Multipath detection in carrier phase differential GPS

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Abstract: A multipath mitigation method using the fault detection and isolation technique is proposed for the CDGPS. The base station is assumed to be immune to the effect of the multipath. With this reasonable assumption, the effect of multipath in moving station is mitigated. For that, the double difference measurement is produced, and then another additional difference between code pseudorange and acclumulated carrier phase is calculated. The test statistic is constituted with those differences. The hypothesis testing is applied to that test statistic. The proposed test statistic makes use of the effect of multipath in code pseudoranges and it does not use time differences. Therefore the detection ability for multipath is improved in most environments. However, the increased number of differences makes the measurement noises larger. The performance of the method is compared with that of the conventional parity space method with code pseudorange.

Keywords: Multipath, Carrier phase differential GPS, Fault detection and isolation, Hypothesis tesing

1. INTRODUCTION

The GPS (Global Positioning System) is a satellite-based radio navigation system. It allows a user to obtain accurate position information anywhere in the globe. The DGPS (differential GPS) is an improved technology to provide more accurate navigation solutions. It needs a base station for difference calculations. The spatially correlated errors are efficiently removed in the calculations. The CDGPS (Carrier phase DGPS) is a more sophisticated method to obtain position solution with errors of a few score centimeters. In that method, it is necessary to obtain the integer ambiguity solutions [1-2]. However, in each method, some error sources make the solution inaccurate and the method difficult. Among those errors, the multipath problem of the signal is one of the most difficult problems.

The multipath propagation of the signal from a GPS satellite occurs when there are objects in the vicinity of a GPS receiver. The objects reflect the direct signal from the satellite therefore multiple arrivals of the signal is detected by the receiver. It makes the measurement of time of arrival inaccurate and causes errors in position solutions. Multipath is a major source of error, especially in high precision GPS differential positioning because the other major errors can be removed by differential calculations. For mitigation of multipath errors, some researches have been achieved. In [3], as signal tracking loop approach, the MEDLL (Multipath Estimating Delay Lock Loop) is proposed. In [4], the multipath effect is mitigated by using a multiantenna system. The effect can be detected by signal to noise ratio and suppressed by differences of measurement [5]. The fault detection and isolation method can be applied to multipath detection and rejection [6-7]. In this approach, a test statistic must be constituted with measurement, and then, with a prespecified threshold, the multipath error can be detected by hypothesis testing.

In this paper, a new multipath mitigation method for CDGPS applications is proposed being based on fault detection and isolation technique. A new test statistic is constituted with triple difference of measurement of code pseudorange and accumulated carrier phase. With that statistic, multipath error can be detected and rejected. The method does not need any additional hardware and makes use of code pseudorange measurement.

In the following section, the multipath mitigation method based on fault detection and isolation theory is explained. The new test statistic is given in section 3, and then the simulation and experimental results are presented in section 4. finally, the conclusion is following.

2. MULTIPATH MITIGATION

In many researches, the multipath is regarded as some fault, and the fault detection and isolation theory is directly applied to multipath mitigation. It is a reasonable approach because the effect of multipath looks like the slowly varying bias or soft fault. In the following subsection, the multipath detection theory is presented and then the multipath isolation is explained in the other subsection.

2.1 Multipath detection

The general case of measurement model is addressed. The measurement equation is

$$z = Hx + w + b. \tag{1}$$

z is the $m \times 1$ vector of measurement and x is n dimensional vector of state. w is the $m \times 1$ measurement noise vector with Gaussian distribution of $E\{w\} = 0$ and $\mathrm{cov}\{w\} = \sigma_w^2 I_{m \times m}$. H is the measurement matrix. The measurement matrix transforms from the state space to the measurement space and its rank is n. b is the $m \times 1$ vector of multipath error.

The least squares estimate of the state vector in model (1) is

$$\hat{x} = H^* z = x + H^*(w + b) \tag{2}$$

where $H^* = (H^T H)^{-1} H^T$ is the pseudo inverse of H. The matrix H^* represents a transformation from the measurement space to the state space. However, in the least squres, the estimate is contaminated by the multipath error, b.

Now, to discriminate the effect of multipath, b from state estimate, consider the transformation P with following properties. The P has a dimension of $(m-n)\times m$, its rank is $m-n$, $PP^T = I_{(m-n)\times(m-n)}$ and $PH = 0$. The matrix P spans the parity space or the null space of H. The P is devised to isolate the effect of multipath in another space which is called parity space. By the transformation, the parity vector is obtained as

$$p = Pz = P(w+b) \tag{3}$$

In this vector, the effect of state vector vanishes and only the effect of multipath remains. Consider another additional transformation P^T. Then, the effect of multipath in parity space is moved into measurement space.

$$f = P^T p = P^T Pz = Sz \tag{4}$$

where $S = P^T P$, $S^2 = S$, and rank of S is $m-n$. The S can be directly calculated from H as following [7]:

$$S = I_{m\times m} - HH^* \tag{5}$$

Finally, by the transformation S, the effect of multipath is discriminated from sound information.

