Optimal Guidance Law Using Exact Linearization (ICCAS 2005)

Takahiro Ogawa, Kenji Uchiyama and Yuzo Shimada

Department of Aerospace Engineering College of Science and Technology, Nihon University, Chiba, Japan
(Tel : +81-47-469-5390: E-mail: getcha920@yahoo.co.jp)

Abstract: In this paper, we present a new guidance law for a reusable launch vehicle (RLV) that lands vertically after reentry. In our past studies, a guidance law was developed for a vertical/soft landing to a target point. The guidance law, which is analytically obtained, can regenerate a trajectory against disturbances because it is expressed in the form of state feedback. However, the guidance law does not necessarily guarantee a vertical/soft landing when a dynamical system such as an RLV includes a nonlinear phenomenon owing to the atmosphere of the earth. In this study, we introduce a design of the guidance law for a nonlinear system to achieve a vertical/soft landing on the ground using the exact linearization method and solving the two-point boundary-value problem for the derived linear system. Numerical simulation confirmed the validity of the proposed guidance law for an RLV in an atmospheric environment.

Keywords: RLV, Vertical/soft landing, Two-point boundary-value problem, Exact linearization

1. INTRODUCTION

One approach to returning a RLV from an orbit to a surface on the earth is to perform a vertical/soft landing. Such a vehicle is also required to perform an inversion maneuver from a nose-first attitude to a tail-first one and a vertical/soft landing to a target point. Furthermore, it is preferable that a trajectory is generated according to various situations, that is, the guidance and control system must have the robustness against unknown disturbances to accomplish the mission of the landing on the ground. Several guidance laws [1]-[3] have been analytically obtained in the form of state feedback to deal with disturbances. Table 1 shows the characteristics of each guidance law for landers.

The NASA Marshall Space Flight Center has studied the possibility and potential problems associated with the guidance and control system for RLVs [1]. They previously proposed a guidance law based on E-Guidance of the NASA Apollo lunar landers for a linear system that does not properly consider its nonlinearity due to the atmosphere. This guidance law called the modified E-Guidance is effective in dealing with the potentially large dispersion from a predefined trajectory because the state quantities of an RLV are successively used for the calculation of control input. The guidance law does not consider optimality.

D'Souza of the C.S.Draper Laboratory has designed a guidance law for a linear system equivalent to the motion of the lunar lander [2]. It was obtained by analytically solving the two-point boundary-value problem at the performance index described by thrust acceleration. It provides an optimal thrust profile for fuel consumption to minimize thrust acceleration. The trajectory generated by the guidance law is calculated in real time corresponding to a situation. However, it does not guarantee a vertical landing for the linear system because it cannot constraint the attitude angle of an RLV at the target point.

In response to this problem, a guidance law that guarantees a vertical/soft landing for a linear system was proposed from the standpoint of minimizing the effect of impact on space vehicles and fuel consumption as much as possible [3]. The optimal guidance law is derived by applying the variation method to a linear system when thrust jerk is treated as control input.

However, a vertical/soft landing cannot be achieved in an atmospheric environment because of the nonconsideration of nonlinearity due to the atmosphere in the vehicle's dynamics.

Then, we analytically design a guidance law for a nonlinear system to overcome this problem by using the method proposed in [4]. Firstly, the linear equation equivalent to the nonlinear motion of an RLV is formulated by the exact linearization. This is a method that linearizes the nonlinear system via state feedback and coordinate transformation without any approximation. Therefore, the derived linear system does not depend on an equilibrium point, and is equivalent to a global nonlinear system. Then, the guidance law that guarantees a vertical/soft landing for a nonlinear system is obtained in a closed-form analytical solution by applying the variation method to the derived linear system.

Numerical simulation is performed to confirm validity of the proposed guidance law to perform a vertical/soft landing to the target point in an atmospheric environment. Furthermore, the robustness of the proposed guidance law is also examined in the simulation.

2.DYNAMICS OF RLV

In this paper, we design a guidance law for an RLV in the final decent phase. Figure 1 shows the descent motion of an RLV. The motion of an RLV is assumed to be expressed by the equation of its center of gravity. The rotation of the earth is not

Table 1 Evaluations of guidance laws (○ with consideration × without consideration)

	Vertical Landing	Soft Landing	Optimality	Nonlinear Compensation
Modified E-Guidance [1]	○	○	×	×
D'Souza [2]	×	○	○	×
Previous our study [3]	○	○	○	×
Present our study	○	○	○	○

considered in its dynamics. The coordinate o-*xz* is regarded as the inertial coordinate system. Its origin is fixed on the ground and is defined to coincide with the terminal point.

