1} & -z^{-1} \end{bmatrix}^{-1} \\ &= \frac{1}{2} \begin{bmatrix} S(z) + S(-z) & z(S(z) - S(-z)) \\ z^{-1}(S(z) - S(-z)) & S(z) + S(-z) \end{bmatrix} \\ &= \begin{bmatrix} S_0(z^2) & S_1(z^2) \\ z^{-1}S_1(z^2) & S_0(z^2) \end{bmatrix}. \end{aligned}$$

The result

$$C(z) = \begin{bmatrix} S_0(z) & S_1(z) \\ z^{-1}S_1(z) & S_0(z) \end{bmatrix}$$

is in the form of pseudocirculant and is consistent with the previous result in Eq. (10).

### 3.3 From block to subband

For the relationship between the block adaptive filters in Fig. 4 and the subband adaptive filters in Fig. 2, it is assume that $P(z)$ is given, then the equivalent $C(z)$ needs to be determined under reasonable conditions. To transmit the near-end voice signal $v(n)$ faithfully, the analysis and the synthesis filter banks in Fig. 2 must form a perfect reconstruction pair. Let $E(z)$ and $R(z)$ be the polyphase matrices of $H(z)$ and $G(z)$ respectively. In this work, it is assume that the $E(z)$ and $R(z)$ has the most simple relation for perfect reconstruction, i.e.,

$$R(z)E(z) = I \ .$$

More general condition for perfect reconstruction can be found in [3]. Insert one pair of $R(z)E(z)$ in the upper path of Fig. 4 in between the decimators and the interpolators. Then, move $E(z)$ to the left of the decimators, combine $E(z)$ with the delay chain results in the analysis filter bank $H(z)$. Similarly, move $R(z)$ to the right of the interpolators, combine $R(z)$ with the delay chain on the output side results in the synthesis filter bank $G(z)$. As a consequence, the upper path of Fig. 2 is obtained from Fig. 4. Using the same technique for the lower path of Fig. 4, it is observed that the matrices of the block and the subband adaptive filters are related by

$$C(z) = E(z)P(z)R(z) \ . \tag{11}$$

## 4. A SIMPLE EXAMPLE

To illustrate the relationship developed in the previous section. The standard procedure for multirate systems [3] is applied to the following simple example. Assume that the fullband adaptive filter is

$$\hat{S}(z) = 4 + 3z^{-1} + 2z^{-2} + z^{-3}$$

and the polyphase components of the analysis filter bank is

$$E(z) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

Using Eq. (10), the equivalent block adaptive filters for alias free are obtained as the polyphase components of $\hat{S}(z)$,

$$P(z) = \begin{bmatrix} 4 + 2z^{-1} & 3 + z^{-1} \\ 3z^{-1} + z^{-2} & 4 + 2z^{-1} \end{bmatrix}.$$

Substitute

$$R(z) = E^{-1}(z) = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

into Eq. (11), we get

$$C(z) = \frac{1}{2}\begin{bmatrix} 11 + 8z^{-1} + z^{-2} & -3 + 2z^{-1} + z^{-2} \\ 3 - 2z^{-1} - z^{-2} & 5 - z^{-2} \end{bmatrix}.$$

Clearly, the pseudocirculant structure does not preserved in the general subband structure. Moreover, the matrix of subband adaptive filters, $C(z)$, is not diagonal.

## 5. CONCLUSIONS

Based on polyphase decomposition, the relationship between fullband and subband adaptive filters is established by using block filter structure as an intermediate stage. In this analysis, conditions for these systems to be equivalent are described and details of the cross-terms in subband structure are revealed.

## REFERENCES


[1] B. Widrow and S. D. Stearns, *Adaptive Signal Processing*, Englewood Cliffs, NJ: Prentice-Hall, 1985.

[2] J. J. Shynk, "Frequency-domain and multirate adaptive filtering," IEEE Signal Processing Magazine, vol. 9, pp. 14-37, Jan. 1992.

[3] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Englewood Cliffs, NJ: Prentice-Hall, 1993.

[4] H. Yasukawa, S. Shimada, and I. Furukawa, "Acoustic echo canceller with high speech quality," in Proc. IEEE ICASSP, pp. 2125-2128, 1987.

[5] W. Kellermann, "Analysis and design of multirate systems for cancellation of acoustical echoes," in Proc. IEEE ICASSP, pp. 2570-2573, 1988.

[6] A. Gilloire and M. Vetterli, "Adaptive filtering in subbands with critical sampling: Analysis, experiments, and application to acoustic echo cancellation," IEEE Trans., Signal Processing, vol. 40, pp. 1862-1875, Aug. 1992.

[7] S. S. Pradhan and V. U. Reddy, "A new approach to subband adaptive filtering," IEEE Trans., Signal Processing, vol. 47, pp. 655-664, Mar. 1999.

[8] M. R. Petraglia, and R. G. Alves, "A new adaptive subband structure with critical sampling," in Proc. IEEE ICASSP, pp. 1861-1864, 1999.

[9] G. A. Clark, S. K. Mitra, and S. R. Parker, "Block implementation of adaptive digital filters," IEEE Trans. Circuits Syst., vol. CAS-28, pp. 584-592, June 1981.

[10] P. P. Vaidyanathan and S. K. Mitra, "Polyphase networks, block digital filtering, LPTV systems, and alias-free QMF banks: A unified approach based on pseudocirculants," IEEE Trans. Acoust., Speech, Signal Processing, vol. 36, pp. 381-191, Mar. 1988.