Now, the test statistic can be constructed:

$$D = f^T f \tag{6}$$

The test statistic is compared with the threshold and hypothesis is tested. If the statistic is larger than the threshold, then the multipath is declared. If the statistic is smaller than the threshold, then there is no multipath.

$$\begin{aligned} D < T : H_0 \qquad & H_0 : \text{No multipath}, \\ D > T : H_1 \qquad & H_1 : \text{Multipath} \end{aligned}$$

The hypothesis test is characterized by false alarm rate and probability of missed detection. False alarm rate is a probability of wrong declaration of multipath and probability of missed detection is a probability of missing the real multipath. Once the false alarm rate is determined, the threshold can be calculated:

$$T = \sigma_w^2 Q^{-1}(P_{FA}, m-n) \tag{7}$$

where σ_w^2 is covariance of measurement noise, P_{FA} is false alarm rate, and $Q^{-1}(P_{FA}, m-n)$ is the inverse function of right-tail probability function which is the probability of exceeding a given value, defined as $1-\Phi(T, m-n)$ where $\Phi(T, m-n)$ is cumulative distribution function for chi-squared random variable T with degrees of freedom, $m-n$ [8].

To summarize, the multipath detection algorithm is as follows:

1) Based on the required P_{FA}, the number of redundant measurement, $m-n$, and the measurement noise variance, σ_w^2, calculate the threshold $T = \sigma_w^2 Q^{-1}(P_{FA}, m-n)$.
2) Calculate the fault vector, $f = Sz$.
3) Calculate the test statistic, $D = f^T f$.
4) If $D > T$, then declare that multipath exists.
5) Repeats 2)~4) for each new measurement vector.

2.2 Multipath isolation

For identification of measurement including multipath error, the maximum likelihood estimation approach is employed. Measurement i is identified as multipath included one if

$$P(p|b_i) = \max_i \{P(p|b_i)\}. \tag{8}$$

It is shown that the i that maximize $P(p|b_i)$ also maximizes f_i^2 / S_{ii} [7]. Thus the multipath identification algorithm is as follows:

1) Compute the matrix $S = I_{m\times m} - HH^*$.
2) Compute the vector $f = Sz$.
3) Compute the quantifies f_i^2 / S_{ii} for $i = 1, \cdots, m$.
4) Find the maximum f_i^2 / S_{ii}, then the measurement source i corresponding to the maximum value is identified as multipath included measurement.

By the procedures, the multipath error is identified and the measurement can be excluded from measurement vector.

3. A NEW TEST STATISTIC

3.1 Derivation

In the CDGPS applications, two types of measurement are presented. One is accumulated carrier phase double difference measurement and the other is code pseudorange double difference measurement. They are given as

$$\Phi_{AB}^{ij} = r_{AB}^{ij} + m_B^{ij} + v_{AB}^{ij} + \lambda N_{AB}^{ij} \tag{9}$$
$$\rho_{AB}^{ij} = r_{AB}^{ij} + M_B^{ij} + \upsilon_{AB}^{ij} \tag{10}$$

where superscripts denotes satellite number, subscript A indicates the base station, and subscript B means the moving station. The operation, $(\cdot)_{AB}^{ij}$, means double difference between receivers and satellites. Φ is accumulated carrier phase measurement, r is actual distance, m is multipath error of phase measurement, v is the Gaussian noise with zero mean and variance σ_Φ^2, λ is wave length of carrier signal, and N is integer ambiguity. ρ is pseudorange measurement by code, M is multipath error of code pseudorange, and υ is the Gaussian noise with zero mean and variance σ_ρ^2. Generally, the base station is installed on relatively good environment, therefore the multipath phenomena are assumed to be observed only in the moving station. Due to this reason, the subscript of multipath m and M has only B. In most CDGPS applications, only (9) is used for positioning.

Now, for a test statistic which is suitable for CDGPS applications, additional difference between (9) and (10) is calculated as

$$\Pi = \Phi_{AB}^{ij} - \lambda N_{AB}^{ij} - \rho_{AB}^{ij} = m_B^{ij} - M_B^{ij} + v_{AB}^{ij} - \upsilon_{AB}^{ij} . \quad (11)$$

With (11), the test statistic is proposed:

$$D = \Pi^T \Pi \quad (12)$$

With D and prespecified threshold, hypotheses are tested. If D is larger than the threshold, then there are multipath errors. If D is smaller than the threshold, then no multipath propagation has occurred.