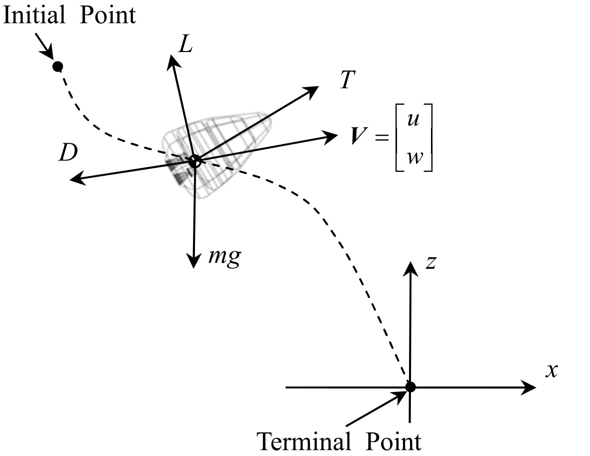

Fig.1 Descent motion of an RLV

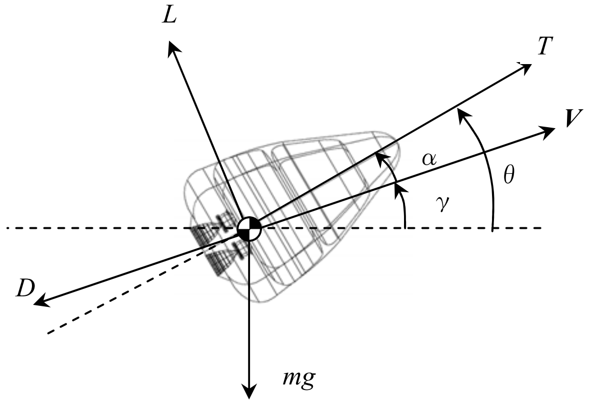

Fig.2 Notations

The equation of motion of an RLV is obtained as

$$m\frac{d}{dt}\begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} -D\cos\gamma - L\sin\gamma + T\cos\theta \\ -D\sin\gamma + L\cos\gamma - mg + T\sin\theta \end{bmatrix} , \tag{1}$$

where u and w denote the x and z-components of velocity $\boldsymbol{V}$, respectively, D the drag, L the lift, T the trust, g the gravitational acceleration, γ the flight path, θ the attitude angle, and m the mass of an RLV.

Thrust acceleration which is the control input of the present controlled system is defined as

$$a_{Tx} = (T/m)\cos\theta , \tag{2a}$$

$$a_{Tz} = (T/m)\sin\theta . \tag{2b}$$

where a_{Tx} and a_{Tz} are thrust acceleration along x and z directions, respectively. In Fig.2, the flight path γ, the angle of attack α , and the attitude angle θ are defined as

$$\cos\gamma = \frac{u}{\sqrt{u^2+w^2}} , \quad \sin\gamma = \frac{w}{\sqrt{u^2+w^2}} , \tag{3}$$

$$\theta = \tan^{-1}\left(\frac{a_{Tz}}{a_{Tx}}\right) , \tag{4}$$

$$\alpha = \theta - \gamma . \tag{5}$$

From Eq.(4), the attitude of an RLV becomes vertical, that is, θ = 90 [deg] when the thrust acceleration a_{Tx} is 0[m/s^2]. Lift and drag are respectively defined as

$$L = \frac{1}{2}\rho\left(u^2+w^2\right)C_L S , \tag{6}$$

$$D = \frac{1}{2}\rho\left(u^2+w^2\right)C_D S . \tag{7}$$

where, ρ is the air density, C_L and C_D the lift and drag coefficients, and S the reference area.

Using these definitions, the equation of motion of an RLV is rewritten as

$$\frac{d}{dt}\begin{bmatrix} x \\ z \\ u \\ w \end{bmatrix} = \begin{bmatrix} u \\ w \\ -\frac{\rho S}{2m}\sqrt{u^2+w^2}\left(C_D u + C_L w\right) + a_{Tx} \\ \frac{\rho S}{2m}\sqrt{u^2+w^2}\left(-C_D w + C_L u\right) - g + a_{Tz} \end{bmatrix} . \tag{8}$$

3.EXACT LINEARIZATION

In the final decent phase, there is a possibility that the position of an RLV is largely deviated from the predefined trajectory due to the guidance error. It is desirable to be capable of recalculating of the trajectory according to the state of an RLV. To design the guidance law that satisfies this requirement, an analytical solution is required for the nonlinear system expressed by Eq.(8). However, it is difficult to derive the analytical solution directly from the nonlinear equation of an RLV.

In response to this problem, we attempt to derive a linear system that is equivalent to the nonlinear motion described by Eq.(8). The linearization using the Taylor series is a conventional method of deriving the linear system around an equilibrium point. However, this linearization method is not suitable for the motion of an RLV because it is difficult to specify the equilibrium state of the system.

Thus, the exact linearization method that linearizes the nonlinear system via coordinate transformation and the linearization feedback without any approximation is adopted to express the motion of an RLV as a linear system.

The motion is rewritten as the following affine system.

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} \tag{9}$$

Here,

$$\boldsymbol{x} = \begin{bmatrix} x & z & u & w & a_{Tx} & a_{Tz} \end{bmatrix}^T$$

$$\boldsymbol{u} = \begin{bmatrix} j_{Tx} & j_{Tz} \end{bmatrix}^T$$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} u \\ w \\ -\frac{\rho S}{2m}\sqrt{u^2+w^2}\left(C_D u + C_L w\right) + a_{Tx} \\ \frac{\rho S}{2m}\sqrt{u^2+w^2}\left(-C_D w + C_L u\right) - g + a_{Tz} \\ 0 \\ 0 \end{bmatrix} \qquad \boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} .$$

The first term on the right side of Eq.(9) denotes a nonlinear function with respect to state quantities. The second term is a linear one with respect to control input. From Eq.(4), it is required that a_{Tx} is 0[m/s^2] at the terminal time to enable a vertical landing. The derivatives of thrust acceleration with respect to time called the trust jerk j_T are considered as control input in Eq.(9) to satisfy the requirement at the terminal time.

In the exact linearization, coordinate transformation is defined as

$$\boldsymbol{\xi} = \boldsymbol{P}(\boldsymbol{x}) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\varphi}(\boldsymbol{x}) \\ L_f \boldsymbol{\varphi}(\boldsymbol{x}) \\ L_f^2 \boldsymbol{\varphi}(\boldsymbol{x}) \end{bmatrix} \in \boldsymbol{R}^6 , \tag{10}$$

where $\boldsymbol{\xi}$ is new state variable vector obtained after coordinate transformation. The operator $L_f\boldsymbol{\varphi}$ denotes the Lie derivative with respect to the function $\boldsymbol{\varphi}(x)$. The nonlinear mapping $\boldsymbol{\varphi}$ is defined as

$$\boldsymbol{\varphi}(\boldsymbol{x}) = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \in \boldsymbol{R}^2 \quad . \tag{11}$$

The transformed state equation is expressed as the following equation with the nonlinear mapping.

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} L_f \boldsymbol{\varphi}(\boldsymbol{x}) \\ L_f^2 \boldsymbol{\varphi}(\boldsymbol{x}) \\ L_f^3 \boldsymbol{\varphi}(\boldsymbol{x}) \end{bmatrix} + \begin{bmatrix} L_g \boldsymbol{\varphi}(\boldsymbol{x}) \\ L_g L_f \boldsymbol{\varphi}(\boldsymbol{x}) \\ L_f L_f^2 \boldsymbol{\varphi}(\boldsymbol{x}) \end{bmatrix} \boldsymbol{u}(t) \tag{12}$$

The linearized system becomes the following linear system called the Brunovsky canonical form.

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \xi_2 \\ \xi_3 \\ \boldsymbol{0}_{2\times 2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0}_{2\times 2} \\ \boldsymbol{0}_{2\times 2} \\ \boldsymbol{I}_{2\times 2} \end{bmatrix} \boldsymbol{u}(t) \tag{13}$$

It is obvious from Eqs.(12) and (13) that the nonlinear mapping $\boldsymbol{\varphi}$ must satisfy

$$\begin{bmatrix} L_g \boldsymbol{\varphi} \\ L_g L_f \boldsymbol{\varphi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0}_{2\times 2} \\ \boldsymbol{0}_{2\times 2} \end{bmatrix} \in \boldsymbol{R}^{4\times 2} \Leftrightarrow$$

$$\begin{bmatrix} \dfrac{\partial \varphi_1}{\partial a_{Tx}} & \dfrac{\partial \varphi_1}{\partial a_{Tz}} \\ \dfrac{\partial \varphi_2}{\partial a_{Tx}} & \dfrac{\partial \varphi_2}{\partial a_{Tz}} \\ -\dfrac{\partial \varphi_1}{\partial u}\dfrac{\partial f_3}{\partial a_{Tx}} - \dfrac{\partial \varphi_1}{\partial w}\dfrac{\partial f_4}{\partial a_{Tx}} & -\dfrac{\partial \varphi_1}{\partial u}\dfrac{\partial f_3}{\partial a_{Tz}} - \dfrac{\partial \varphi_1}{\partial w}\dfrac{\partial f_4}{\partial a_{Tz}} \\ -\dfrac{\partial \varphi_2}{\partial u}\dfrac{\partial f_3}{\partial a_{Tx}} - \dfrac{\partial \varphi_2}{\partial w}\dfrac{\partial f_4}{\partial a_{Tx}} & -\dfrac{\partial \varphi_2}{\partial u}\dfrac{\partial f_3}{\partial a_{Tz}} - \dfrac{\partial \varphi_2}{\partial w}\dfrac{\partial f_4}{\partial a_{Tz}} \end{bmatrix} = \boldsymbol{0} \quad . \tag{14}$$

Therefore, we defined $\boldsymbol{\varphi}$ that satisfies Eq.(14) as

$$\boldsymbol{\varphi}(\boldsymbol{x}) = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} x \\ z \end{bmatrix} \quad . \tag{15}$$

Eq.(14) is also obtained from the Frobenius theorem [5].