3.2 Analysis of the test statistic

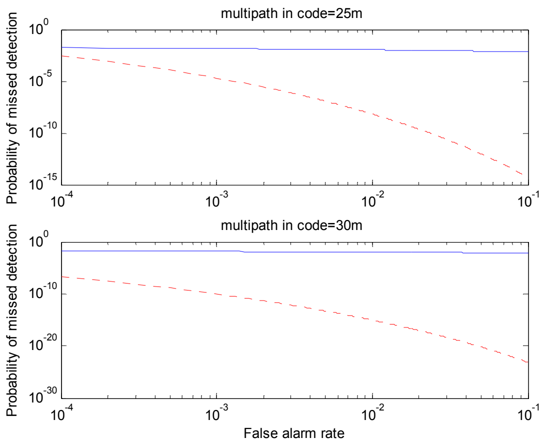
The measurement model (11) is constituted by additional difference between code double difference and accumulated carrier phase double difference. In the model, the measurement has only multipath effect except the inevitable measurement noise. Moreover it has multipath effect of code measurement, which is usually larger than that of carrier phase. Therefore, the test statistic is more sensitive to multipath effect.

The test statistic, D, is chi-squared random variable. If no multipath exists, it is central chi-squared variable. If the measurement has multipath errors, it is noncentral chi-squared variable. The characteristics of the test statistic are compared with that of conventional parity vector method which uses only carrier phase double difference measurement.

Fig. 1 shows the test statistic operating characteristics (TOC) for 1 degree of freedom with respect to the multipath error magnitude of code measurement. The effects are assumed to be 25m~40m and those of carrier phase measurement are assumed to be 0.2m.

TOCs of 1, 2, 3, and 4 degrees of freedom are shown in fig. 2. The multipath errors of codes are assumed to be 30m and those of carrier phase are assumed to be 0.2m

The solid lines are the TOCs of normal parity method. The dotted lines are the TOCs of the test statistic (12). Due to the calculation of double difference, n degrees of freedom need 4+n common visible satellites of the base and moving station.



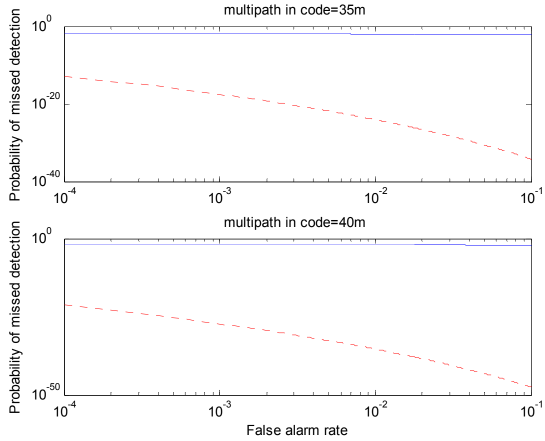


Fig. 1 TOCs for multipath errors of 25m~40m in codes

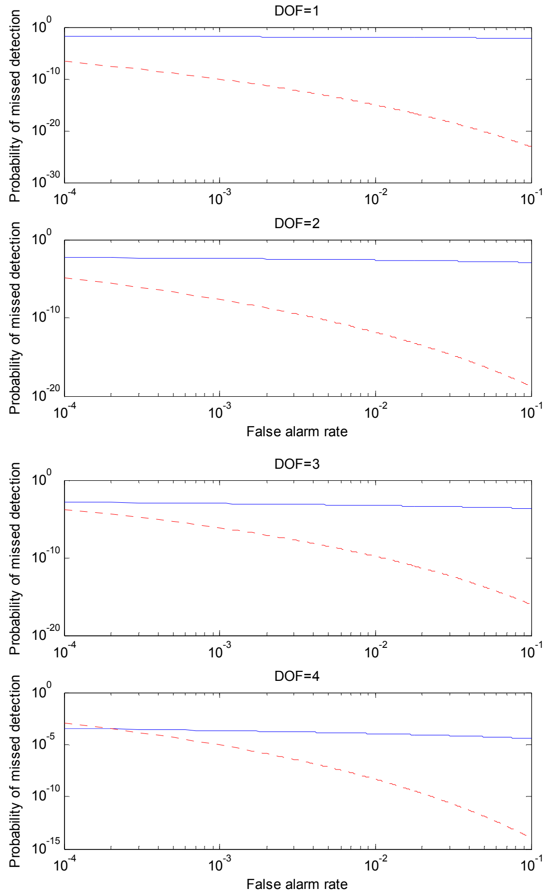


Fig. 2 TOCs for 1,2,3, and 4 degrees of freedom

4. SIMULATION AND EXPERIMENTAL RESULTS

The performance of the proposed statistic is demonstrated by computer simulation and experiment.