The transformed state variables are defined from Eqs.(10) and (15) as

$$\boldsymbol{\xi} = \begin{bmatrix} \xi_{11} \\ \xi_{12} \\ \xi_{21} \\ \xi_{22} \\ \xi_{31} \\ \xi_{32} \end{bmatrix} = \begin{bmatrix} x \\ z \\ u \\ w \\ -\dfrac{\rho S}{2m}\sqrt{u^2+w^2}\left(C_D u + C_L w\right) + a_{Tx} \\ \dfrac{\rho S}{2m}\sqrt{u^2+w^2}\left(-C_D w + C_L u\right) - g + a_{Tz} \end{bmatrix} . \tag{16}$$

From Eq.(15), the transformed state equation Eq.(12) is rewritten as

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \xi_2 \\ \xi_3 \\ L_f^3 \boldsymbol{\varphi}(\boldsymbol{x}) \end{bmatrix} + \begin{bmatrix} \boldsymbol{0}_{2\times 2} \\ \boldsymbol{0}_{2\times 2} \\ L_f L_f^2 \boldsymbol{\varphi}(\boldsymbol{x}) \end{bmatrix} \boldsymbol{u}(t) \quad . \tag{17}$$

The nonlinearity term in the equation of $\boldsymbol{\xi}_3$ is not removed as shown in Eq.(17) by only coordinate transformation. Consequently, the control input $\boldsymbol{u}$ is defined as the following equation to linearize Eq.(17) by defining a new control input $\boldsymbol{v}$ and the state feedback.

$$\boldsymbol{u} = -\left\{L_f L_f^2 \boldsymbol{\varphi}\right\}^{-1} L_f^3 + \left\{L_f L_f^2 \boldsymbol{\varphi}\right\}^{-1} \boldsymbol{v} \tag{18}$$

$$\left\{L_g L_f^2 \boldsymbol{\varphi}\right\}^{-1} = \frac{1}{\dfrac{\partial \dot{u}}{\partial a_{Tx}}\dfrac{\partial \dot{w}}{\partial a_{Tz}} - \dfrac{\partial \dot{u}}{\partial a_{Tz}}\dfrac{\partial \dot{w}}{\partial a_{Tx}}} \begin{bmatrix} \dfrac{\partial \dot{w}}{\partial a_{Tz}} & -\dfrac{\partial \dot{u}}{\partial a_{Tz}} \\ -\dfrac{\partial \dot{w}}{\partial a_{Tx}} & \dfrac{\partial \dot{u}}{\partial a_{Tx}} \end{bmatrix}$$

where

$$\frac{\partial \dot{u}}{\partial a_{Tx}} = -\frac{\rho S}{2m}\sqrt{u^2+w^2}\left(\frac{\partial C_D}{\partial a_{Tx}}u + \frac{\partial C_L}{\partial a_{Tx}}w\right) + 1 \quad ,$$

$$\frac{\partial \dot{w}}{\partial a_{Tz}} = \frac{\rho S}{2m}\sqrt{u^2+w^2}\left(-\frac{\partial C_D}{\partial a_{Tz}}w + \frac{\partial C_L}{\partial a_{Tz}}u\right) + 1 \quad ,$$

$$\frac{\partial \dot{u}}{\partial a_{Tz}} = -\frac{\rho S}{2m}\sqrt{u^2+w^2}\left(\frac{\partial C_D}{\partial a_{Tz}}u + \frac{\partial C_L}{\partial a_{Tz}}w\right) \quad ,$$

$$\frac{\partial \dot{w}}{\partial a_{Tx}} = \frac{\rho S}{2m}\sqrt{u^2+w^2}\left(-\frac{\partial C_D}{\partial a_{Tx}}w + \frac{\partial C_L}{\partial a_{Tx}}u\right) \quad .$$

The first term of the equation with respect to $\boldsymbol{\xi}_3$ in Eq.(17) is calculated as

$$L_f^3 \boldsymbol{\varphi} = \begin{bmatrix} \dfrac{\partial \dot{u}}{\partial z} & \dfrac{\partial \dot{u}}{\partial u} & \dfrac{\partial \dot{u}}{\partial w} \\ \dfrac{\partial \dot{w}}{\partial z} & \dfrac{\partial \dot{w}}{\partial u} & \dfrac{\partial \dot{w}}{\partial w} \end{bmatrix} \begin{bmatrix} w \\ -\dfrac{\rho S}{2m}\sqrt{u^2+w^2}\left(C_D u + C_L w\right) + a_{Tx} \\ \dfrac{\rho S}{2m}\sqrt{u^2+w^2}\left(-C_D w + C_L u\right) - g + a_{Tx} \end{bmatrix}$$

where

$$\frac{\partial \dot{u}}{\partial u} = -\frac{\rho S}{2m}\left\{\frac{u\left(C_D u + C_L w\right)}{\sqrt{u^2+w^2}} + \sqrt{u^2+w^2}\left(C_D + \frac{\partial C_D}{\partial u}u + \frac{\partial C_L}{\partial u}w\right)\right\},$$

$$\frac{\partial \dot{u}}{\partial w} = -\frac{\rho S}{2m}\left\{\frac{w\left(C_D u + C_L w\right)}{\sqrt{u^2+w^2}} + \sqrt{u^2+w^2}\left(C_L + \frac{\partial C_D}{\partial w}u + \frac{\partial C_L}{\partial w}w\right)\right\},$$

$$\frac{\partial \dot{w}}{\partial u}=\frac{\rho S}{2m}\left\{\frac{u(-C_D w+C_L u)}{\sqrt{u^2+w^2}}+\sqrt{u^2+w^2}\left(C_L-\frac{\partial C_D}{\partial u}w+\frac{\partial C_L}{\partial u}u\right)\right\},$$
$$\frac{\partial \dot{w}}{\partial w}=\frac{\rho S}{2m}\left\{\frac{w(-C_D w+C_L u)}{\sqrt{u^2+w^2}}+\sqrt{u^2+w^2}\left(-C_D-\frac{\partial C_D}{\partial w}w+\frac{\partial C_L}{\partial w}u\right)\right\},$$
$$\frac{\partial \dot{u}}{\partial z}=-\frac{S}{2m}\sqrt{u^2+w^2}\left\{\frac{\partial \rho}{\partial z}\cdot(C_D u+C_L w)+\rho\cdot\left(\frac{\partial C_D}{\partial z}u+\frac{\partial C_L}{\partial z}w\right)\right\},$$
$$\frac{\partial \dot{w}}{\partial z}=\frac{S}{2m}\sqrt{u^2+w^2}\left\{\frac{\partial \rho}{\partial z}\cdot(-C_D w+C_L u)+\rho\cdot\left(-\frac{\partial C_D}{\partial z}w+\frac{\partial C_L}{\partial z}u\right)\right\}-\frac{\partial g}{\partial z}.$$

It is assumed that the air density ρ and the gravity acceleration g are functions of z, and the lift and drag coefficients, C_L and C_D, are functions of u, w, a_{Tx}, a_{Tz}, and z.

The term, $\{L_g L_f^2 \varphi\}^{-1}$, must be nonsingular because the control input $\boldsymbol{v}$ defined in Eq.(18) cannot calculate if the term expresses a singular matrix. It is assumed that the following equation which is expressed as the requirement that the determination of $\{L_g L_f^2 \varphi\}^{-1}$ is not equal to zero, is always satisfied in the final decent phase.

$$\left|L_g L_f^2 \boldsymbol{\varphi}\right| \neq 0 \Leftrightarrow \frac{\partial \dot{u}}{\partial a_{Tx}}\frac{\partial \dot{w}}{\partial a_{Tz}}-\frac{\partial \dot{u}}{\partial a_{Tz}}\frac{\partial \dot{w}}{\partial a_{Tx}} \neq 0 \tag{19}$$

Eq.(17) can be expressed as the following linear system.

$$\frac{d\boldsymbol{\xi}}{dt}=\boldsymbol{A\xi}+\boldsymbol{Bv}$$
$$\frac{d}{dt}\begin{bmatrix}\xi_{11}\\ \xi_{12}\\ \xi_{21}\\ \xi_{22}\\ \xi_{31}\\ \xi_{32}\end{bmatrix}=\begin{bmatrix}0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}\xi_{11}\\ \xi_{12}\\ \xi_{21}\\ \xi_{22}\\ \xi_{31}\\ \xi_{32}\end{bmatrix}+\begin{bmatrix}0&0\\0&0\\0&0\\0&0\\1&0\\0&1\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix} \tag{20}$$

Eq.(20) is equivalent to the nonlinear system described in Eq.(9). From Eqs.(9), (16), and (20), the control input $\boldsymbol{v}$ in the derived system is represented as

$$\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\frac{d}{dt}\begin{bmatrix}\xi_{31}\\ \xi_{32}\end{bmatrix}=\frac{d^2}{dt^2}\begin{bmatrix}u\\ w\end{bmatrix}=\frac{d^3}{dt^3}\begin{bmatrix}x\\ z\end{bmatrix} \tag{21}$$

The control input $\boldsymbol{v}$ is treated as the jerk of an RLV.

4.GUIDANCE LAW

In this section, we design the guidance law by analytically solving the two-point boundary-value problem for the derived linear system.

Performance index is defined as

$$J=\Gamma t_f+\frac{1}{2}\int_0^{t_f}\left(v_1^2+v_2^2\right)dt, \tag{22}$$

where Γ denotes the weighting coefficient on the terminal time. The coefficient Γ enables the control input to adjust fuel consumption indirectly. The performance index represents a trade-off between the minimum time problem and the minimum jerk one. The jerk which can be specified as the effect of the destructive power on payloads in an RLV should be limited within a certain range. Therefore, a minimum jerk is desirable for an RLV to accomplish its mission.