In simulation, the 7 satellites are assumed to be visible by the base and moving stations. Therefore, the 6 double difference measurement is available and this also means that the statistic has 3 degrees of freedom. The other parameters are given as followings:

$$P_{FA} = 10^{-4}, \quad \sigma_\rho^2 = 1.2^2, \quad \sigma_\Phi^2 = 0.05^2 .$$

For the comparison, the parity space method with only carrier phase measurement is simulated to the same situation. In fig. 1, the each test statistic is plotted. The above one is of the only carrier phase measurement and the below one is the result of the statistic. The solid lines are the thresholds of each method.

Fig. 4 and 5 are the positioning results of each case. The points are position results without multipath mitigation methods and the circles are the results with the mitigation methods. The performance of the only carrier phase measurement is shown in fig. 4. On the other hand, fig. 5 represents the improvement by the proposed test statistic. The latter shows much better results. These results are summarized in table 1.

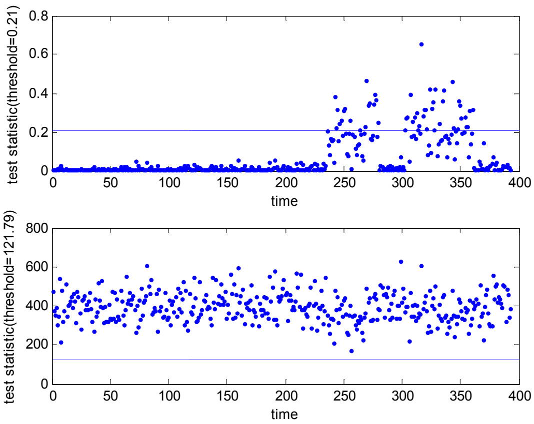


Fig. 3 Test statistics

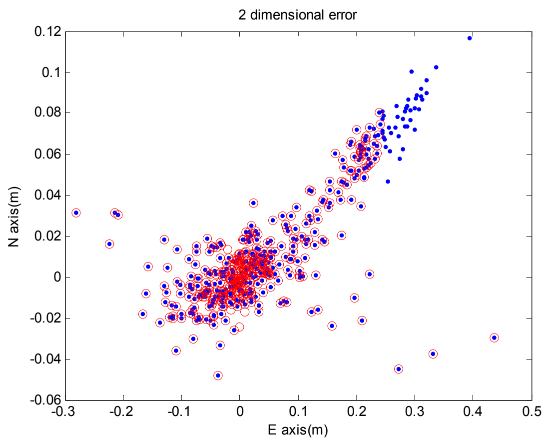


Fig. 4 Parity space method with carrier phase measurement.

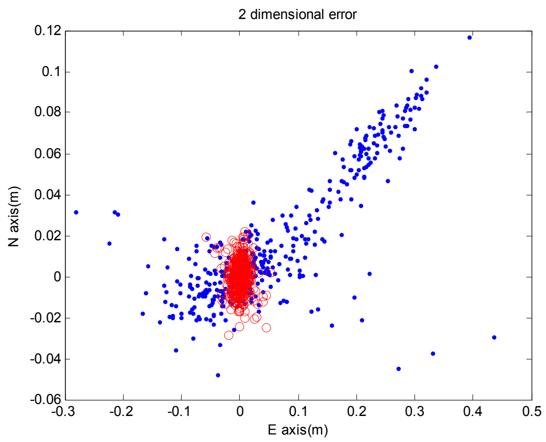


Fig. 5 The proposed test statistic

Table 1 Positioning accuracy

	No mitigation methods	With carrier measurement only	The proposed statistic
RMS of horizontal errors	14.3cm	10.3cm	1.5cm

The experimental result is illustrated in fig. 6. The RMS error in horizontal positioning is improved form 12.6cm to 1.5cm by the statistic.

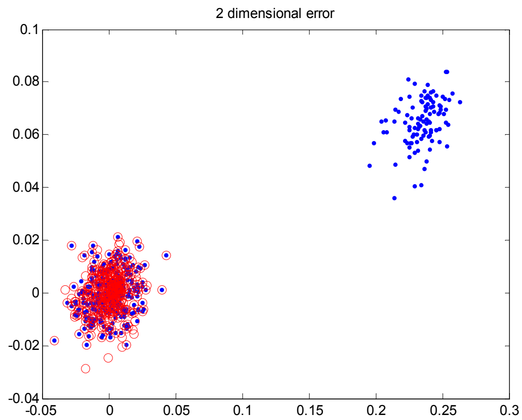


Fig. 6 Experimental result

5. CONCLUSION

The multipath problem in CDGPS applications is addressed. Based on the fault detection and isolation theory, a new test statistic is proposed and applied to the problem. The basic fault detection and isolation theory is explained and the derivation of the test statistic is presented. The properties of the statistic are analyzed. The performance is compared and demonstrated by computer simulation and experiment.

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