To achieve a vertical/soft landing to the target point, the terminal state is constrained as

$$\boldsymbol{x}(t_f)=\begin{bmatrix}0&0&0&0&0&a_{Tz}(t_f)\end{bmatrix}^T$$
$$\Leftrightarrow \boldsymbol{\xi}(t_f)=\begin{bmatrix}0&0&0&0&0&a_{Tz}(t_f)-g\end{bmatrix}^T. \tag{23}$$

The guidance law is obtained by analytically solving the two-point boundary-value problem.

$$\begin{cases}v_1=-\dfrac{9\xi_{31}}{t_{go}}-\dfrac{36\xi_{21}}{t_{go}^2}-\dfrac{60\xi_{11}}{t_{go}^3}\\[2ex] v_2=-\dfrac{9\xi_{32}}{t_{go}}-\dfrac{36\xi_{22}}{t_{go}^2}-\dfrac{60\xi_{12}}{t_{go}^3}+\dfrac{3\xi_{32f}}{t_{go}}\end{cases} \tag{24}$$

Here, t_{go} is regarded as the time to go, which is the time between the current time and the terminal time. Time to go is calculated to solve

$$\begin{aligned}&\Gamma t_{go}^6-\frac{1}{2}\left(9\xi_{31}^2+9\xi_{32}^2-6\xi_{32f}\xi_{32}+9\xi_{32f}^2\right)t_{go}^4\\&-24\left(3\xi_{31}\xi_{21}+3\xi_{32}\xi_{22}-2\xi_{32f}\xi_{22}\right)t_{go}^3\\&-36\left(8\xi_{21}^2+8\xi_{22}^2+5\xi_{31}\xi_{11}+5\xi_{32}\xi_{12}-5\xi_{32f}\xi_{12}\right)t_{go}^2\\&-1440\left(\xi_{21}\xi_{11}+\xi_{22}\xi_{12}\right)t_{go}-1800\left(\xi_{11}^2+\xi_{12}^2\right)=0\end{aligned} \tag{25}$$

Control input, which depends only on the current state, terminal state, and time to go is derived in a closed form. This guidance law is attractive in terms of computational speed because it generates the trajectory using the current state without any iterative computations.

The proposed guidance law guarantees a vertical/soft landing to the target point in an environment with an atmosphere because the nonlinearity due to the lift and drag is taken into account for the guidance system. Figure 3 shows the block diagram of the guidance system for an RLV. This guidance system has the robustness against uncertainties due to the atmosphere.

5.NUMERICAL SIMULATION

The numerical simulation is performed to confirm the validity of the proposed guidance law to achieve a vertical/soft landing to the target point under arbitrary initial conditions.

The initial and terminal conditions are

$$\boldsymbol{x}(t_0)=\begin{bmatrix}-10000&5000&124&-30&10&5\end{bmatrix}^T \tag{26}$$
$$\boldsymbol{x}(t_f)=\begin{bmatrix}0&0&0&0&0&20\end{bmatrix}^T \tag{27}$$

The other parameters are as follows: an initial mass m_0 of 71.6×10^3 kg, a reference area S of 150 m^2, and a weighting coefficient Γ of 0.1. The lift and drag coefficients, C_L and C_D, obtained by CFD2000 (Adaptive Research Corporation) are used in the dynamics of an RLV, that is, Eq.(9). To confirm the robustness against the error of coefficient, the simulation is performed under following two conditions of the guidance system.

Case 1: The lift and drag coefficients C_L and C_D in the linearization feedback Eq.(18) accurately correspond to them in the dynamics Eq.(9).
Case 2: C_L and C_D in Eq.(18) have a 50% error to them in Eq.(9).

The solid and dotted lines denote the simulation results in "without error" and "with error" cases, respectively. The derivatives of C_L, C_D, ρ, and g shown in Eq.(18) are required to linearize the nonlinear system Eq.(9). In the simulation, the derivatives of C_L and C_D in Eq.(18) with respect to a_{Tx} and a_{Tz} are assumed to be sufficiently small in this simulation.

Figure 4 shows the trajectory of an RLV in the final decent phase. In this figure, the solid and dotted lines indicate that an RLV reached the target point (x=0m, z=0m) at the terminal time using the proposed guidance law even if uncertainties due to the air data exists in the guidance system.

Figures 5 and 6 illustrate the time histories of the directional velocities u and w. The results show that both of the velocities u and w at the terminal time are 0m/s. This means that a soft landing to the target point is achieved.

Figures 7 and 8 show the time histories of thrust acceleration a_{Tx} and a_{Tz}. Each value at the terminal time agree with the terminal condition in Eq.(27). It is clear that a vertical landing is achieved since the thrust acceleration a_{Tz} is zero at the terminal time. This can be also shown in Figure 9 which represents the time history of the attitude angle θ. It can be observed from Figure 9 that the inversion maneuver from the nose-first attitude to the tail-first one is performed under the proposed guidance law. Figure 10 shows the time history of the mass. The fuel consumptions are about 10[ton] in each case.

These results indicate the proposed guidance law is useful in achieving a vertical/soft landing to the target point in an atmospheric environment. Furthermore, the results show that the proposed guidance law also has robustness against uncertainties.

The linear system Eq.(20) derived by using the exact linearization may not be exactly equivalent to the nonlinear one Eq.(9) due to uncertainties and the assumption that their derivatives in Eq.(18) can be neglected. However, if the air data in Eq.(18) is not estimated accurately, this guidance law dose not guarantee the realization of a vertical/soft landing in any situations.

6.CONCLUSION

A new guidance law for an RLV in the final descent phase was presented. The linear system equivalent to the nonlinear one expressed as the motion of an RLV was formulated by the exact linearization method. The guidance law was obtained to analytically solve the two-point boundary-value problem for the derived linear system. The proposed guidance law has the robustness against uncertainties due to the atmosphere because it regenerates the trajectory on-line in response to it. Numerical simulation confirmed that the proposed guidance law has the ability to realize a vertical/soft landing to the target point in an atmospheric environment and robustness against uncertainties.

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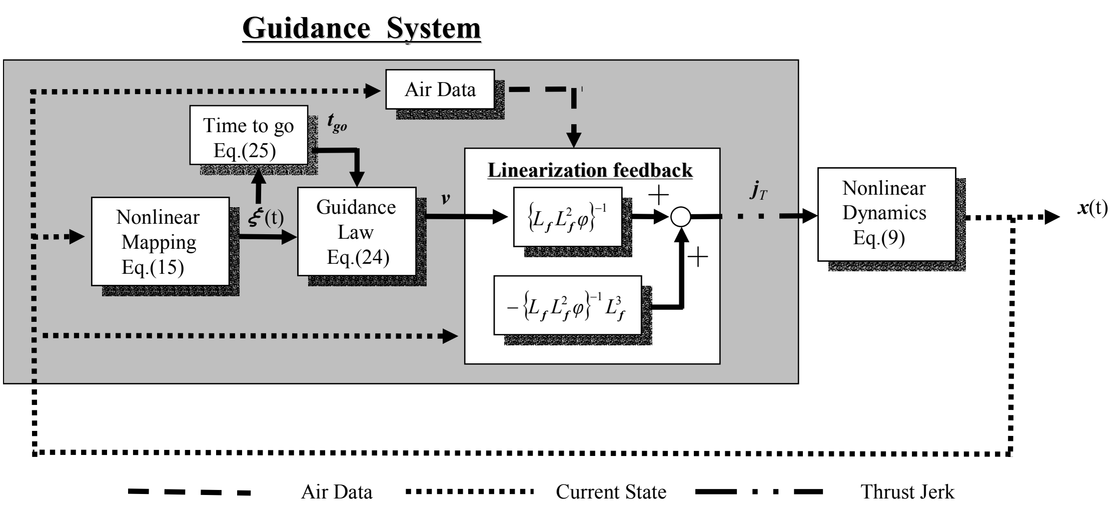

Fig.3 The block diagram of the proposed guidance system

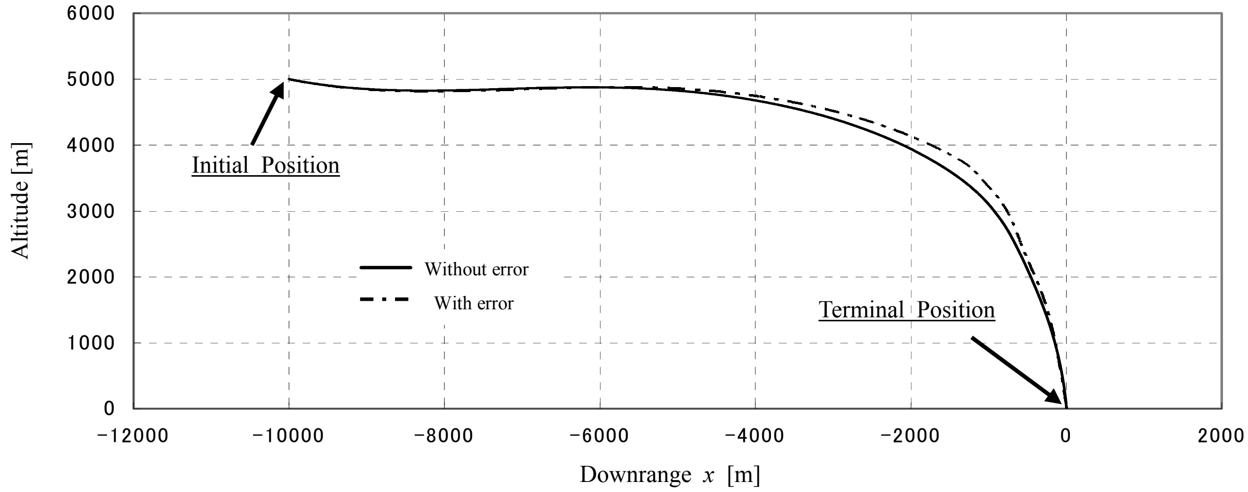

Fig.4 Trajectory

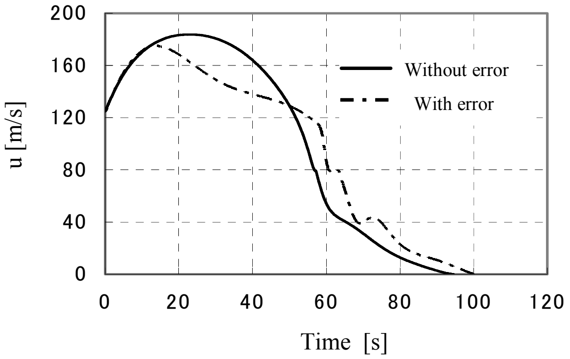

Fig.5 Time history of velocity u

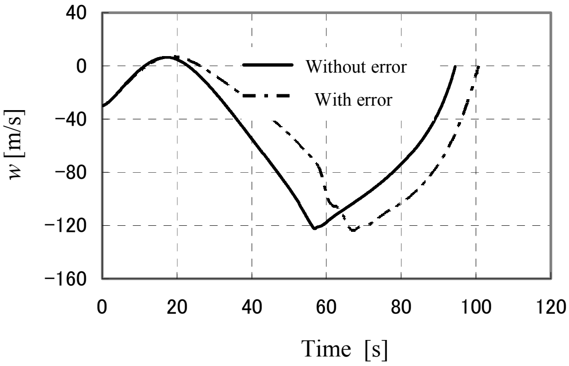

Fig.6 Time history of velocity w

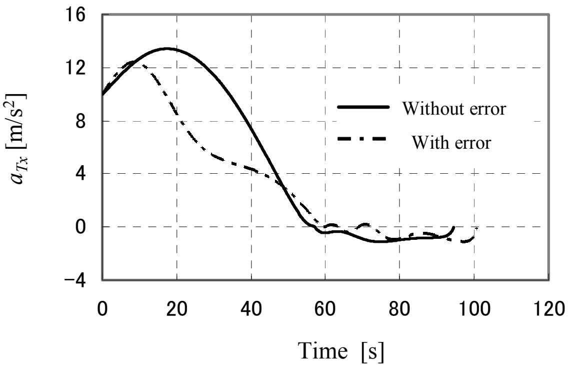

Fig.7 Time history of thrust acceleration a_{Tx}

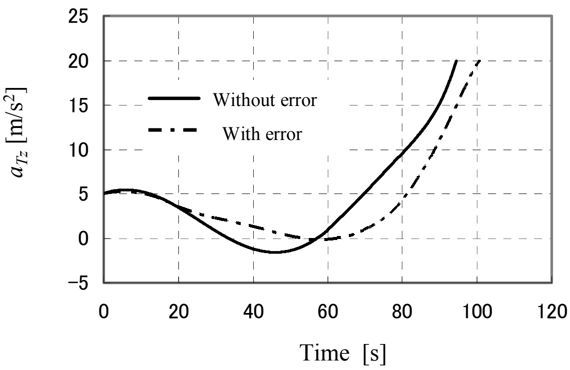

Fig.8 Time history of thrust acceleration a_{Tz}

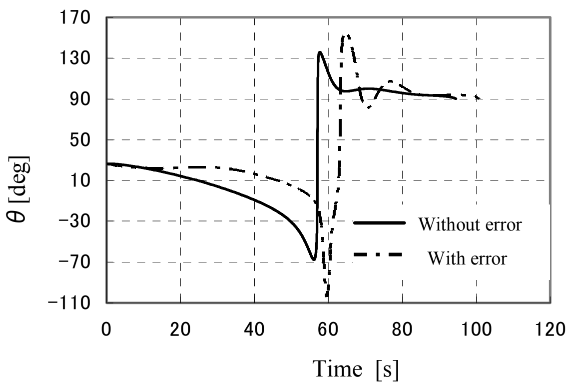

Fig.9 Time history of attitude angle θ

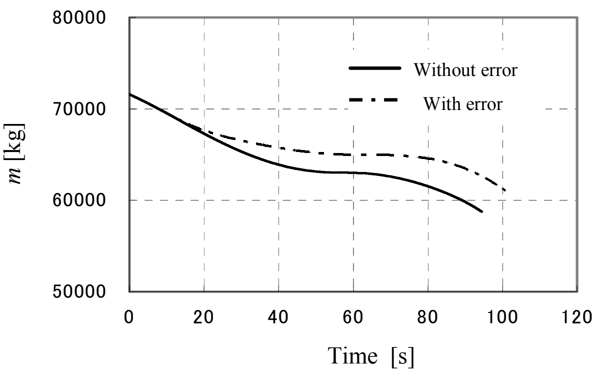

Fig.10 Time history of mass m